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THE
Progressive Commercial
Arithmetic

FOR

COMMERCIAL SCHOOLS, HIGH SCHOOLS
AND ACADEMIES,

BY

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FOURTEENTH EDITION

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PREFACE TO PART I.

The original plan of the Progressive Commercial Arithmetic contemplated a work for advanced commercial and other classes.


The first five editions began with Part II of the present edition, omitting the elementary work included under the fundamental topics, Notation, Numeration, Addition, Subtraction, Multiplication and Division.

But a demand has arisen for a more complete text for use in Normal and other classes, where a general and full discussion of the elementary processes is desirable.

To meet this demand the present edition has been prepared. It includes the work hitherto published, adding a full preliminary treatment of the fundamental topics, including a large number of drill problems and exercises, both oral and written.

It is believed that the practical and original character of the matter in Part I will promptly appeal to the progressive teacher, and that the book in its present form will better meet the needs of all schools that have classes in Commercial Arithmetic.

The study of Part I may be omitted, if, on account of limited time, or for any other reason, it is found desirable to commence with the more advanced topics.



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PART I.

ARITHMETIC.

INTRODUCTION.

The idea of *number* is present in every sound human mind. Perhaps there is no better way of defining number than to say that it is the idea suggested by the question, "how many?"

NOTE.—The student, in his thinking, should see a clear distinction between numbers themselves and the conventional characters, or nine digits, used to represent them. The *number* six is entirely different from the *figure* 6. The child, the savage, or the ignorant man may comprehend numbers, although he may not have the least knowledge of the figures that are used by the arithmetician.

The idea of number begins with the idea of *units*, or single things of which number is composed, and which are necessarily thought of when we ask or answer the question, "how many?"

The unit of a number may consist of a group or groups of other units. Thus, in speaking of a quantity of eggs, we may say, "there are six 30-dozen cases," the unit of the number *six* being *one* 30-dozen case. Each case may then be thought of as 30 units of *one dozen* eggs each. Finally, we may think of each dozen as a number, the unit of which is *one egg*.

After some experience with numbers in their application to the things about us, we are enabled to conceive of the *numbers themselves*, and without any thought of their application to objects. Thus we may think of 6, or 10, or 15, without thinking of any particular unit as the number basis. Numbers thus thought of or expressed are called **Abstract Numbers**.

When we apply numbers directly to definite objects, as in the expressions, *seven dollars*, *five sheep*, *eight dozen eggs*, etc., the numbers are said to be **Concrete**.

NOTE.—Travelers report that the Terra del Fuegians, and certain tribes of the Australian Aborigines, are unable to count beyond five, the number of fingers on one hand. It is a curious historical fact that the fingers (digits) were primarily used by all peoples as a means of indicating number. This is why all number systems use *five* and *ten* as the basis in forming number groups.

There is much difference among individuals in the ability to recognize numbers at sight,

and without stopping to count. Some persons cannot recognize more than five objects without counting, while others can recognize eight or nine. It is an interesting experiment to test this power in the case of a class of students, by arranging irregular groups of from five to ten objects, as beans, bits of chalk, kernels of corn, etc., allowing the students to name at sight, as far as they can, the number in each group. The power to see instantly the number of objects in a group is susceptible of cultivation.

Arithmetic, as a *science*, treats of the principles and relations of numbers, and the accepted methods of computing by means of the figures or characters that represent them. As an *art*, it teaches us to apply these principles and methods to the common affairs of life.

The Fundamental Processes of Arithmetic are *Notation*, *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

THE REPRESENTATION AND EXPRESSION OF NUMBERS.

Notation is the method of indicating numbers by means of letters, figures, or other characters.

The representation of numbers by means of special symbols is of very ancient origin—perhaps as old as language itself. Among most ancient peoples numbers were represented by letters of the alphabet. In the Greek and the Hebrew notations, each of the nine digits was denoted by a letter, while other letters represented groups of 10's, as 20, 30, 40, 50, 100, etc. In the Roman system the grouping was carried still further, and fewer characters were used. But far superior to all other systems is the one now in general use, and known as the **Arabic**. This method of number notation probably originated in India in very ancient times. A knowledge of it was acquired by the Arabian mathematicians, and it began to come into use in Europe about the thirteenth century.

The distinctive features of the Arabic system are:

1. The employment of the nine characters or "digits," 1, 2, 3, 4, 5, 6, 7, 8, 9, to denote, respectively, the first nine numbers.
2. The variation of the unit values of these digits according to the position they occupy in the number.
3. The use of the tenth character, 0, to indicate the absence or negation of numerical value.

The unit values of the digits in an Arabic number increase

in a tenfold ratio from right to left. Thus, in the number 357, the unit value of the 7 is 1, of the 5 is 10, and of the 3 is 100. Now, if we place a cipher to the right of the number (3570), the unit value of each digit is increased tenfold, since the order of each digit from the right of the number has been increased one place.

By the *order* of a digit is meant the value of its units, as indicated by its position in the number.

The orders for the first sixteen positions are named as follows:

Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
5	3	6	7	4	7	8	5	2	3	7	1	6	4	3	2

For convenience in reading numbers, the orders are grouped into *periods* of three figures each, and in reading, the unit value of the right hand figure in each period is named, instead of naming the unit value of all the figures. Thus, in reading the above number, 5367478523716432, we do not read, "5 *quadrillions*, 3 *hundred trillions*, 6 *tens trillions*," etc., but we separate the number into periods of three figures each, beginning at the right, thus, 5,367,479,523,716,432, and then read 5 *quadrillion*, 367 *trillion*, 478 *billion*, 523 *million*, 716 *thousand*, 432, the unit value of the unit's period not being expressed.

Numeration is the process of giving expression in words to numbers that are indicated by figures, letters, or other characters.

To read Arabic numbers readily, one must be able:

(1) To read at sight any group of three figures.

(2) To name the periods, in their proper order, from highest to lowest.

NOTE.—Strictly speaking there is no limit to the number of periods; but numbers beyond the sixth period, or Quadrillions, are rarely used. The student should learn the periods at least this far, and be able to name them from left to right.

Exercise.

Read at sight the following number groups:

123	101	020	001	905	090	909
328	025	008	075	770	202	060
469	606	206	010	080	003	100
510	011	110	050	709	030	005

In the following numbers the periods are indicated at the head of the columns. Practice reading them rapidly from left to right until you can name the period without looking at the column head. If a period is occupied by ciphers, pass it without mention. Notice that the left-hand period may have less than three figures. Do not name the unit value of the first, or unit's, period.

NOTE.—In reading numbers it is now customary to omit the word “and.” Say, “five hundred seventy-six,” not, “five hundred *and* seventy-six,” “Six hundred two,” not “six hundred *and* two.”

It is also customary, especially in business offices, to read thousands and hundreds together, as so many hundreds. Thus, instead of saying, “one thousand seven hundred sixty-five,” we may say, “seventeen hundred sixty-five, or we may read, 7429, “seventy four hundred twenty-nine.”

Read the following:

QUAD.	TRILL.	BILL.	MILL.	THOUS.	.
160	045	001	206	570	002
14	120	000	567	010	206
506	200	014	000	005	700
870	002	569	703	000	504
9	000	000	010	000	530
	53	010	278	005	500
16	000	000	050	005	020
		500	000	030	003
120	601	250	006	310	037

In the following numbers the periods are indicated by a separating point. Read them.

20,120,160,130,701
 150,000,627,100,034
 573,020,050,006
 5,006,279,130,162
 300,000,017,235,837
 20,371,500,006,008
 40,271,638,798,580,367

Separate the following numbers into periods, beginning at the right, and read them :

127003710025673
 800000670001256
 1127000008670000124
 372607165002016000
 530802016300000167

Read the following without pointing off the periods:

1261025	2701630346	400210103
3702010	2000016307	500160710
5000120	4020300061	82000102
31000100	25702016730	3000001200

WRITTEN EXERCISE IN ARABIC NOTATION.

Express in figures the following numbers:

1. Nine hundred, one.
2. One hundred, nine.
3. Two thousand, ten.
4. Eleven hundred, eleven.
5. Sixteen hundred, one.
6. Eighty thousand, ten.
7. Four hundred thousand, eleven.
8. Sixty-two million, twenty-six.
9. Forty-eight hundred, sixty.
10. Three million, forty.
11. Forty million, three.
12. Eleven million, twenty thousand, four.
13. Seven hundred one million, forty thousand, eleven.
14. Ninety million, ten thousand, one.
15. Fourteen billion, seven hundred million, forty thousand, five.
16. Eight billion, eighty.
17. Forty billion, forty million, forty thousand, forty.
18. Seven hundred twenty billion, three thousand, ten.
19. Sixty-five million, eight hundred, four.
20. Ten billion, twenty.
21. One hundred billion, sixteen million, one hundred, ten.

22. One hundred one billion, one hundred ten million, one hundred thousand, ten.
23. Nineteen trillion, fifty billion, forty thousand, four.
24. Seven hundred one trillion, twenty million, fifteen thousand, two hundred fifty.
25. Eight hundred sixty quadrillion, ten trillion, one billion, eleven thousand.

THE ROMAN NOTATION.

This system was in general use throughout Europe until about the thirteenth century, and it is still used to indicate the hours on clock-dials, in numbering the chapters and other divisions of books, and for a few other minor purposes.

In the Roman Notation, all numbers are represented by means of seven letters of the alphabet, and the combinations formed by them.

These seven letters and the numbers represented by them are as follows:

I=One.
 V=Five.
 X=Ten.
 L=Fifty.
 C=One Hundred.
 D=Five Hundred.
 M=One Thousand.

The principles governing the combining of these number-letters are as follows:

1. When a letter is followed by the same letter, or a letter denoting a lower number, the number value of the combination is equal to the sum of the letters. Thus, VI=six; XX=twenty; XVIII=eighteen.

2. When a letter is preceded by a letter denoting a smaller number, the value of the combination is equal to the difference between the numbers. Thus, IV=four; XL=forty; XC=ninety; CD=four hundred; CM=nine hundred.

3. A dash (—) placed over a letter or combination of letter increases their value a thousand-fold. Thus, \overline{V} =5000. \overline{XVCC} =15200.

Two or more dashes are used to indicate the multiplying of a number by a million, a billion, etc. Thus $\overline{\text{X}}$, $\overline{\text{VI}}$, XL =10,006,040.

Since, at the present time, Roman numerals are rarely of never used to indicate large numbers, the study of this notation is a matter of historical, rather than practical, importance. Familiarity with the combinations in the following table will enable the student to read any Roman numeral that he is likely to meet with in modern works.

TABLE OF ROMAN NUMERALS.

I,	1	XV,	15	CC	200
II,	2	XVI,	16	CCC,	300
III,	3	XVII,	17	CD,	400
IV,	4	XVIII,	18	D,	500
V,	5	XIX,	19	DC,	600
VI,	6	XX,	20	DCC,	700
VII,	7	XXX,	30	DCCC,	800
VIII,	8	XL,	40	CM,	900
IX,	9	L	50	M,	1000
X,	10	LX,	60	MM,	2000
XI,	11	LXX,	70	$\overline{\text{X}}$,	10000
XII,	12	LXXX,	80	$\overline{\text{L}}$,	50000
XIII,	13	XC,	90	$\overline{\text{C}}$,	100000
XIV,	14	C,	100	$\overline{\text{M}}$,	1000000

ADDITION.

How many units in the numbers 2, 3, and 4?

Mary spent 3 cents for a pencil, 1 cent for a pen, and 4 cents for a tablet, how many cents did she spend for all? How many letters are required to write the words: "See what God hath wrought?"

The operation of finding a number that is equal to two or more given numbers is called **Addition**.

The numbers to be added are called **Addends**.

The result of the Addition is called the **Sum**.

Addition is indicated by the sign called **Plus (+)**, which, when written between two numbers, implies that they are to be added.

The **Sign of Equality (=)** when written between two numbers, or combinations of numbers, implies that they are equal. Thus the expression, " $9+6=15$," is read, "nine plus six equals fifteen," (or "nine and six are fifteen"), and implies that the two numbers, 9 and 6, are together equal to 15, or, what is the same thing, that the sum of 9 and 6 is 15.

In mathematics, the expression of equality between two or more quantities is called an **Equation**.

PRINCIPLES.

1. Only like numbers, or those having the same kind of units, can be added.

2. The result of an addition is the same, regardless of the order in which the numbers are added.

METHODS AND EXERCISES.

The result of an addition may be found by counting the individual units of the numbers to be added, or by a shorter process, which depends upon a knowledge of the *Addition Table*. This is a table which exhibits the several sums obtained by adding consecutively each of the numbers less than ten to itself, and to every other number less than ten.

In the following form of the Addition Table the several sums are shown at the junctions of the vertical and horizontal columns.

ADDITION TABLE.

1	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

Before one can add with any degree of accuracy or facility, he must know the above table perfectly. He should also know all combinations in the Addition Table that are required to produce any sum given in the table. These combinations are shown in the following table, and they should be thoroughly mastered.

TABLE OF COMBINATIONS.

$\left. \begin{array}{l} 1+3 \\ 2+2 \end{array} \right\} = 4$	$\left. \begin{array}{l} 1+4 \\ 2+3 \end{array} \right\} = 5$	$\left. \begin{array}{l} 1+5 \\ 2+4 \\ 3+3 \end{array} \right\} = 6$	$\left. \begin{array}{l} 1+6 \\ 2+5 \\ 3+4 \end{array} \right\} = 7$	$\left. \begin{array}{l} 1+7 \\ 2+6 \\ 3+5 \\ 4+4 \end{array} \right\} = 8$
$\left. \begin{array}{l} 1+8 \\ 2+7 \\ 3+6 \\ 4+5 \end{array} \right\} = 9$	$\left. \begin{array}{l} 1+9 \\ 2+8 \\ 3+7 \\ 4+6 \\ 5+5 \end{array} \right\} = 10$	$\left. \begin{array}{l} 2+9 \\ 3+8 \\ 4+7 \\ 5+6 \end{array} \right\} = 11$	$\left. \begin{array}{l} 3+9 \\ 4+8 \\ 5+7 \\ 6+6 \end{array} \right\} = 12$	$\left. \begin{array}{l} 4+9 \\ 5+8 \\ 6+7 \end{array} \right\} = 13$
$\left. \begin{array}{l} 5+9 \\ 6+8 \\ 7+7 \end{array} \right\} = 14$	$\left. \begin{array}{l} 6+9 \\ 7+7 \end{array} \right\} = 15$	$\left. \begin{array}{l} 7+9 \\ 8+8 \end{array} \right\} = 16$	$\left. \begin{array}{l} 8+9 \end{array} \right\} = 17$	$\left. \begin{array}{l} 9+9 \end{array} \right\} = 18$

Numbers to be added are usually written in vertical columns, units of the same order being written in the same column.

MENTAL EXERCISE.

Read at sight, and as rapidly as possible, the results of the following additions. Read first from right to left, then from the top to the bottom, and then from the bottom to the top.

$\begin{array}{r} 2 \\ 9 \\ \hline 11 \end{array}$	$\begin{array}{r} 8 \\ 6 \\ \hline 14 \end{array}$	$\begin{array}{r} 5 \\ 3 \\ \hline 8 \end{array}$	$\begin{array}{r} 6 \\ 4 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ 6 \\ \hline 15 \end{array}$	$\begin{array}{r} 3 \\ 7 \\ \hline 10 \end{array}$	$\begin{array}{r} 4 \\ 2 \\ \hline 6 \end{array}$	$\begin{array}{r} 7 \\ 8 \\ \hline 15 \end{array}$	$\begin{array}{r} 9 \\ 9 \\ \hline 18 \end{array}$	$\begin{array}{r} 7 \\ 2 \\ \hline 9 \end{array}$
$\begin{array}{r} 6 \\ 2 \\ \hline 8 \end{array}$	$\begin{array}{r} 5 \\ 9 \\ \hline 14 \end{array}$	$\begin{array}{r} 6 \\ 6 \\ \hline 12 \end{array}$	$\begin{array}{r} 5 \\ 2 \\ \hline 7 \end{array}$	$\begin{array}{r} 3 \\ 6 \\ \hline 9 \end{array}$	$\begin{array}{r} 9 \\ 7 \\ \hline 16 \end{array}$	$\begin{array}{r} 4 \\ 6 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ 5 \\ \hline 14 \end{array}$	$\begin{array}{r} 7 \\ 4 \\ \hline 11 \end{array}$	$\begin{array}{r} 5 \\ 5 \\ \hline 10 \end{array}$
$\begin{array}{r} 3 \\ 9 \\ \hline 12 \end{array}$	$\begin{array}{r} 5 \\ 4 \\ \hline 9 \end{array}$	$\begin{array}{r} 6 \\ 8 \\ \hline 14 \end{array}$	$\begin{array}{r} 9 \\ 3 \\ \hline 12 \end{array}$	$\begin{array}{r} 4 \\ 5 \\ \hline 9 \end{array}$	$\begin{array}{r} 9 \\ 2 \\ \hline 11 \end{array}$	$\begin{array}{r} 3 \\ 5 \\ \hline 8 \end{array}$	$\begin{array}{r} 8 \\ 9 \\ \hline 17 \end{array}$	$\begin{array}{r} 2 \\ 6 \\ \hline 8 \end{array}$	$\begin{array}{r} 6 \\ 9 \\ \hline 15 \end{array}$
$\begin{array}{r} 6 \\ 7 \\ \hline 13 \end{array}$	$\begin{array}{r} 9 \\ 1 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ 8 \\ \hline 17 \end{array}$	$\begin{array}{r} 8 \\ 5 \\ \hline 13 \end{array}$	$\begin{array}{r} 4 \\ 7 \\ \hline 11 \end{array}$	$\begin{array}{r} 7 \\ 5 \\ \hline 12 \end{array}$	$\begin{array}{r} 9 \\ 9 \\ \hline 18 \end{array}$	$\begin{array}{r} 6 \\ 4 \\ \hline 10 \end{array}$	$\begin{array}{r} 7 \\ 9 \\ \hline 16 \end{array}$	$\begin{array}{r} 5 \\ 8 \\ \hline 13 \end{array}$
$\begin{array}{r} 9 \\ 4 \\ \hline 13 \end{array}$	$\begin{array}{r} 8 \\ 7 \\ \hline 15 \end{array}$	$\begin{array}{r} 2 \\ 8 \\ \hline 10 \end{array}$	$\begin{array}{r} 7 \\ 6 \\ \hline 13 \end{array}$	$\begin{array}{r} 5 \\ 8 \\ \hline 13 \end{array}$	$\begin{array}{r} 5 \\ 6 \\ \hline 11 \end{array}$	$\begin{array}{r} 8 \\ 6 \\ \hline 14 \end{array}$	$\begin{array}{r} 2 \\ 7 \\ \hline 9 \end{array}$	$\begin{array}{r} 4 \\ 9 \\ \hline 13 \end{array}$	$\begin{array}{r} 6 \\ 9 \\ \hline 15 \end{array}$

CONSTANT UNIT FIGURE.

The unit figure obtained by adding numbers having the same unit figure is constant. Thus, since the unit figure of the sum of 6 and 9 (15) is 5, the unit figure obtained by adding any two numbers ending respectively in 6 and 9 will be 5; as, $19+6$, $26+9$, $49+6$, $39+16$, etc. In like manner, the unit figure for 4 and 7 is 1; for 8 and 5 is 3; for 9 and 3 is 2; and so on.

MENTAL EXERCISE.

Name as rapidly as you can the unit figures produced by adding the following pairs of numbers. Do not name the result; merely give its unit figure.

6 and 9	19 and 7	54 and 27
4 and 6	26 and 9	39 and 26
7 and 8	47 and 4	45 and 29
6 and 7	16 and 8	98 and 17
5 and 6	27 and 5	26 and 55
3 and 8	84 and 6	44 and 58
9 and 9	28 and 9	17 and 46
7 and 4	58 and 5	53 and 68
2 and 8	36 and 7	25 and 37
9 and 5	49 and 2	28 and 52

Name rapidly the results of the following:

<u>16</u>	<u>8</u>	<u>27</u>	<u>19</u>	<u>35</u>	<u>29</u>	<u>9</u>	<u>28</u>	<u>8</u>	<u>59</u>
<u>7</u>	<u>14</u>	<u>5</u>	<u>6</u>	<u>8</u>	<u>7</u>	<u>34</u>	<u>7</u>	<u>15</u>	<u>6</u>
<u>14</u>	<u>33</u>	<u>6</u>	<u>9</u>	<u>38</u>	<u>4</u>	<u>67</u>	<u>55</u>	<u>27</u>	<u>58</u>
<u>8</u>	<u>7</u>	<u>56</u>	<u>42</u>	<u>3</u>	<u>76</u>	<u>9</u>	<u>7</u>	<u>4</u>	<u>6</u>
<u>26</u>	<u>15</u>	<u>47</u>	<u>8</u>	<u>9</u>	<u>19</u>	<u>28</u>	<u>3</u>	<u>44</u>	<u>29</u>
<u>7</u>	<u>9</u>	<u>6</u>	<u>22</u>	<u>64</u>	<u>7</u>	<u>5</u>	<u>79</u>	<u>8</u>	<u>9</u>

CONTINUOUS ADDITIONS.

By a *continuous addition* is meant the successive addition of the same number, as $6+7+7+7+7$ or $2+5+5+5+5$, etc. Rapid continuous additions are a most helpful means of aiding one to add rapidly and accurately.

NOTE. In adding orally a series of numbers, simply announce results; do not say, "5 and 7 are 12 and 5 are 17 and 5 are 22," etc.; say, "5, 12, 17, 22."

MENTAL EXERCISE.

1. Begin with 6 and add 5's until you get a result of 56.
2. Commencing with 9, add 7's until you reach a number whose unit figure is 9.
3. Commencing with 4, add 6's until you have a sum greater than 100.
4. Commence with 2 and add 9's continuously until the sum exceeds 100.
5. Beginning with 5, add 7's until a number is obtained whose unit figure is 5.

In the following exercises continue the additions until the results exceed 100:

1. Commence with 3 and add 8's.
2. " " 9 " " 6's.
3. " " 1 " " 7's.
4. " " 3 " " 4's.
5. " " 7 " " 9's.
6. " " 6 " " 8's.
7. " " 3 " " 7's.
8. " " 2 " " 3's.
9. " " 5 " " 8's.
10. " " 4 " " 7's.

ADDING SINGLE COLUMNS.

In the following exercise, add the numbers one at a time, commencing at the bottom and announcing the result as each number is added. After recording the final result, verify by adding the column again, beginning at the top. Add rapidly and with certainty, and allow nothing to divert the attention until the result is obtained.

WRITTEN PRACTICE.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4	7	3	7	9	8	4	3	4	3	9
3	4	6	2	2	4	3	9	7	6	5
8	5	2	9	3	2	9	8	6	0	8
6	6	8	8	6	9	7	4	3	7	6
3	3	4	6	8	8	8	6	5	2	7
5	2	9	3	4	7	9	2	8	5	4
4	4	6	8	9	9	7	9	2	6	9
2	9	5	4	7	8	6	8	7	9	3

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
8	7	9	7	3	2	5	9	6
5	6	7	3	9	9	9	2	9
6	5	5	8	4	3	3	7	5
7	3	3	6	8	8	6	3	7
4	4	2	5	6	4	7	5	2
9	6	4	9	5	7	2	8	8
3	2	6	2	2	6	4	4	4
6	9	8	4	4	5	9	1	9
2	8	3	6	7	8	1	6	3
4	3	5	3	9	3	6	9	7
6	6	9	8	3	9	7	7	6
8	7	7	7	6	2	3	3	2
9	5	6	6	4	7	8	6	8
5	4	2	2	8	3	6	7	1
8	8	6	5	9	5	9	9	5
3	3	5	9	1	9	2	4	6
0	7	8	3	7	6	7	1	9
7	2	9	7	6	8	8	6	8
6	6	3	4	8	4	6	8	6
9	4	8	9	4	7	3	5	9

ADDING NUMBERS OF TWO OR MORE ORDERS.

Carrying the Tens.—In adding numbers composed of two or more orders, the unit's column is added first, and if the sum of this column exceeds 9, the unit figure is recorded and the tens are "carried," or added with the numbers of the next column. In like manner, if the sum of the ten's column exceeds 9, the hundreds are carried to the next column, and so on with the remaining columns.

WRITTEN PRACTICE.

Copy neatly the following numbers, add them, and prove the results by adding from top to bottom:

(21)	(22)	(23)	(24)	(25)	(26)
364	327	643	842	634	3426
742	645	864	375	784	3548
436	327	963	842	643	7263
743	846	327	963	426	4986
<u>375</u>	<u>364</u>	<u>842</u>	<u>643</u>	<u>962</u>	5432
					8374
					<u>3652</u>

(27)	(28)	(29)
4	4983649	9
26	837463	84
342	54289	356
5463	7436	8974
79834	842	765
378426	97	98
<u>5897634</u>	<u>8</u>	<u>6</u>

(30)	(31)	(32)	(33)	(34)
21524	34253	22736	13125	12567
13065	25321	16853	24143	45526
43753	12579	40632	13560	30415
10476	40055	12345	30234	57939
44342	54204	43210	45746	21473
13526	13652	25607	25612	65114
26437	31148	34051	14200	13496
<u>10596</u>	<u>26554</u>	<u>15779</u>	<u>17488</u>	<u>28744</u>

RECORDING PARTIAL ADDITIONS.

In adding numbers composed of several columns of figures, accountants are liable to be interrupted before the result is obtained, and thereby have to do their work all over. To

obviate this, the sum of the several columns may be recorded separately, as illustrated in the accompanying example.

46976	45	EXPLANATION.—The first number, 45, is the footing of the first column, giving us 5 to write under units' column, and 4 to carry to tens' column. 49 is the total of the second column, including the 4 tens carried. After the footings of all the columns are found and proven, the result is read, beginning with the last number and reading up the units' column (274695). This result is then written as the required footing.
58764	49	
74256	56	
9845	54	
47876	27	
895		
6438		
29645		
<u>274695</u>		

Add the following, recording separate results, as illustrated in the foregoing example:

(35)	(36)
56432156	\$1242.46
75439965	1154.22
28365283	2425.12
96543216	3204.25
76896543	1115.38
76987654	2250.50
76345923	1425.42
91683468	1350.24
<u>23456837</u>	1556.18
	2345.15
	2412.20
	<u>3525.32</u>

GROUPING.

The learner may greatly increase the rapidity of his additions through the process known as *grouping*. This consists in learning to recognize at a glance the sums of various groups of figures, and to add them as one number. The ability to do this depends upon practice and the possession of a "quick eye." Begin by learning to recognize all the groups of two figures whose sum is 10. These are as follows:

{ 1	{ 2	{ 3	{ 4	{ 5
{ 9	{ 8	{ 7	{ 6	{ 5

The "nine groups" are also easy to add, since adding nine to any number diminishes the unit figure by one.

The nine groups are $\begin{Bmatrix} 1 \\ 8 \end{Bmatrix}$ $\begin{Bmatrix} 2 \\ 7 \end{Bmatrix}$ $\begin{Bmatrix} 3 \\ 6 \end{Bmatrix}$ $\begin{Bmatrix} 4 \\ 5 \end{Bmatrix}$

In the first two of the following examples the number-groups are inclosed in braces; treat each of these groups as a single number. In the remaining examples form your own groups.

(37)	(38)	(39)	(40)	(41)	(42)
$\begin{Bmatrix} 3 \\ 6 \\ 7 \\ 8 \\ 2 \\ 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 5 \\ 4 \\ 7 \\ 9 \\ 1 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 7 \\ 8 \\ 3 \\ 4 \\ 6 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 9 \\ 3 \\ 6 \\ 5 \\ 8 \\ 7 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 8 \\ 2 \\ 6 \\ 3 \\ 4 \\ 1 \end{Bmatrix}$	\$257.32
$\begin{Bmatrix} 7 \\ 2 \\ 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 5 \\ 4 \\ 7 \\ 9 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 8 \\ 3 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 6 \\ 5 \\ 8 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 2 \\ 6 \\ 3 \end{Bmatrix}$	242.28
$\begin{Bmatrix} 8 \\ 2 \\ 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 7 \\ 9 \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 3 \\ 4 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 5 \\ 8 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 6 \\ 4 \\ 1 \end{Bmatrix}$	420.50
$\begin{Bmatrix} 8 \\ 2 \\ 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 9 \\ 1 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 4 \\ 6 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 5 \\ 8 \\ 7 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 3 \\ 4 \\ 1 \end{Bmatrix}$	134.36
$\begin{Bmatrix} 2 \\ 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 9 \\ 1 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 6 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 5 \\ 8 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 4 \\ 1 \end{Bmatrix}$	175.25
$\begin{Bmatrix} 5 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$	200.00
$\begin{Bmatrix} 3 \end{Bmatrix}$	$\begin{Bmatrix} 6 \end{Bmatrix}$	$\begin{Bmatrix} 2 \end{Bmatrix}$	$\begin{Bmatrix} 7 \end{Bmatrix}$	$\begin{Bmatrix} 1 \end{Bmatrix}$	324.14
$\begin{Bmatrix} 7 \\ 2 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 2 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 3 \\ 5 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 6 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 7 \\ 6 \end{Bmatrix}$	455.50
$\begin{Bmatrix} 7 \\ 2 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 2 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 3 \\ 5 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 6 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 7 \\ 6 \end{Bmatrix}$	255.75
$\begin{Bmatrix} 3 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 8 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ 4 \end{Bmatrix}$	125.80
$\begin{Bmatrix} 7 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 3 \end{Bmatrix}$	137.25
$\begin{Bmatrix} 6 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 9 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 9 \end{Bmatrix}$	143.56
$\begin{Bmatrix} 3 \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 8 \end{Bmatrix}$	$\begin{Bmatrix} 9 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 7 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 9 \\ 1 \end{Bmatrix}$	
$\begin{Bmatrix} 0 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 8 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 5 \end{Bmatrix}$	
$\begin{Bmatrix} 7 \\ 9 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 6 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 5 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 8 \end{Bmatrix}$	
$\begin{Bmatrix} 9 \\ 2 \end{Bmatrix}$	$\begin{Bmatrix} 6 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 5 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 5 \end{Bmatrix}$	$\begin{Bmatrix} 8 \\ 2 \end{Bmatrix}$	
$\begin{Bmatrix} 6 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 3 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 4 \\ 7 \end{Bmatrix}$	$\begin{Bmatrix} 5 \\ 4 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ 7 \end{Bmatrix}$	
$\begin{Bmatrix} 4 \end{Bmatrix}$	$\begin{Bmatrix} 7 \end{Bmatrix}$	$\begin{Bmatrix} 7 \end{Bmatrix}$	$\begin{Bmatrix} 4 \end{Bmatrix}$	$\begin{Bmatrix} 7 \end{Bmatrix}$	
				$\begin{Bmatrix} 8 \end{Bmatrix}$	

(43)	(44)
\$524.25	\$1242.36
247.19	1154.22
325.30	2425.12
432.75	3204.25
157.45	1115.38
284.50	2250.50
356.15	1425.42
575.25	1350.24
142.75	1556.18
381.54	2345.15
434.34	2412.20
150.50	3525.32

ADDING HORIZONTAL COLUMNS.

It is frequently desirable to add numbers that are written in horizontal columns, and without copying them.

Find the results of the following additions without copying the numbers:

WRITTEN PRACTICE.

45. $8+7+6+14+27+46=?$
46. $42+6+19+21+35+26=?$
47. $\$1.15+\$5.00+\$3.75+\$2.50+\$1.25=?$
48. $65¢+\$2.45+75¢+\$10.00+\$7.30+85¢=?$
49. $765+528+326+467=?$
50. $\$2.45+40¢+15¢+\$2.00+\$4.45+\$5.00+60¢=?$
51. $\$21.50+\$19.00+75¢+\$35.25+\$4.00+\$13.00+5.00=?$
52. $\$9.40+\$7.15+\$21.25+\$8.00+65¢+\$3.20+\$5.00=?$
53. $\$8.05+\$41.50+\$11.00+25¢+70¢+\$9.65+\$14.00+\$1.35=?$
54. $\$20.00+\$1.40+\$28.50+90¢+\$9.25+\$1.15+\$82.00=?$

WRITTEN PROBLEMS.

55. The following are the items of cost for Mr. Granger's new residence property: Cost of lot, \$1875; paid for lumber, \$626.06; carpenter work, \$314.25; masonry, \$175.20; plastering, \$85.00; plumbing and wiring, \$93.05; windows and doors, \$45.10; painting, \$76.00; paper hanging, \$41.75. What was the total cost of the property?

56. Frank's yearly allowance for college expenses is as follows: Tuition and fees, \$150; board, \$175; clothes, \$75; books, \$25; laundry, \$30; stationery, \$5; car fare, \$35; lectures and amusements, \$25; incidentals, \$15. What was his total yearly allowance?

57. A dealer delivered ten loads of coal with recorded weights as follows: 2142 lbs., 1956 lbs., 2047 lbs., 2265 lbs., 2143 lbs., 1989 lbs., 2008 lbs., 2106 lbs., 1989 lbs., 2047 lbs. What was the total weight?

58. The seven townships of Jefferson county show school enrollments as follows: Merrimac, 567; Clinton, 906; Walnut, 1146; Coldwater, 428; Marion, 1019; Rock, 479; Harding, 1428. Find the total enrollment for the county.

59. Find the sum of all the odd numbers from 0 to 50.

60. Find the sum of all the even numbers from 0 to 51.

61. Find the sum of:

One hundred twenty-five.

Three hundred thousand, two hundred sixty-four

One thousand, seven.

Fifty thousand, five hundred twenty-six.

Twenty-seven.

Three hundred ninety-six.

Seven hundred seventy-five.

Three thousand, three hundred thirty-three.

62. Add:

7 hundred 25.

3 thousand, 7 hundred 45.

8 thousand, 9 hundred 64.

5 thousand, 9 hundred 84.

9 thousand, 3 hundred 26.

8 thousand, 17.

4 thousand, 37.

8 hundred 24.

3 thousand, 28.

63. The following table represents the total daily attendance of the public schools of Hamilton Township for the ten school months of 1899-1900. Copy carefully this tabulation on a sheet of blank paper, add the columns and fill in the total for each month and for each school. The total attendance for the year is entered in the last space to the right, and it should be proved by adding the totals for the different months, also for the several schools.

ADDITION

23

[illegible]

SUBTRACTION.

What is the sum of 4 and 3? If, from a pile of 7 pennies, 3 pennies are taken away, how many will remain? If from the sum of 8 and 6, 6 be taken away, what will remain? If 8 be taken from the sum of 9 and 8, what number will remain? If from the sum of any two given numbers we take one of them away, what remains? If from a basket containing 12 apples we remove 5 apples, what remains?

DEFINITIONS.

Subtraction is the process of finding the excess of one number over another of the same kind, or of finding the difference between two numbers.

The **Minuend** is the greater of the two numbers.

The **Subtrahend** is the less number.

The **Difference**, or **Remainder**, is the result of the subtraction, or the excess of the Minuend over the Subtrahend.

The **Sign of Subtraction** is the dash ($-$); it is read "minus," and when written between two numbers implies that the number to the right of the sign is to be subtracted from the number to the left. Thus, $12-4=8$.

PRINCIPLES.

1. The Subtrahend and Minuend must be like numbers.
2. The Minuend is equal to the sum of the Subtrahend and Remainder.
3. Increasing or diminishing both Minuend and Subtrahend by the same number does not affect the value of the Remainder.

If from the sum of two numbers one of them be subtracted, the result is the remaining number. It is therefore evident, (1) that subtraction is the reverse process of addition, and, (2) that to subtract readily one must be thoroughly familiar with the addition table.

Thus, if one knows that $7+8=15$, he will also know that $15-7=8$, and that $15-8=7$.

MENTAL EXERCISE.

Read the results, as rapidly as you can, of the following indicated subtractions:

12	16	18	17	9	8	11	13	15	18	11	9	7	12
<u>6</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>7</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>3</u>	<u>7</u>
17	13	16	16	14	9	13	14	18	17	10	12	10	14
<u>9</u>	<u>5</u>	<u>7</u>	<u>8</u>	<u>6</u>	<u>5</u>	<u>8</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>8</u>	<u>4</u>	<u>6</u>	<u>7</u>
12	13	16	14	13	11	13	10	17	12	14	13	15	11
<u>8</u>	<u>9</u>	<u>8</u>	<u>8</u>	<u>6</u>	<u>7</u>	<u>7</u>	<u>3</u>	<u>9</u>	<u>5</u>	<u>5</u>	<u>4</u>	<u>6</u>	<u>5</u>

Without copying the numbers, write as rapidly as you can the results of the following subtractions:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
67	96	26	43	85	28	64	79	87	58
<u>24</u>	<u>32</u>	<u>14</u>	<u>21</u>	<u>52</u>	<u>17</u>	<u>43</u>	<u>54</u>	<u>35</u>	<u>26</u>
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
84	96	37	48	56	27	75	59	37	65
<u>41</u>	<u>52</u>	<u>25</u>	<u>13</u>	<u>25</u>	<u>13</u>	<u>52</u>	<u>24</u>	<u>16</u>	<u>23</u>

GENERAL EXPLANATION OF THE SUBTRACTION PROCESS.

First Example Where the digits of the minuend are all greater than the corresponding digits of the subtrahend, the subtraction process is quite simple, and the result can be ascertained by a mere inspection of the numbers. In this case, we may begin the subtraction either at the left or right.

Second Example In this example some of the digits of the subtrahend are greater than the corresponding digits of the minuend. In such cases the subtraction necessarily is commenced at the unit order. The number of units indicated in the unit's column of the minuend, 3, is less than the unit's number in the subtrahend, 7; we therefore add 10 to the minuend figure,

making 13 units, from which we subtract the 7, writing the result, 6, in the unit's place of the remainder. We may now assume that the 10 which we added to the unit figure of the minuend was "borrowed," or taken away from the tens' column, and we may consider that the 4 tens have been reduced to 3, from which we now take the 1 ten in the subtrahend, writing the remainder, 2, in the tens' column of the remainder. Or we may apply Prin. 3, and add, or "carry," as it is sometimes called, 1 ten to the tens of the subtrahend, making 2 tens, which we subtract from the 4 tens of the minuend, leaving 2 tens to write in the remainder as before. Both of these methods are employed, but the result is the same in either case.

Proceeding in the same way in the case of the hundreds, we add 10 hundreds (1 thousand) to the 6 in the minuend, making 16, from which we subtract the 7 of the subtrahend, writing the result 9 in the remainder.

Then, either considering the 7 thousand of the minuend diminished to 6, or the 2 thousand of the subtrahend increased to 3, we subtract, writing the result, 4, in the remainder

PROOF OF SUBTRACTION.

Since the remainder is the difference between the subtrahend and the minuend, the minuend is equal to the sum of the subtrahend and remainder; hence,

TO PROVE SUBTRACTION.—*Add the Subtrahend to the Remainder; if the sum is equal to the Minuend, the subtraction is correct.*

WRITTEN EXERCISE.

Solve the following and prove your results.

1. $9124 - 857 = ?$
 2. $2147 - 1429 = ?$
 3. $24582 - 9385 = ?$
 4. $82430 - 29572 = ?$
 5. $940367 - 47038 = ?$
 6. $102012 - 87965 = ?$
 7. $720164 - 385726 = ?$
 8. $80000127 - 5000864 = ?$
 9. $\$17965.42 - \$12867.92 = ?$
 10. $\$100046.21 - \$74506.38 = ?$
 11. $\$1006.20 - \$95.87 = ?$
 12. $2964301 - 780067 = ?$
 13. $42716301 - 743986 = ?$
- Find the difference between—
14. 4275 and 958.
 15. 1256 and 8794.
 16. $\$2796.42$ and $\$37601.15$.
 17. $\$1215627$ and $\$862479$.
 18. 3874216 and 982475.
 19. 10201670 and 34876216.
 20. 937624 and 1000000.

COMPLEMENTARY NUMBERS.

Complementary Numbers are any two numbers whose sum is equal to a unit of the next higher order. Thus, 6 and 4, 7 and 3, 8 and 2, etc., are complementary numbers, as the sum of each of these pairs is 10; and 36 and 64, 27 and 73, 42 and 58, are complementary, as the sum of each pair is 100.

A number is said to be the *complement* of another when the

sum of the two is a unit of the next higher order; thus, 7 is the complement of 3, 26 is the complement of 74, and 364 is the complement of 636.

In all complementary numbers of more than one figure, the sum of the unit figures is 10, while that of the other corresponding orders is 9.

Example:

$$\begin{array}{r} 4632 \\ 5368 \\ \hline 999_{10} = 10000 \end{array} \qquad \begin{array}{r} 247 \\ 753 \\ \hline 99_{10} = 1000 \end{array}$$

By applying the foregoing principle, a little practice will enable one to name, at sight, the complement of any number. The ability to do this is very useful in the business office in making change, etc.

MENTAL EXERCISE.

Name as rapidly as you can the complements of the following numbers: 36, 26¢, 64, 73, 28¢, 83, 42, 34, 65¢, 48, \$26, 36, 57, \$21, 32, 63, 76, 19, 83, 55, 22, 31, 43, 82, 53.

Give the complements of the following: 125, 236, 328, 475, 643, 764, 238, 753, 146, 321, 816, 458, 735, 593, 637.

In case a ten-dollar bill is offered in payment, name the amount of change required for each of the following amounts: \$2.75, \$1.45, \$3.56, \$8.27, \$3.42, \$6.29, \$4.53, \$2.35, \$5.75, \$8.21, \$6.15, \$4.38, \$7.23, \$2.91, \$3.54, \$2.74, \$8.75, \$4.18, \$3.37, \$2.85.

MENTAL EXERCISE.

In each of the following problems, find the change required by subtracting the sum of the items from the sum paid. Where possible, find the result mentally.

Items.	Paid.	Items.	Paid.
1. 15¢, 12¢, 50¢,	\$1	8. 75¢, 24¢, 47¢,	\$2
2. 24¢, 36¢, 18¢,	\$1	9. \$1.25, 60¢, 22¢,	\$5
3. 45¢, 15¢, 36¢,	\$2	10. \$3.25, 50¢, \$1.25,	\$5
4. 21¢, 18¢, 12¢,	\$1	11. \$4 60, \$2.25, \$1.20	\$10
5. 45¢, 55¢, 48¢,	\$2	12. \$1.20, \$1.50, \$2,	\$5
6. 23¢, 34¢, 25¢,	\$1	13. \$3.60, \$4, 55¢,	\$10
7. 63¢, 8¢, 4¢,	\$1	14. \$4.25, \$5.50, \$8.20,	\$20

WRITTEN PROBLEMS.

21. What number must be added to 98647 to give a result of 102613?

22. From a carload of coal containing 46975 lbs., 29874 lbs. has been sold; how much coal remains in the car?

23. On Jan. 1, Carter & Stone's bank balance amounted to \$28596.15. On Feb. 1 their balance was \$17524.98; what was the amount of decrease during January?

24. The consumption of water by the town of Clyde for June, 1900, amounted to 13,247,162 gallons. The consumption for July was 15,964,386; what was the increase in consumption over the previous month?

25. The purchase price of the Brockton Street Car Plant was \$275,946.25, of which sum, \$198,596.48 has been paid. What sum remains unpaid?

26. In 1890 the unsold government land in one of the Western States was 526,516 acres. In 1900 the unsold land was 359,879 acres. What number of acres was sold during the decade?

27. What number must be subtracted from 98601 to give a remainder of 29748?

WRITTEN PROMISCUOUS PROBLEMS.

28. On Nov. 1, Fay & Brookings' balance at the Mercantile Bank was \$947.60. On Nov. 10 they deposited \$256.75, and on Nov. 27, \$425.83. During the month they withdrew on their checks, \$1159.25. What is their bank balance on Dec. 1?

29. The school population of Jackson County for 1900 was 21568. Of this number of children, 2594 are in Holton, 2862 are in Bridgeport, and 1846 in Vinton, the three leading towns in the county. What is the school population of the remainder of the county?

30. The total school appropriation of the town of Lawn-dale for 1900 is \$12,538.67. During the year, school warrants were issued as follows: teachers' salaries, \$8528.50; salaries of other officers, \$2847.50; repairs, \$469.15; fuel and incidentals, \$1087.25; library, \$287.60. What is the amount of the deficiency for the year?

31. The outlays of the Oakdale Manufacturing Co. for the year ending July 1, 1900, were as follows: wages and salaries, \$21,564.42; materials and equipment, \$19462.75; advertising, \$3586.15; other expenses, \$642.90. If the gross receipts were \$58962.70, what were the net receipts for the year?

ACCOUNTS.

In bookkeeping the various charges and credits relating to one person or firm, or to one class of transactions, are assembled in groups called *accounts*.

The accompanying form represents an account with Hall & Morton. That part of the account at the left of the central dividing line is called the *debit side* of the account, and that part to the right is called the *credit side*. The various sums or items on the left are called *debits*, and those on the right are called *credits*.

The difference between the sums of the debits and credits is called the *balance* of the account.

To *close* or *balance* an account, is to enter the balance on the lesser side and rule up the account, as shown in the accompanying form.

When an account is properly closed, the totals or footings of the two sides should be equal.

HALL & MORTON.

1900				(DEBITS)				1900				(CREDITS)			
Jan...	2	Mdse.....	...	1	2	6	50	Jan..	6	Cash.....	...	5	0	00	
" ..	8	"	2	5	4	25	" ..	12	Mdse.....	...	1	6	2	25
" ..	21	Cash.....	...	7	5	00		" ..	29	"	6	4	90	
Feb..	4	Mdse.....	...	1	1	5	75	Mar. 1	Balance..	...	5	1	2	90	
" ..	16	"	4	5	10		
" ..	21	Cash.....	...	5	0	00		
" ..	23	Mdse.....	...	1	2	3	45	
				7	9	0	05					7	9	0	05

WRITTEN PRACTICE.

In the following examples only the money columns of the accounts are shown. Prepare similarly ruled forms for these accounts, copy them neatly, after which ascertain the balances, and close and rule up the several accounts:

SUBTRACTION

(32)

M. S. ROBINSON.

349	60	592	11
74	68	821	65
148	24	273	87
205	87	21	59
		143	25
		65	27
		9	46

(33)

C. W. MARSH.

142	50	253	40
275	25	19	28
8	15	446	27
34	68	35	92
	45		
519	12		

(34)

CASH.

79	54	62	95
427		321	60
4	49	11	45
37	58	10	
96		50	
125		43	69
9	14	27	53
329	50	148	65
49	23		
75	80		

(35)

CHESTER I. LONG.

48	75	79	50
15	25	25	
98		367	98
114	60	14	46
85	07	5	
9	25		94
2	46		
300			
48	76		

(36)

C. B. RENFRO & CO.

857	28	185	90
9	46	72	47
350		9	80
1754	87	1246	70
957	20	189	45
89	56	968	
		40	08
		3475	62
		8	97
		400	
		98	58

(37)

BILLS RECEIVABLE.

624	80	250	
3658	70	742	80
459	25	624	80
97	15	75	
500		146	
1076	28	97	15
746	25	2500	
19	15		
5			
253	96		
874	28		

Prepare account-forms for the following schedule of items, and close and rule up the several accounts. In entering the items omit dollar marks.

(38) C. J. RAMSDELL.

DEBITS. \$1940.00, 75¢, \$142.60, \$189.15, \$47.25, \$54.96, 98¢, \$78.00.

CREDITS. \$44.60, \$15.00, \$175.00, \$34.75, \$5.00, \$21.40.

(39) CASH.

DEBITS. \$39.65, \$147.19, \$278.47, \$435.86, \$79.28, \$164.00, \$358.60, \$29.56, \$85.90.

CREDITS. \$57.24, \$94.15, \$68.75, \$29.00, \$58.64, 47¢, \$2.78, \$13.09.

(40) BILLS PAYABLE.

DEBITS. \$75.00, \$54.28, \$47.90, \$175.00, \$243.80, \$67.00, \$48.50, \$374.19, \$59.60, \$178.50.

CREDITS. \$178.50, \$75.00, \$59.60, \$175.00, \$243.80, \$54.28, \$47.90, \$67.00, \$374.19, \$427.95, \$67.75.

MULTIPLICATION.

At 3 cents each, what is the cost of 7 pears?

SOLUTION.—If the pears were bought one at a time, we should have to pay out 3 cents *seven times*, and the cost would equal $3+3+3+3+3+3+3=21$ cents. If, when we add the seven 3's, we remember the result, the next time we have a similar problem we would not need to add, but would simply say, "The pears will cost 7 times 3 cents, or 21 cents."

The **Multiplication Table** is a table showing the results of adding equal numbers. The process of finding the sum of several equal numbers by means of this table is called **Multiplication**.

DEFINITIONS.

In multiplication, the several numbers to be added are thought of and spoken of as one number, which is called the **Multiplicand**. The number that indicates the number of times the multiplicand is taken, is called the **Multiplier**.

The result of the multiplication is called the **Product**.

Thus in the expression, "5 times 9 are 45," 9 is the *multiplicand*, 5 is the *multiplier*, and 45 is the *product*.

The multiplier and multiplicand are also called "factors."

The sign of multiplication is (\times), and when written between two numbers indicates that either one is to be taken as an addend as many times as there are units in the other. Thus, the expression " $4 \times 6 = 24$ " may be read "four times six are twenty-four," or "four multiplied by six equals 24."

Continued Multiplication is the process indicated by joining a series of numbers with the multiplication sign, as, $5 \times 2 \times 3 \times 4$. The result of a continued multiplication is obtained by multiplying any of the numbers by any other, and this product by the third, and so on; the result being the same, regardless of the order in which the numbers are taken.

The number of distinct multiplications indicated in any continued multiplication is equal to one less than the num-

ber of factors. Thus, to find the product of $5 \times 7 \times 8 \times 9$, three distinct multiplications are necessary.

PRINCIPLES.

1. The multiplier is always an abstract number.
2. The multiplicand and product are like numbers.
3. Either term of the multiplication is contained in the product as many times as there are units in the other term.

MENTAL EXERCISE.

1. Prove by addition that 8 times 7 is equal to 7 times 8.
2. What is the product of $2 \times 3 \times 5 \times 7$?
In how many ways can you get the result?
3. Find, by addition, the cost of 5 acres of land at \$25 per acre.
4. Name the multiplier, the multiplicand, and the product in the following problem: What is the total length of seven boards, if each board is 9 ft. long?
5. Find, by subtraction, the number that must be multiplied by 7, to give 91 as a product.
6. Find, by addition, the number that 14 must be multiplied by to give 84 as a product.
7. Find, by addition, the product of $3 \times 4 \times 5$.
8. How many distinct multiplications are necessary to find the product of $6 \times 7 \times 8 \times 4 \times 12$?
9. Prove that the product of $6 \times 3 \times 5 \times 2$ is the same regardless of the order in which the several multiplications are performed.
10. If $2 \times 4 \times (x) = 72$, find x by subtraction.

THE MULTIPLICATION TABLE.

The Multiplication Table, as ordinarily presented, includes the numbers from 1 to 12, the combinations under each of the several numbers being placed in separate groups.

The Multiplication Table here presented contains all combinations from " 1×1 " to " 25×25 ." To use this table, find the junction of the required columns; thus, if we wish to know the product of 12 times 19, we find it to be 228, which is at the junction of the 12th and 19th columns.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

SUPPLEMENTARY MULTIPLICATION TABLE.

As a means of enlarging the student's knowledge of the composition of the lesser numbers, we here present an extension of the Multiplication Table, which includes all possible multiplication combinations, the results of which do not exceed 100. Thus the "2 line" is extended to 2×50 ; the "3 line," to 3×33 ; the "4 line," to 4×25 , and so on.

A knowledge of this supplementary table will prove of the greatest advantage in the student's subsequent arithmetical work, and its thorough mastery is strongly advised.

NOTE.—The table, as here presented, is shortened by omitting the reverse combinations, the "4 line" beginning with "4x4," the "5 line," with "5x5," etc. The student should be required to give the results of the several combinations, entirely regardless of the order in which the factors are presented.

SUPPLEMENTARY MULTIPLICATION TABLE.

$1 \times 2 = 4$	$1 \times 43 = 86$	$4 \times 4 = 16$	$6 \times 6 = 36$
$1 \times 3 = 6$	$1 \times 44 = 88$	$4 \times 5 = 20$	$6 \times 7 = 42$
$1 \times 4 = 8$	$1 \times 45 = 90$	$4 \times 6 = 24$	$6 \times 8 = 48$
$2 \times 5 = 10$	$2 \times 46 = 92$	$4 \times 7 = 28$	$6 \times 9 = 54$
$2 \times 6 = 12$	$2 \times 47 = 94$	$4 \times 8 = 32$	$6 \times 10 = 60$
$2 \times 7 = 14$	$2 \times 48 = 96$	$4 \times 9 = 36$	$6 \times 11 = 66$
$2 \times 8 = 16$	$2 \times 49 = 98$	$4 \times 10 = 40$	$6 \times 12 = 72$
$2 \times 9 = 18$	$2 \times 50 = 100$	$4 \times 11 = 44$	$6 \times 13 = 78$
$2 \times 10 = 20$		$4 \times 12 = 48$	$6 \times 14 = 84$
$2 \times 11 = 22$		$4 \times 13 = 52$	$6 \times 15 = 90$
$2 \times 12 = 24$	$3 \times 3 = 9$	$4 \times 14 = 56$	$6 \times 16 = 96$
$2 \times 13 = 26$	$3 \times 4 = 12$	$4 \times 15 = 60$	
$2 \times 14 = 28$	$3 \times 5 = 15$	$4 \times 16 = 64$	
$2 \times 15 = 30$	$3 \times 6 = 18$	$4 \times 17 = 68$	
$2 \times 16 = 32$	$3 \times 7 = 21$	$4 \times 18 = 72$	$7 \times 7 = 49$
$2 \times 17 = 34$	$3 \times 8 = 24$	$4 \times 19 = 76$	$7 \times 8 = 56$
$2 \times 18 = 36$	$3 \times 9 = 27$	$4 \times 20 = 80$	$7 \times 9 = 63$
$2 \times 19 = 38$	$3 \times 10 = 30$	$4 \times 21 = 84$	$7 \times 10 = 70$
$2 \times 20 = 40$	$3 \times 11 = 33$	$4 \times 22 = 88$	$7 \times 11 = 77$
$2 \times 21 = 42$	$3 \times 12 = 36$	$4 \times 23 = 92$	$7 \times 12 = 84$
$2 \times 22 = 44$	$3 \times 13 = 39$	$4 \times 24 = 96$	$7 \times 13 = 91$
$2 \times 23 = 46$	$3 \times 14 = 42$	$4 \times 25 = 100$	$7 \times 14 = 98$
$2 \times 24 = 48$	$3 \times 15 = 45$		
$2 \times 25 = 50$	$3 \times 16 = 48$		
$2 \times 26 = 52$	$3 \times 17 = 51$		
$2 \times 27 = 54$	$3 \times 18 = 54$	$5 \times 5 = 25$	$8 \times 8 = 64$
$2 \times 28 = 56$	$3 \times 19 = 57$	$5 \times 6 = 30$	$8 \times 9 = 72$
$2 \times 29 = 58$	$3 \times 20 = 60$	$5 \times 7 = 35$	$8 \times 10 = 80$
$2 \times 30 = 60$	$3 \times 21 = 63$	$5 \times 8 = 40$	$8 \times 11 = 88$
$2 \times 31 = 62$	$3 \times 22 = 66$	$5 \times 9 = 45$	$8 \times 12 = 96$
$2 \times 32 = 64$	$3 \times 23 = 69$	$5 \times 10 = 50$	
$2 \times 33 = 66$	$3 \times 24 = 72$	$5 \times 11 = 55$	
$2 \times 34 = 68$	$3 \times 25 = 75$	$5 \times 12 = 60$	
$2 \times 35 = 70$	$3 \times 26 = 78$	$5 \times 13 = 65$	$9 \times 9 = 81$
$2 \times 36 = 72$	$3 \times 27 = 81$	$5 \times 14 = 70$	$9 \times 10 = 90$
$2 \times 37 = 74$	$3 \times 28 = 84$	$5 \times 15 = 75$	$9 \times 11 = 99$
$2 \times 38 = 76$	$3 \times 29 = 87$	$5 \times 16 = 80$	
$2 \times 39 = 78$	$3 \times 30 = 90$	$5 \times 17 = 85$	
$2 \times 40 = 80$	$3 \times 31 = 93$	$5 \times 18 = 90$	
$2 \times 41 = 82$	$3 \times 32 = 96$	$5 \times 19 = 95$	$10 \times 10 = 100$
$2 \times 42 = 84$	$3 \times 33 = 99$	$5 \times 20 = 100$	

MENTAL PROBLEMS FOR ANALYSIS.

1. What will 17 lemons cost at 5 cents each?

ANALYSIS.—They will cost 17 times 5 cents, or 85 cents.

NOTE.—In giving the analysis of problems in multiplication, do not reverse the terms in the expression. Take for the multiplicand the number that is of the same denomination as the required product. Thus, in the foregoing problem it would be illogical to say "five times seventeen," instead of "seventeen times five," notwithstanding that both expressions involve the same numerical result.

2. Allowing 8 rails to the panel, how many rails will be required for 12 panels of fence?

3. If 15 quires of paper are used in one copy of a certain book, how many quires will be used in an edition of 3000 copies?

4. How many days are there in 13 weeks?

5. Find the value of 5 acres of land at \$17 an acre.

6. How many eggs in 2 cases containing 30 dozen each?

7. If there are 16 ounces in a pound, find the weight, in ounces, of 6 pounds of nails.

8. What is the value of 13 cases of berries at \$3 each?

9. Find the cost of 5000 ft. of lumber at \$19 per thousand.

10. What is the value of 11 yards of ribbon at 12 cents a yard?

11. How far will a man travel in 9 hours at the rate of 60 miles an hour?

12. At 25 cents an hour, what can a man earn in 3 days of 8 hours each?

13. What is the distance around a square field that measures 17 rods on each side?

14. If a boy runs 12 feet per second, how far can he run in a minute?

15. In an orchard there are 12 rows of trees with 9 trees in each row; how many trees in the orchard?

16. A room is 60 ft. long and 40 ft. wide; how many square feet of floor space in the room?

NOTE.—The square feet in any surface is equal to the product of the length by the width. The analysis is as follows:

"If the surface were 60 feet long and 1 foot wide, the surface would equal 60 square feet; therefore, a surface 60 feet long and 40 feet wide will contain 40 times 60 square feet, or 2400 square feet.

17. Find the cost of 8 quarts of berries at 5 cents a pint.

18. What is the length of 5 panels of fence, the length of a panel being 16 ft.?

19. Mary's flower bed is 14 ft. long and 6 feet wide, how many square feet in its surface?

20. What is the value of 5 dozen eggs, each egg being worth 2 cents?

21. A certain room is 16 feet long and 10 feet wide, what is the entire distance around the room? How many square feet in the floor?

22. Find the cost of 15 tons of coal at \$5 per ton.

A **Multiple** of a number is any number that is obtained by multiplying it by another whole number. Thus, 24, 36, 48, and 60 are each multiples of 12; and 16, 24, 32 and 40 are multiples of 8.

MENTAL EXERCISE.

1. Name, in order, all of the multiples of 12 less than 100. Of 13; of 18; of 16.

2. Of what numbers is 60 a multiple? 72? 93? 85? 68?

3. Name a number that is a multiple of 5 and 12. Of 9 and 15; of 18 and 4; of 7 and 13.

4. Name all multiples, less than 100, of the following numbers: 7, 14, 15, 17, 19, 21, 29.

WRITTEN EXERCISE.

Multiply 5679 by 6.

In complete form, the operation would be recorded as follows:

$$\begin{array}{r}
 5679 \\
 6 \\
 \hline
 54 \\
 42 \\
 36 \\
 30 \\
 \hline
 34074
 \end{array}$$

each figure of the multiplicand being multiplied by the multiplier, the several results being written in proper order and added.

In practice, however this process is shortened by performing the additions mentally as the multiplication proceeds, only the general product being recorded, thus:

$$\begin{array}{r} 5679 \\ 6 \\ \hline 34074 \end{array}$$

In this process, the unit figure of the first multiplication, 4, is written, and the tens are "carried," or added in with the next ten's product, and so on, until all of the digits have been multiplied.

1. Multiply each of the following numbers by 4: 964; 7446; 9241; 24563; 9768.

2. Multiply the following by 5: 2046; 7487; 92507; 70906; 19867.

3. Multiply the following by 6: 7438; 20906; 38720; 84769; 372067.

4. Multiply the following by 7: 42876; 89735; 48738; 90764; 58678.

5. Multiply the following by 8: 9675; 6958; \$97.65; \$189.48; \$9476.28.

6. Multiply the following by 9: 87¢; \$9.85; \$597.62; \$9839.46; 46758.

MENTAL EXERCISE.

Perform mentally as many as you can of the following multiplications. If you have any difficulty, study the Supplementary Multiplication Table.

- | | | |
|-----------------------|-----------------------|-------------------------|
| 1. $13 \times 3 = ?$ | 11. $18 \times 4 = ?$ | 21. $350 \times 2 = ?$ |
| 2. $15 \times 6 = ?$ | 12. $23 \times 3 = ?$ | 22. $1900 \times 4 = ?$ |
| 3. $19 \times 2 = ?$ | 13. $19 \times 5 = ?$ | 23. $18 \times 30 = ?$ |
| 4. $16 \times 6 = ?$ | 14. $15 \times 5 = ?$ | 24. $260 \times 2 = ?$ |
| 5. $14 \times 5 = ?$ | 15. $36 \times 2 = ?$ | 25. $1700 \times 5 = ?$ |
| 6. $13 \times 6 = ?$ | 16. $28 \times 3 = ?$ | 26. $160 \times 30 = ?$ |
| 7. $29 \times 2 = ?$ | 17. $14 \times 6 = ?$ | 27. $300 \times 48 = ?$ |
| 8. $24 \times 3 = ?$ | 18. $18 \times 5 = ?$ | 28. $130 \times 60 = ?$ |
| 9. $17 \times 4 = ?$ | 19. $13 \times 7 = ?$ | 29. $2800 \times 3 = ?$ |
| 10. $46 \times 2 = ?$ | 20. $23 \times 4 = ?$ | 30. $470 \times 2 = ?$ |

- | | | |
|--------------------------|----------------------------|------------------------------|
| 31. $1300 \times 40 = ?$ | 55. $2114 \times 3 = ?$ | 79. $\$13.25 \times 3 = ?$ |
| 32. $380 \times 20 = ?$ | 56. $1729 \times 30 = ?$ | 80. $\$15.18 \times 5 = ?$ |
| 33. $1700 \times 30 = ?$ | 57. $3213 \times 3 = ?$ | 81. $504 \times 19 = ?$ |
| 34. $140 \times 300 = ?$ | 58. $1319 \times 50 = ?$ | 82. $1700 \times 40 = ?$ |
| 35. $180 \times 40 = ?$ | 59. $4728 \times 20 = ?$ | 83. $\$1316 \times 50 = ?$ |
| 36. $120 \times 12 = ?$ | 60. $16140 \times 6 = ?$ | 84. $204 \times 23 = ?$ |
| 37. $160 \times 50 = ?$ | 61. $\$1.90 \times 5 = ?$ | 85. $320 \times 30 = ?$ |
| 38. $500 \times 19 = ?$ | 62. $\$18.25 \times 3 = ?$ | 86. $17019 \times 30 = ?$ |
| 39. $700 \times 130 = ?$ | 63. $\$16.40 \times 2 = ?$ | 87. $141517 \times 5 = ?$ |
| 40. $29 \times 30 = ?$ | 64. $\$13.18 \times 4 = ?$ | 88. $12018 \times 4 = ?$ |
| 41. $1314 \times 3 = ?$ | 65. $\$12.29 \times 3 = ?$ | 89. $\$190.29 \times 3 = ?$ |
| 42. $1912 \times 5 = ?$ | 66. $\$6.48 \times 2 = ?$ | 90. $\$240.45 \times 2 = ?$ |
| 43. $1713 \times 4 = ?$ | 67. $\$7.15 \times 5 = ?$ | 91. $\$180.15 \times 5 = ?$ |
| 44. $2648 \times 2 = ?$ | 68. $\$28.17 \times 3 = ?$ | 92. $\$305 \times 16 = ?$ |
| 45. $1815 \times 5 = ?$ | 69. $\$47.25 \times 2 = ?$ | 93. $\$507.03 \times 13 = ?$ |
| 46. $918 \times 3 = ?$ | 70. $\$15.17 \times 5 = ?$ | 94. $\$306.04 \times 14 = ?$ |
| 47. $514 \times 6 = ?$ | 71. $\$14.18 \times 4 = ?$ | 95. $\$1825 \times 30 = ?$ |
| 48. $1918 \times 4 = ?$ | 72. $\$26.45 \times 2 = ?$ | 96. $\$13.15 \times 60 = ?$ |
| 49. $2946 \times 2 = ?$ | 73. $\$18.32 \times 3 = ?$ | 97. $\$25016 \times 30 = ?$ |
| 50. $1525 \times 3 = ?$ | 74. $\$12.16 \times 6 = ?$ | 98. $\$140.15 \times 5 = ?$ |
| 51. $1327 \times 3 = ?$ | 75. $\$8.19 \times 5 = ?$ | 99. $\$1700 \times 40 = ?$ |
| 52. $2319 \times 4 = ?$ | 76. $\$27.16 \times 3 = ?$ | 100. $\$181619 \times 4 = ?$ |
| 53. $1131 \times 3 = ?$ | 77. $\$19.25 \times 2 = ?$ | |
| 54. $1618 \times 5 = ?$ | 78. $\$47.28 \times 2 = ?$ | |

WRITTEN EXERCISE.

Multiply 4678×537 .

4678
 537
32746
 14034
 23390
2512086

Beginning with units, multiply successively by the figures of the multiplier, writing the several products so that figures of the same order may stand in the same column. The several products are then added, the result being the final product.

7. Multiply 7464 by 29.
8. Multiply 9348 by 58.
9. Multiply 7468 by 89.
10. Multiply 567 by 246.
11. Multiply 946 by 708.
12. Multiply 1234 by 567.
13. Multiply $\$87.46$ by 987.
14. Multiply $\$679.43$ by 369.

Find the required products for the following continued multiplications:

$$15. \quad 64 \times 87 \times 9 = ?$$

$$16. \quad 9 \times 8 \times 6 \times 7 \times 15 = ?$$

$$17. \quad 47 \times 102 \times 64 = ?$$

$$18. \quad 3 \times 9 \times 6 \times 54 \times 42 = ?$$

$$19. \quad 35 \times 127 \times 86 = ?$$

$$20. \quad 53 \times 12 \times 43 \times 124 = ?$$

CONTRACTIONS IN MULTIPLICATION.

A contraction is a shorter method or process for obtaining a result than the one ordinarily used.

A number of contractions are employed in Multiplication, the following cases being among the most useful:

CASE I.—Where there are ciphers at the Right of the Multiplier or the Multiplicand. If either factor is 10, 100, 1000, 10000, etc., the product is obtained by writing the ciphers at the right of the other factor. Thus, $10 \times 56 = 560$; $157 \times 1000 = 157000$.

If either factor consists of other figures than 1, with ciphers to the right, multiply regardless of the ciphers, and annex them to the product, thus:

562	234000
2400	16
<hr/> 2248	<hr/> 1404
1124	234
<hr/> 1348800	<hr/> 3744000

If both factors have ciphers at the right, multiply together the remaining digits, and annex as many ciphers as there are at the right of both factors, thus:

$$\begin{array}{r}
 243000 \\
 2500 \\
 \hline
 1215 \\
 486 \\
 \hline
 607500000
 \end{array}$$

WRITTEN EXERCISE.

21. Multiply 768 by 1000.
22. Multiply 346 by 4600.
23. Multiply 1,234,000 by 75.
24. Multiply 346,700 by 9000.
25. Multiply 106,000 by 30,400.
26. Multiply 740,000 by 28,000.

CASE II.—When the Multiplier is Composed of Factors.

Multiply 567 by 48.

567 Since the multiplier equals 6×8 , we may employ the
 6 continued multiplication $567 \times 6 \times 8$, instead of multi-
 3402 plying by 48, thus avoiding the necessity of adding
 8 partial products.
 27216

WRITTEN EXERCISE.

In the same way, find the required products for the following multiplications, using the factors indicated:

27. $2587 \times 24 (= 3 \times 8) = ?$
28. $9376 \times 63 (= 7 \times 9) = ?$
29. $75296 \times 36 (= 12 \times 3) = ?$
30. $\$875.75 \times 72 (= 8 \times 9) = ?$

Find the following products, using factors of the multiplier:

31. 6741×18 .
32. 54967×36 .
33. 82973×48 .
34. $\$458.95 \times 56$.

CASE III.—When one Part of the Multiplier is a Multiple of Another Part.

Multiply 1254 by 637.

1254 Since 63 is 9 times 7, we multiply the first partial product by 9 instead of con-
 637 tinuing the multiplication in the usual way.

8778
 79002
 798798

Multiply 2351 by 856.

2351 Since 56 is 7 times 8, we first multiply by 8, then multiply this partial product
 856 by 7.

18808
 131656
 2012456

Multiply 120304 by 48246.

120304	Since 24 is 4 times 6, we first multiply by 6, and
48246	this product by 4, for the second partial product.
<u>721824</u>	Since 48 is 2 times 24, we multiply the second
2887296	partial product by 2. The sum of the three
5774592	partial products is the complete product. We
<u>5804186784</u>	could also find the third partial product by
	multiplying the first one by 8.

WRITTEN EXERCISE.

Perform the following multiplications, shortening the work, wherever practical, by means of any of the foregoing contractions:

35. $3564 \times 287 = ?$
36. $1356 \times 4900 = ?$
37. $2456 \times 2408 = ?$
38. $27800 \times 540 = ?$
39. $124760 \times 56287 = ?$
40. $345200 \times 7208 = ?$
41. $796500 \times 24000 = ?$
42. $324610 \times 9072 = ?$

WRITTEN PROBLEMS.

43. Find the value of 267 tons of coal at \$4.75 per ton.
44. If there are 16 lots to the block, how many lots are there in the Adams Addition to the town of Mayfair, which extends 7 blocks in one direction and 5 blocks in the other? What is the total value of the Addition, if the average value of the lots is \$175 each.
45. On a Dakota ranch there are 2180 sheep. If the fleeces average 8 pounds each, what is the value of the wool clip when the price of the wool is 18 cents a pound?
46. In a certain cornfield there are 469 rows, and 528 hills in each row; how many hills of corn in the field?
47. In the town of Belleville, each of the 6 street cars makes 32 trips per day, and the receipts of each car per trip average \$1.45. What are the total receipts for one year of 365 days?

48. When the price of lumber is \$18.75 per thousand feet, what is the value of 187 thousand feet?

49. There are 320 rods in one mile; how many rods in 129 miles?

50. A pile of iron rails weighing 65 lbs. to the yard contains 148 rails, each 8 yards long; what is the weight of the pile?

51. The net cost of making a certain book was found to be 59 cents per copy. What was the cost of an edition of 7500 copies?

52. Scott & Co., contractors, employ 75 men at wages averaging \$1.75 per day. What is the amount of the pay-roll for one month of 26 working days?

PROMISCUOUS WRITTEN PROBLEMS.

53. Mr. Bennet bought three tracks of land, including respectively, 325, 79, and 146 acres, at \$17.50 per acre. He afterwards sold the larger tract at \$20 per acre, and the other tracts at \$16.25 per acre. Did he lose or gain on the speculation, and how much?

54. John Clark sold to Carter Bros. 79 bushels of potatoes at 45 cents per bu. and 125 lbs. of butter at 17 cents per lb. He received in payment groceries amounting to \$17.25, and \$20 in cash. How much do Carter Bros. still owe him?

55. Clarence earns \$9.50 a week. He estimates his weekly expenses as follows: Board, \$3.50; laundry, 40 cents; books and newspapers, 50 cents; incidentals, 75 cents. Allowing \$50 per year for clothing, what should he save in one year (52 weeks)?

56. At 35 cents a rod, what will it cost to plant a hedge around a field 65 rods long and 48 rods wide?

57. A lumber dealer sold to a farmer 34 thousand feet of lumber at \$16.25 per thousand, receiving in payment 7 tons of hay at \$9.50 a ton, 16 cords of wood at \$3.75 a cord, and a check for the balance. What was the amount of the check?

58. A grocer bought from a huckster 357 lbs. butter @ 13 cents, and 175 doz. eggs @ 9 cents. He sold the butter @ 16

cents and the eggs @ 12 cents. What was his gain on the purchase?

NOTE.—The character (@) signifies "each" or "apiece," and is used to indicate the market price per pound, per bushel, per doz., etc.

59. A dealer gained \$75.90 on 55 tons of baled hay, which he bought at \$7.75 per ton. For what sum did he sell it?

60. Miller & Brown, real estate dealers, bought a farm containing 375 acres for \$9500. They expended on the property for repairs and buildings, \$758.25, and for taxes \$102.15. They sold wood from the land to the amount of \$95.25, and then sold the entire property for cash at \$31.50 per acre. What did they gain through the speculation?

61. A contractor engaged to build a cement walk 155 feet long and 6 feet wide, for \$95. He then sublet the work at 9¢ per square foot. How much did he gain on the contract?

DIVISION.

Walter having 12 plums gives 2 to Henry; how many remain? He then gives 2 to Frank, and 2 to Chester; how many are left? How many times can he give 2 away? In the same way successively take 3 out of 15 until nothing is left. How many subtractions are required? How many times can 4 be taken out of 20? Out of 12? Out of 28? How many 5's in 35? In 45?

How many times can you take 3 out of 14? What number will remain?

If Ethel has \$25, how many books can she buy at \$5 each, and what sum will she have remaining? How often can 9 be taken out of 50, and what number will remain?

A knowledge of what table will enable you to determine the number of 9's in 63, the number of 7's in 42, the number of 8's in 56, etc., without resorting to subtraction?

If 5 pencils cost 30 cents, what do they cost apiece? What would they cost at 1 cent each? How many times is the cost at 1 cent each contained in 30 cents? How much can be paid for each pencil in order to expend the whole of 30 cents?

If a seven-acre lot of land is sold for \$56, what is the price per acre? How many times is 2 contained in 10? In 14? In 20?

When we divide a number by 2, we get as a result what part of the number? What is one-half of 16? Of 24? Of 42?

What part of a number do we get when we divide it by three? By 4? By 5? By 12? By 20?

If 5 coats cost \$40, 1 coat costs what part of \$40? What sum does one coat cost?

Twelve books cost \$36, what do they cost apiece?

DEFINITIONS.

Division is the process of finding the number of times one number is contained in another, or of finding one of the equal

parts of a number. The first of these processes is termed **Rational Division**; the second, **Partative Division**.

RATIONAL DIVISION.

A **Ratio** is the result obtained by dividing or measuring one number or quantity by another of like kind. Thus, the ratio of \$12 to \$4 is 3; the ratio of 24 men to 12 men is 2; the ratio of 5 acres to 10 acres is $\frac{1}{2}$, and so on.

In arithmetic, a ratio is indicated by a colon (:) written between the numbers to be compared. Thus, $15:5=3$ indicates that 15 is measured (divided) by 5, and the result is 3. The expression is read, "the ratio of 15 to 5 is 3."

MENTAL EXERCISE.

What is the ratio of \$24 to \$8? 42 sheep to 6 sheep? 63 days to 9 days? 51 acres to 17 acres? 48 men to 16 men? 6 apples to 2 apples? 2 apples to 6 apples?

NOTE.—There are always two ways of expressing the ratio, or relative value of two numbers, accordingly as the one or the other is taken as the basis of the comparison: thus, in comparing 12 and 4, we may say "12 is three times 4," or we may say "4 is one-third of 12." In like manner there are two ways of stating a ratio, viz.: " $12:4=3$," or " $4:12=\frac{1}{3}$."

State the value of the following ratios:

\$25 : \$5; 8 men : 24 men; 32 cents : 2 cents; 100 pounds : 25 pounds; 68 acres : 17 acres; 19 tons : 76 tons; \$12 : \$108; 78 miles : 26 miles; 91 gallons : 13 gallons; 90 feet : 18 feet.

MENTAL PROBLEMS IN RATIONAL DIVISION.

At 15 cents each, how many pineapples may be bought for 75 cents?

ANALYSIS.—As many pineapples may be bought as 15 cents is contained times in 75 cents, or 5; hence, 5 pineapples may be bought.

Analyze the following:

1. If there are 8 pints in one gallon, how many gallons in 56 pints?
2. How many weeks are there in 84 days?
3. At \$3 a case, how many cases of eggs can be bought for \$81?
4. If a train is running at the rate of 18 miles an hour, in what time will it run 72 miles?

5. Chester wishes to invest \$96 in chickens at \$3 a dozen; how many dozen can he buy?

6. At 30 cents each, how many music books can be bought for \$6.30?

7. Fred planted 85 hills of potatoes. If there are 17 hills in each row, there were how many rows?

8. If 40 arithmetics may be packed in a box of given size, how many similar boxes will be required for an edition of 6000 books?

9. How many feet in 96 inches?

10. If 20 tons of coal may be loaded on one car how many cars will be required for 780 tons?

11. Of a field containing 80 acres Henry plowed 16 acres; what part of the field did he plow?

ANALYSIS.—Since 16 acres is contained in 80 acres 5 times, he plowed one-fifth of the field.

12. Henry had \$84 and has spent \$14; what part of the whole amount has he spent?

13. 8 days is what part of 8 weeks?

14. From a bin of wheat containing 560 bushels 140 bushels were sold; what part of the wheat was sold?

15. Arthur has 96 cents; how many pencils can he buy at 4 cents each? After he has purchased 6 pencils, what part of the remainder of his money will be required to purchase two more?

PARTATIVE DIVISION.

When a number or quantity is divided into two equal parts, the parts are called halves. If it is divided into three equal parts, the parts are called thirds; if in four equal parts, fourths, and so on.

The division of a number into equal parts is expressed by writing it above a horizontal line and the number by which it is divided below the line. Thus $\frac{12}{4}$ indicates that 12 is to be divided into four equal parts. The expression is read, "12 fourths," and its value is 3.

MENTAL EXERCISE.

Give the value of the following:

$\frac{21}{7}$; $\frac{42}{6}$; $\frac{51}{17}$; $\frac{24}{6}$; $\frac{50}{10}$; $\frac{63}{9}$; $\frac{26}{16}$; $\frac{72}{12}$; $\frac{85}{17}$; $\frac{65}{13}$; $\frac{64}{16}$; $\frac{81}{9}$; $\frac{76}{4}$; $\frac{91}{7}$.

What is one-fifth of 90? One-twelfth of 72? One-ninth of 81? One-fifteenth of 60? One-thirteenth of 78? One-twenty-ninth of 58? One-nineteenth of 95? One-seventh of 84?

MENTAL PROBLEMS IN PARTATIVE DIVISION.

Frank sold 6 bushels of apples for \$9; what was the price per bushel?

ANALYSIS.—Each bushel was sold for one-sixth of \$9, or \$1.50.

Analyze the following:

1. Henry earned \$45 in 15 days; what did he earn in a day?
2. A train ran 87 miles in 3 hours; what was the rate of speed?
3. Fred planted 85 hills of potatoes. If there were 5 rows, there were how many hills in each row?
4. If seven hogs weighed 840 pounds, what was their average weight?
5. Carl raised 150 bushels of potatoes on a six-acre lot; what was the yield per acre?
6. A carload of coal containing 13 tons was sold for \$52; what was the price per ton?
7. If there are 18 rows in an orchard containing 720 trees, there are how many trees in each row?
8. A company of men packed 750 tons of ice in 15 days; what was the average quantity packed per day?
9. \$108 was paid for a cement walk 2 yards wide and 60 yards long; what was the price per square yard?
10. To fence a lot 20 rods long and 15 rods wide, costs \$84; what was the cost of the fence per rod?

DIVISION IN GENERAL.

The **Dividend** is the number to be divided.

The **Divisor** is the number by which we divide.

The **Quotient** is the number which expresses the number of times the Divisor is contained in the Dividend, or the value of the equal parts into which the Dividend is divided.

The **Remainder** is the undivided portion of the Dividend.

EXPLANATION.—When we divide 47 by 8, 47 is the *dividend*, 8 is the *divisor*, 5 is the *quotient*, and 7 is the *remainder*.

METHODS OF INDICATING DIVISION.

The General Sign of Division (\div) is a combination of the colon (:), which indicates Rational Division, and the horizontal line ($—$), which indicates Partative Division.

When written between two numbers the General Sign indicates that the number to the left of the sign is to be divided by the number to the right. Thus, $24 \div 6 = 4$.

The General Sign of Division may be used to indicate either Rational or Partative Division; or the general process of division irrespective of its application.

Division is also indicated by writing the divisor to the left of the dividend with a curved line between them, the quotient being written below the dividend, thus:

$$\begin{array}{r} 7 \overline{)21} \\ 3 \end{array}$$

Or in Long Division, to the right of the dividend, thus:

$$\begin{array}{r} 7 \overline{)21} 3 \\ 21 \\ \hline \end{array}$$

NOTE.—In Algebra, and in some works on arithmetic, the divisor is written to the *right* of the dividend, and the quotient beneath the divisor, thus: $\begin{array}{r} 21 \overline{)7} \\ 21 \overline{)3} \end{array}$

PRINCIPLES.

1. A ratio can exist only between like numbers.
2. In Rational Division, the quotient is an abstract number.
3. In Partative Division, the divisor is an abstract number, and the quotient is of the same kind as the dividend.
4. In any division, the dividend is equal to the product of the divisor and quotient, plus the remainder, if there be one.

RELATION OF DIVIDEND, DIVISOR, AND QUOTIENT.

A change in the dividend alone produces a like change in the quotient; a change in the divisor alone produces an opposite change in the quotient; the same change in both dividend and divisor does not affect the value of the quotient.

EXPLANATION.—If the dividend alone be multiplied or divided, the quotient will be correspondingly multiplied or divided; but if the divisor alone be multiplied, the quotient is divided; or, if the divisor be divided, the quotient is multiplied. If both dividend and divisor are either multiplied or divided by the *same number*, the quotient remains the same.

The foregoing law has many important applications in arithmetic, and should be thoroughly understood.

DIVISION PROCESSES.

There are two forms of the division process, called, respectively, *Long Division* and *Short Division*.

Long Division is the full, or complete, form of the division, all the steps or processes of the division being recorded.

Example.

$$25521 \div 7 = ?$$

7)25521(3645

21

45

42

32

28

41

85

6

EXPLANATION.—Taking enough of the left hand digits of the dividend to contain the divisor, we find that 25 (thousands) contains 7, 3 (thousands) times. 3 (thousands) times 7 gives 21 (thousands), which, subtracted from 25 (thousands), leaves 4 (thousands). Annexing the next figure and dividing, we find that 7 is contained in 45 (hundreds) 6 (hundreds) times. Multiplying and subtracting as before, we have left the remainder, 3 (hundreds). Bringing down the next figure of the dividend, we have 32 (tens), which, divided by 7, gives us 4 (tens). Multiplying, subtracting, and bringing down the next figure, as before, we have 41, which, divided by 7, equals 5. Again multiplying and subtracting, we have a final remainder of 6.

Short Division is merely an abbreviated form in which the various multiplications and subtractions are performed mentally, thus:

$$\begin{array}{r} 7 \overline{)25521} \\ 3645 + 6 \text{ rem.} \end{array}$$

EXPLANATION.—Ascertaining mentally that 7 is contained in 25, 3 times, with 4 remaining, we write the 3 as the first figure of the quotient, and *mentally prefix* the 4 to the next figure, which gives 45 as the dividend in the next step, and so on.

Short Division can be employed only when the divisor is a number small enough for us to perform the multiplications and subtractions of the process mentally. The student should never employ Long Division unless the divisor exceeds 12.

MENTAL EXERCISE.

Give the results at sight of the divisions here indicated. Read the results from top to bottom, and then from left to right:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2)26	2)52	2)72	4)84	13)26	13)39	31)62	15)75
3)45	2)34	5)75	7)91	14)42	13)78	14)70	15)90
2)38	3)54	2)74	5)85	29)58	17)68	24)72	31)93
3)39	2)56	2)78	3)87	32)64	23)69	29)87	19)95
2)54	4)60	4)76	6)84	18)36	18)90	27)81	14)98
2)28	3)57	5)85	5)90	17)51	18)54	16)64	47)94
3)42	5)65	3)81	2)92	13)65	19)57	6)90	16)80
2)46	2)64	2)82	3)93	17)34	18)72	23)92	22)88
4)52	4)68	2)84	2)94	16)48	26)52	14)42	36)72
2)32	3)69	4)72	4)96	14)56	21)63	13)91	32)96
2)58	3)63	5)90	6)96	19)38	17)85	26)78	39)78
3)51	5)70	6)78	7)98	21)84	15)60	24)96	13)52

Give the quotients and remainders of the following:

(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
7)24	8)44	8)41	6)71	7)55	26)53	13)5	15)100
5)36	7)62	7)60	12)59	12)82	15)76	14)79	21)100
8)42	6)43	6)51	11)60	19)97	13)68	18)91	12)1000
9)56	5)52	12)62	9)89	3)53	14)58	14)100	18)100
5)38	9)49	8)20	6)70	18)59	24)100	16)100	17)100

WRITTEN PRACTICE.

1. Divide 1978 by 7.
2. Divide 8976 by 6.
3. Divide 10274 by 8.
4. Divide 7861247 by 4.
5. Divide 20761201 by 5.
6. Divide 12172946 by 9.
7. Divide \$8673.57 by 3.
8. Divide \$407232.45 by 9.
9. Divide 9706421 by 11.
10. Divide 201634596 by 12.
11. $21786 \div 19 = ?$
12. $87463 \div 28 = ?$
13. $10271086 \div 146 = ?$
14. $94207658 \div 67 = ?$
15. $560217563 \div 496 = ?$
16. $85205617 \div 649 = ?$
17. $417601924 \div 4567 = ?$
18. $93681596 \div 837 = ?$
19. $170215862 \div 299 = ?$
20. $37021675 \div 7019 = ?$

CONTRACTIONS IN DIVISION.

CASE I.—Where the Divisor is 10, 100, 1000, etc.

$56743 \div 1000 = 56.743$. Where the divisor consists of the digit, 1, with ciphers annexed, the result is obtained by merely pointing off from the left of the dividend as many figures as there are ciphers in the divisor. The figures thus pointed off comprise the remainder; the quotient consists of the remaining figures. Thus, in the accompanying example, the quotient is 56, and the remainder is 743.

MENTAL EXERCISE.

Give at sight the quotients and remainders for the following divisions:

1. $124763 \div 100$.
2. $96538 \div 10$.
3. $2016043 \div 1000$.
4. $4873620 \div 10000$.
5. $1862746 \div 1000$.
6. $9021638 \div 100000$.
7. $4267358602 \div 1,000,000$.

CASE II.—Where the Divisor consists of other digits than 1 with Ciphers at the Right.

$$246713 \div 1700 = ?$$

$$17 \overline{)00}246713(145$$

$$\begin{array}{r} 17 \\ \underline{76} \\ 68 \\ \underline{87} \\ 85 \\ \underline{213} \end{array}$$

Cut off the ciphers, of the divisor, and from the right of the dividend, an equal number of figures. Divide the remaining figures of the dividend by the divisor, exclusive of its ciphers. To the first remainder, if there be one, annex the figures cut off from the right of the dividend, for a complete remainder.

WRITTEN EXERCISE.

21. $24678 \div 1400 = ?$
22. $18276201 \div 9000 = ?$
23. $74216730 \div 870000 = ?$
24. $8726013 \div 52000 = ?$
25. $674120167 \div 24600 = ?$

PROBLEMS FOR MENTAL ANALYSIS.

NOTE—In analyzing these problems, the student should carefully distinguish between those involving Rational, and those involving Partative Division.

1. A train moving at the rate of 17 miles per hour will require how long to run 85 miles?
2. A lady received 3 pounds of coffee for 7 doz. eggs @ 9c. What was the coffee worth per pound?
3. Four men built 60 rods of fence in 3 days. What was the average number of rods per day for each man?
4. If cranberries are worth 15c. a quart, how many quarts should be given in exchange for 5 pounds of butter @ 18c.?
5. A torpedo boat running at the rate of 24 knots an hour, will steam how far in 15 minutes?
6. A grocer sells lemons at 20c. a dozen and oranges at 30c. At these prices, how many oranges are worth the same as 36 lemons?
7. Henry and his 5 brothers share equally a legacy of \$18000. Henry invests his share in land at \$15 an acre. How many acres does he purchase?
8. Five men united in a speculation, buying 5000 bushels of grain @ 65c. and selling it at 75c. If the expenses were \$100 and they shared the profits equally, what sum did each gain?
9. In a certain town the water tax is 13 cents per thousand

gallons, with 15 cents per quarter for meter rent. If Mr. A's quarterly water bill is \$2.75, how much water was used?

10. Five heirs inherit an estate valued at \$60,000, which they are to share equally. Afterwards another heir is found; to what amount will this discovery diminish the share of each of the other five?

WRITTEN PROBLEMS.

26. It requires about 8 minutes for light to pass from the sun to the earth, a distance of 92,500,000 miles. What is the approximate velocity of light per minute? What is the velocity per second?

27. During 18 days in the month of May, the total attendance at the Fairview school amounted to 576 days. What was the average daily attendance? If there are 36 pupils enrolled, what was the average number of days each pupil attended during the month?

28. A tract of land containing 2529 acres was sold for \$42360.75; what was the price per acre?

29. By a drop of 7 cents in the market price of wheat, a speculator lost \$1330; how many bushels did he buy?

30. The product of two numbers is 40548. One of the numbers is 327; what is the other?

31. Mr Howland sold his spring wool clip at 17 cents a pound, receiving \$400.86. How many pounds did he sell?

32. A steam pump with a capacity of 575 gallons a minute, will require how long to fill a tank holding 32775 gallons.

33. By a rise of 25 cents per bushel in the market price, the value of a farmer's potato crop is increased \$198.75. How many bushels did he raise?

34. If a cruiser can sustain an average speed of 19 knots an hour, how many days and hours will she require to cover a distance of 5662 knots?

35. Mr. L. finds that he spends for tobacco an average of \$2.75 per month. In how many years and months would this expenditure amount to enough to purchase a \$500 piano?

COMBINATIONS OF ARITHMETICAL PROCESSES.

It is often desirable to express in one formula, or statement, several arithmetical operations; also to indicate the order in which these operations are to be performed.

This is accomplished by means of the four arithmetical signs $+$, $-$, \times , and \div , used in connection with the comma, the parenthesis (), and the vinculum .

Thus, the expression $6+7, \times 9$ indicates that 6 and 7 are to be added, and the sum multiplied by 9, the result being 117; while the expression $6, +7 \times 9$ indicates that the product of 7×9 , or 63, is to be added to 6, the result being 69.

The parenthesis is used to inclose one or more indicated operations, the result of which is to be considered as one quantity. Thus, $5+(12 \div 3)+8-(2 \times 3)$ indicates that the quotient of $12 \div 3$, or 4, is to be added to 5, that 8 is to be added to this, and that from this result, 17, the product of 2×3 , or 6, is to be subtracted, giving the result, 11.

The vinculum may be used instead of, or in connection with, the parenthesis. Thus, $27-\overline{24 \div (12-8)}$ indicates that the whole quantity, 24 divided by the quantity $12-8$, is to be subtracted from 27.

NOTE.—Division may also be conveniently indicated by writing the divisor under the dividend with a horizontal line between them. Thus the last formula may be written:

$$27-\frac{24}{12-8}$$

MENTAL EXERCISE.

Verify the following:

1. $(17-8) \div 3 = 3.$

2. $\overline{51 \div 17} + 9, -(2 \times 4) = 4.$

3. $34 \div (15 + \overline{(3 \times 2) \div 3}) = 2.$

4. $\frac{(75 \div 15) + 9}{2} = 7.$

5. $(98-7) \div (\overline{8-4} \times \overline{28-9}) = 7.$

Find the results of the following:

1. $54 \div (12 + \overline{78 \div 13}) + 15 - (8-4) = ?$

2. $\frac{(25-6) \times (52 \div 13)}{(3 \times 13) - 37} = ?$

$$3. (7+8+9, \div 6) \times (56 \div 19-5) \times 6 = ?$$

$$4. \frac{48}{17+7} \times (\overline{52 \div 2}, -(4+8, \times 2)) = ?$$

$$5. ((3 \times 12) - 24) \times (76 \div 19) + 15 = ?$$

WRITTEN EXERCISE.

Mr. A. sold some cattle for \$950 and some sheep for \$375. With a part of the money he bought 3 cows at \$35 each and with the remainder bought 8 horses. What was the average cost of the horses?

FORMULA FOR SOLUTION.

$$(\$950 + \$375) - (3 \times \$35) \div 8 = \text{Average cost of horses.}$$

In a like manner indicate the formulas for the solution of the following:

1. A man bought a house and lot for \$1500, expended \$250 for repairs, and then sold the property for \$2000, investing his gains in corn at 25c. a bushel. How many bushels did he buy?

2. A certain tank holds 40,000 gallons. It has a supply pipe that brings in 150 gallons per minute, and a discharge pipe that carries away 175 gallons per minute. If the tank is full, and both pipes are left open, how many hours will be required to empty the tank?

3. A man bought a farm for \$5000, 60 head of cattle at \$25 each, and 15 horses at \$75 each. He then sold the farm and stock for \$8000. What sum did he gain?

4. If there are 160 square rods in an acre, what is the value at \$25 an acre of a field 96 rods long and 70 rods wide?

5. Mr. Markham earns \$2.75 per day and works an average of 5 days each week. His weekly expenses are as follows: Board, \$3.75; laundry, 50c.; clothing, \$2; other expenses, \$1. The remainder of his earnings he deposits in bank. What sum should he have to his credit at the end of one year, or 52 weeks?

6. Mr. L. sold 965 bushels of wheat at 75 cents, receiving in payment 5 milch cows, a town lot valued at \$125, 140 stock hogs worth \$2.50 each, and cash \$26. What was the average cost of the cows?

7. Ellerton is one of five heirs who inherits an estate con-

sisting of 640 acres of land valued at \$45 per acre. If Ellerton has other property valued at \$2250, cash \$500, and owes debts to the amount of \$3000, what is his present worth after his debts are paid?

8. A residence property rents for \$35 per month. The annual taxes average \$75 and the repairs \$50. The owner pays the water rate of \$1.75 per month; what is the net income from the property during a period of five years?

9. A committee has a certain sum with which to purchase a carpet for a lodge room. They find that if they purchase carpeting at \$1.10 per yard, they will lack \$15.60 of having enough money to pay for it, but if they purchase a grade at 95 cents per yard, they will have \$8.40 of their fund unexpended. How many yards are required?

10. Walter Bright bought a 30-acre wood lot at \$26 per acre. He paid \$275 for fencing the lot, sold from it 160 cords of wood at \$1.50 per cord, and then sold the lot for \$950. What did he gain by the speculation?

After writing the formulas required by the foregoing problems find the result of each.

PROMISCUOUS WRITTEN PROBLEMS.

1. A mill owner sold 260,000 shingles at \$2.55 a thousand and with the money bought 17 acres of land. What did the land cost an acre?

2. A man paid \$80.75 for a cement walk 5 feet wide and 153 feet long. If there are 9 square feet in a square yard, what did the walk cost per square yard?

3. An orchard contains 26 rows of trees, with 42 trees in each row. The apple crop from the orchard was sold for \$4095. What was the value of the yield per tree?

4. A real estate dealer sold a tract of 278 acres of land for \$12510, receiving \$13 per acre more than it cost him. What did he pay per acre for the tract?

5. A farmer sold his wheat crop at 85c. per bushel, and with the money bought 124 stock hogs at \$4.25 each. How many bushels of wheat did he sell?

6. A retailer's sales for one week were as follows: Mon-

day, \$356.90; Tuesday, \$410.50; Wednesday, \$387.45; Thursday, \$274.15; Friday, \$334.93; Saturday, \$512.29. What was the amount of the average daily sales?

7. Lillian finds that if she buys a certain number of yards of dress goods at \$1.25 a yard, she will have \$1.50 remaining. But if she buys the same number of yards of a quality that costs \$1.44 a yard, she will lack 78 cents of having enough money to pay for it. How much money has she?

8. A locomotive runs at the rate of 4 rods per second. If there are 320 rods in a mile, how many miles and rods will the locomotive run in an hour.

9. A man bought 16480 pounds of wool @ 19c., and sold it @ 26c. He then invested the gains of this speculation in pork at 8c. a pound, which he afterwards sold at 11c. a pound. What did he gain on the pork?

10. Walter's expense account while attending Marion College from Jan. 1 to June 1, 1900, was as follows: Board and room, \$75.50; tuition and books, \$62.85; clothing and laundry, \$41.15; incidentals, \$7.95. What were his average monthly expenses?

11. The items of cost for an edition of 6000 books are as follows: paper, \$318.25; printing and binding, \$1345.80; packing, \$15.95. If the books are sold at 50c. each, what is the gain on each book?

12. A speculator bought a piece of city property for \$2650 sold the property at an advance of \$206 over the purchase price, and invested the money in wheat at 68c. a bushel. He afterward sold the wheat at 74c. a bushel, paying commission and other charges to the amount of \$62.75. How much was his original capital increased?

13. A buyer for a department store is allowed a certain sum with which to purchase bicycles. He bought 55 bicycles at \$27 each, and had \$15 remaining. Supposing he had bought bicycles at \$32 each, how many could he have purchased, and what sum would remain unexpended?

14. Benton & Sons sold 528 bbls. of flour at \$3.75 a barrel, and invested the proceeds in pickled pork at \$17 a barrel. How

many barrels of pork did they buy, and what sum remained unexpended.

15. A contractor engages to build a house for \$2875, provided he uses lumber costing \$18.50 per thousand feet; but if lumber worth \$20 per thousand is used, he will ask \$2945.50. If the cheaper grade is selected, what will be the amount of the lumber bill?

16. On a field of wheat containing 75 acres, a farmer harvested 1397 bushels; on a second field of 64 acres, he harvested 1123 bushels; and on a third field of 57 acres, he harvested 1008 bushels. What was the average yield per acre for the whole crop?

17. Five men bought a paper mill for \$47500. They operated the mill for one year at a total outlay of \$48560.75. During the time they sold paper to the amount of \$69744.25. They then sold the mill for \$50000. What was the amount of each man's share of the gain?

18. A cyclist rode 4 hours and 20 minutes at the rate of 15 miles an hour. He then rode back over the same road in just 5 hours. At what average rate per hour did he ride on his return trip.

19. Mrs. B finds that if she selects for a new gown material that costs \$1.25 per yard, the whole cost of the garment will be \$27.85; but if she selects material worth \$1.48, the cost will be \$30.61. What is the cost of the gown aside from the cloth?

20. By the terms of Mr. Beverly's will, his estate, which is valued at \$375000, is to be divided as follows: his wife is to receive \$25,000; the eldest son, \$20,000; each of the three remaining children, \$10,000. After paying these and the other special bequests to the amount of \$5750, the remainder of the estate is to be divided equally among the five heirs. What is the value of each one's interest in the undivided portion of the estate?

Progressive Commercial Arithmetic.

PART II.

PREFACE TO PART II

A school text book should be brief, small in volume, but so arranged as to call forth great mental activity on the part of the student.

A text book in Arithmetic is calculated to develop the mental faculties, and prepare the student to comprehend and solve all problems that may arise in business intercourse. No young person can hope to be able to do this by merely learning the rules, working the examples after the models given, and proving the work correct by the printed answer. He should be independent of rules; the principles should be so firmly established in his mind that there will be no need of rules, nor of reference to the model forms of solving problems.

A book may be plainly written, its problems and exercises may be well selected; and yet, if the student meets the greater part of his work labeled with topic, case, section, and rule, he will not develop his reasoning faculties to any great extent, but will degenerate into a mere machine. Examples in business do not come to us numbered, and with them a corresponding list of tested answers. The man of business must work his examples, and he must be *certain of their accuracy*, he must prove them for himself.

Our experience warrants us in advising students to devote little time to the learning of rules in arithmetic. Master principles and you can formulate your own rules. Rules committed to memory may be forgotten, but principles thoroughly mastered never are forgotten.

It is our earnest desire that the student who completes a course in the Progressive Commercial Arithmetic may realize mental development, vigorous thought, and accurate results.

PLAN OF PART II

We assume at the outset, that those who study this book have had at least rudimentary drill in the fundamental operations of arithmetic, that they can write and read numbers without difficulty, and that they can add, subtract, multiply, and divide with ordinary accuracy.

Short methods have been introduced wherever their use would be likely to prove a saving of time to the accountant.

The authors realize the great loss to students in recent years, because of the absence of mental arithmetic from the course of study, and they have made provision for extended intellectual drill in every process introduced in the new text. By this mental process, the student is led to the development of his own method and rule, without memorizing those given in the text.

Mental drill has been provided for in every topic treated, and it is hoped that these exercises may be dwelt upon by the student, until he experiences rapidity of thought, correctness of reasoning or analysis, and accuracy of result.

It has been the aim to avoid all arithmetical puzzles and catch questions, and to group together such problems as the business man meets in his practical duties.

A large variety of practical methods of estimating measurements are given in connection with denominate numbers, in preparing which the authors have consulted mechanics, tradesmen, and business men freely, in order to get at the practical usages of the present time. The problems are grouped together after each general topic, instead of sorting and grouping them under their respective cases.

The percentage, interest, and discount methods have been treated with great thoroughness, because they enter so largely into modern business problems. Very little theorizing has been given by the authors, so that there might be room and time for a great variety of mental and written problems relating to every process that will be required in practical life.

The rules and explanations are not given, when it is thought that the average student can prepare them for himself. The plan of the work is inductive; its aim is to lead the student to the mastery of principles, instead of the memorizing of rules and processes.

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THE FUNDAMENTAL PROCESSES.

It is assumed by the authors that the student knows how to read and write numbers, and to add, subtract, multiply, and divide. Skill in these fundamental processes can only be acquired by daily practice, just as the musician depends on running the scales to give him skill in playing or singing.

Classes should have daily exercise in these processes until accuracy and rapidity have been acquired.

The greater proportion of all business calculations involve addition, and to be rapid and accurate in adding columns of figures is of more importance to the accountant than all the other operations of arithmetic combined.

To be successful in any mental process requires systematic action of the mind; to be expert in addition requires system; and it is our present object to offer a few suggestions that may aid the student in developing this quality.

Care should be exercised in writing figures plainly, so that the accountant may not be required to halt in his addition to decipher an obscure figure. The figures most frequently mistaken for each other, are 3 for 5, 7 for 1, 9 for 7, and 6 for 0.

Figures not only represent an absolute value in themselves, but they also have a relative value depending upon their position in columns. The student should form the habit of writing his figures in columns, so that nothing may hinder him in the rapid combination of numbers in a column.

There are various expedients suggested by commercial teachers for developing rapidity in addition of columns. We will only present a few of these, leaving the teacher to introduce other drills that may seem necessary.

Rapid addition of a column of figures is merely the rapid combination of these figures into amounts, the same as we combine letters into words when we read. In learning to read, the student spells out each word, letter by letter, and his progress in reading is slow; but continued practice enables him to combine letters into words at a glance, and to group words into sentences as rapidly as he can call them out.

The same is true of reading figures in a column. The student may be trained to combine figures into amounts, as rapidly as his eye can run up or down the column. To acquire this speed, the student should have daily drill in adding columns of figures. This is best accomplished by means of a class drill for fifteen minutes or more each day.

BLACKBOARD DRILLS.

A series of numbers should be called out by the teacher, as he writes them in columns on the blackboard. Each student should have pencil and tablet for this exercise, and should be required to write down the numbers as they are called by the teacher. Each student should add the columns on his tablet as rapidly as possible, and announce the result when called upon by his teacher.

The class should then be required to add the columns in concert, adding just rapidly enough to spur up the slowest pupil to his best possible speed.

The columns should be added from bottom to top, and afterward, as proof of the work, in reverse order. Adding a column twice in the same direction with the same result is not a proof of its correctness, because the mind is more liable to make mistakes in the same combination of figures, than it would be if the order were reversed and new combinations formed.

The number to be carried from one column to another, should be added at the beginning of the column, in order that the mind may be relieved from any further thought concerning it.

Inasmuch as the accountant is liable to interruption before completing his addition, it is better to write the total of each column on a separate paper, so that he may resume the adding at any time and prove any column he desires.

Among the various methods used for retaining the carrying figure, we prefer the following:

OPERATION.

3193847 36
693428 34
5143952 37
8188437 28
255263 44
4342135 27
931684 22

EXPLANATION.—The first number, 36, is the footing of the first column, giving us 6 to write under unit's column, and 3 to carry to ten's column. 34 is the total of the second column, including the 3 tens carried from units. After the totals of all the columns are found and proven, we can place the result under the columns added, using the last amount 22 entire, and the right hand figures of the other amounts.

22748746

In adding columns we should group the numbers mentally, and merely announce results.

To illustrate, in the example just given, the first column should be added, calling the results at once; 9, 12, 19, 21, 29, 36, instead of thinking in the roundabout method, 4 and 5 are

9, and 3 are 12, and 7 are 19, and 2 are 21, and 8 are 29, and 7 are 36.

After the student has added the above example on his tablet, and the class has added the example on the board in concert, up and down the columns, the teacher can use the same example for a number of very profitable concert drills in all of the fundamental processes, in all cases insisting upon quick responses in order to stimulate every student to rapidity in his calculations. As a drill in combining two or more numbers at a glance, the teacher should take the figures in the two right hand columns and require the students to call their sum. In the first two columns the students would call out as the several amounts 12, 8, 9, 10, 7, 10, 11. The second and third columns can be treated in the same manner, and thus other columns can be used, until every student is able to combine two numbers at a glance. Then the figures in three columns may be used, and the student be required to combine three numbers at a glance. Afterward four columns may be used, and finally the student may be led to combine the six or seven horizontal numbers as rapidly as they are pointed out.

For drill in subtraction, the two columns may be used, requiring the students to call out quickly the difference between two numbers as they are pointed out. In using the first two columns for drill in subtraction, the students would call out as remainders, 4, 2, 3, 4, 3, 6, 3. The product of each two may also be required as a rapid drill in multiplication. In a multiplication drill on the same columns, the students would call out as products, 32, 15, 18, 21, 10, 16, 28. The larger of the two numbers may be used for a dividend and the smaller for a divisor, and the resulting quotient and remainder may be required. In a division drill on the same columns, the student would call out quotient 2, quotient 1 and remainder 2, quotient 2, quotient 2 and remainder 1, quotient 2 and remainder 1, quotient 4, quotient 1 and remainder 3.

Such a drill as we have just described should have a regular place for a few minutes daily in the school program. The teacher can vary the exercise and introduce other calculations to break up the monotony.

COMBINATIONS.

$$\begin{array}{l} 2 \} 3 \} 4 \} \\ 3 \} 6 \} 2 \} 7 \\ \quad 5 \} 1 \} \end{array}$$

$$\begin{array}{l} 4 \} \\ 2 \} 7 \} 3 \} 9 \\ \quad 3 \} 6 \} \\ \quad 2 \} \end{array}$$

$$\begin{array}{l} 4 \} \\ 6 \} 5 \} \\ \quad 4 \} 10 \\ \quad 2 \} \end{array}$$

$$\begin{array}{l} 8 \} 9 \\ 3 \} 1 \} \end{array}$$

$$\begin{array}{l} 4 \} 6 \} \\ 6 \} 5 \} 2 \} 9 \\ \quad 2 \} 7 \} \end{array}$$

After the students have had training in combining several numbers at a glance, they can be required to group two or more figures occurring in a column, and add them as one figure. Such combinations will not increase the speed, if they are allowed to run into large numbers. In the first column given in the example, the student would add the groups as four figures, calling the results, 9, 19, 28, 35. These combination drills cannot be used very extensively in the concert adding, as different students would see different combinations in the columns.

ADDITION. SUBTRACTION.

$$\begin{array}{r} 34263 \\ 350270 \\ \hline 34263 \end{array}$$

$$\begin{array}{r} 52437 \\ 316007 \\ \hline 52437 \end{array}$$

$$\begin{array}{r} 47592 \\ 263570 \\ \hline 47592 \end{array}$$

$$\begin{array}{r} 34287 \\ 215978 \\ \hline 34287 \end{array}$$

$$\begin{array}{r} 59742 \\ 181691 \\ \hline 59742 \end{array}$$

$$\begin{array}{r} 37426 \\ 121949 \\ \hline 37426 \end{array}$$

$$\begin{array}{r} 84523 \\ 350270 \\ \hline 84523 \end{array}$$

For young students who are not accurate in addition and subtraction, we would suggest daily practice at the blackboard, or with pencil tablets, in adding columns of figures, and as a drill in subtraction and a proof of accuracy, subtracting in succession the numbers that were added. The student can construct a table of numbers for this drill, making as many columns with as many figures in a column as the teacher may require, omitting ciphers and 1's and 2's.

Another excellent drill, that should be used daily until students have acquired accuracy, is to add a series of numbers both horizontally and vertically. The sum of the totals in the horizontal adding should equal the sum of the totals in the vertical adding.

$$\begin{array}{r} 324 + 625 + 463 + 527 = 1939 \\ 462 + 375 + 836 + 472 = 2145 \\ 379 + 643 + 728 + 645 = 2395 \\ 643 + 372 + 643 + 528 = 2186 \\ 584 + 642 + 936 + 364 = 2526 \\ \hline 2392 + 2657 + 3606 + 2536 = 11191 \end{array}$$

As a drill in writing numbers in columns, the teacher should call off a series of amounts in dollars and cents, requiring each student to record them accurately and rapidly. The student should be cautioned against reversing figures, which he is likely to do in rapid writing. The decimal points separating dollars from cents should always be in line. The number of figures in each amount should vary, as in the following examples:

\$129.54	\$1237.52	\$273 22	\$317.29
375.16	5675.29	76.53	6.34
92.31	378.87	9.29	29.17
14 56	419.29	.37	186.93
7.85	75.34	.05	4.52
23.78	9.23	9.34	.99
<u>156.15</u>	<u>.67</u>	<u>87.55</u>	<u>7.55</u>

MENTAL DRILLS.

It affords excellent drill in addition and subtraction to require the students to count by 2's or 3's, or any given number, up to 100, and to reverse by subtracting some other number.

EXAMPLES.

1. Start with 1 and add by 2's up to 99, and reverse by subtracting 3's.
2. Start with 2 and add by 3's up to 98, and reverse by subtracting 4's.
3. Use 1, 2, 3, and 4 each as starting figures, and add by 4's to the number nearest 100, and reverse by subtracting 5's.
4. Use 1, 2, 3, 4, and 5 each as starting figures, and add by 5's and reverse by subtracting 6's.

The teacher can readily construct any number of examples affording excellent drill in these combinations.

DRILL IN COMPLEMENTS.

The complement of any number is the difference between that number and the next denomination; as, the complement of 7 is 7 from 10, or 3; the complement of 4 is 6; of 24 is 76; of 43 is 57; of \$2.43 is \$7.57. It illustrates the work of a cashier in making change; as, 84 cents, the cost of a bill of goods, from \$1 leaves 16 cents.

Give the complements of the following:

25 (75)	28	26	\$7.26	\$5.15
65 (35)	68	66	7.66	3.10
5 (95)	8	6	1.06	4.52
85	88	86	6.85	7.56
35	38	36	2.25	8.25
45	48	46	3.50	9.95
75	78	76	6.30	4.16
15	18	16	2.99	7.29
55	58	56	3.84	5.24

Another form of mental drill that may be used with good effect, is for the teacher to call out several amounts in a bill of items, and require the amount of change to be returned from a bank bill or coin; as, the items in a bill were 25, 15, and 37 cents; how much change should be given back from \$1?

An excellent mental drill in both multiplication and addition, is to require the student to multiply by two or three numbers and combine the partial products mentally, writing only the complete product. This process is sometimes called Cross Multiplication, and although the student may at first do more rapid and accurate work by the ordinary method, yet this cross multiplication is so valuable in strengthening the memory, that we would advise daily drill in it until the student becomes skillful in its use.

EXAMPLE.

Multiply 37 by 45.

SOLUTION

$$\begin{array}{r} 37 \\ 45 \\ \hline 1665 \end{array}$$

EXPLANATION.—5 times 7 are 35 units. Write the 5 units and carry the 3 tens. 15 tens (5×3 tens) plus 3 tens plus 28 tens (4 tens ×7) are 46 tens. Write the 6 tens and carry the 4 hundreds. 12 hundreds (4 tens ×3

tens) plus 4 hundreds are 16 hundreds, which should be written in full.

This method is of great practical value in billing, as it enables the billing clerk to extend the amounts at once, without writing the partial products on a separate paper.

EXAMPLES IN CROSS MULTIPLICATION.

24×36=?	22×13=?	22×32=?
62×15=?	15×24=?	38×19=?
21×32=?	28×16=?	36×34=?
25×17=?	54×31=?	22×18=?
26×34=?	85×33=?	42×71=?
21×31=?	17×24=?	38×21=?

DIVISIBILITY OF NUMBERS.

1. Any even number is divisible by 2. Thus, 2, 4, 6, 8, 10, etc., are divisible by 2.

2. Any number is divisible by 3, if the sum of its digits is divisible by 3. Thus, 3372 is divisible by 3, because $3+3+7+2=15$ is divisible by 3.

3. Any number is divisible by 4, whose last two figures is divisible by 4. Thus, 1124 and 1512 are divisible by 4, because 24 and 12 are each divisible by 4.

4. Any number is divisible by 5, whose last figure is 5 or 0. Thus 1275 and 3940 are divisible by 5.

5. Any even number is divisible by 6, if the sum of its digits is divisible by 6.

6. Any number is divisible by 8, if its last three figures is divisible by 8. Thus, 9824 and 53120 are divisible by 8, because 824 and 120 are divisible by 8.

7. Any number is divisible by 9, if the sum of its digits is divisible by 9. Thus, 41203404 is divisible by 9, because $4+1+2+0+3+4+0+4=18$ is divisible by 9.

8. Any number is divisible by 10, if its last figure is 0. Thus 980, 150, and 1650 are divisible by 10.

CANCELLATION.

1. **Cancellation** is the process of rejecting factors from numbers sustaining the relationship of dividend and divisor.

EXAMPLE.—What is the quotient of $25 \times 6 \times 8 \times 9 \times 27 \times 5$ divided by $4 \times 3 \times 3 \times 5 \times 4 \times 2 \times 9 \times 6$?

SOLUTION.

EXPLANATION.—

$$\frac{25 \times \overset{2}{\cancel{6}} \times \overset{3}{\cancel{8}} \times \overset{3}{\cancel{9}} \times \overset{3}{\cancel{27}} \times 5}{4 \times \overset{2}{\cancel{3}} \times \overset{3}{\cancel{3}} \times \overset{3}{\cancel{5}} \times 4 \times \overset{2}{\cancel{2}} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{6}}} = \frac{75}{4}, \text{ or } 18\frac{3}{4}$$

Place the dividend above and the divisor below a horizontal line; cancel all factors common to both dividend and divisor. The product of the numbers above will be the new dividend, and the product of the numbers below

will be the new divisor. If the dividend is larger than the divisor, divide the dividend by the divisor, and the remainder, if any, may be written above the divisor as on preceding page.

WRITTEN PRACTICE.

1. What is the quotient of $24 \times 36 \times 6 \times 7 \times 5$ divided by $18 \times 4 \times 8 \times 14$?

2. Divide $72 \times 210 \times 95 \times 60 \times 42$ by $21 \times 19 \times 12 \times 10 \times 15 \times 7$.

3. Divide the product of $84 \times 1728 \times 48 \times 75$ by the product of $144 \times 21 \times 15 \times 36$.

4. $(16 \times 15 \times 17 \times 36) \div (8 \times 5 \times 24 \times 6) = ?$

5. A farmer brought to market 15 turkeys averaging 12 pounds each, which he sold at 9 cents per pound. He took his pay in sugar at 4 cents per pound. How many pounds did he get?

6. A man sold 32 bushels of oats at 33 cents per bushel and took his pay in beef at 11 cents per pound. How many pounds did he receive?

7. How many bushels of potatoes, at 75 cents per bushel, can be had for 24 sacks of corn, each containing 2 bushels, at 50 cents per bushel?

8. How many pieces of cloth, each piece containing 48 yards, at 10 cents a yard, should be received for 2 loads of oats of 60 bushels each at 25 cents a bushel?

9. A farmer brought to market 10 loads of rye, averaging 16 bags of 2 bushels each. The merchant paid for the rye with 12 barrels of sugar of 225 pounds each at 5 cents per pound. What price per bushel was allowed for the rye?

FACTORING.

2. A **Factor** is a part of a number, one that repeated a certain number of times will produce a certain number. Thus, 4 and 5 are factors of 20; 2, 2, 2, 2, and 2 are factors of 32; 4 and 8 are factors of 32.

3. A **Prime Number** is one that cannot be separated into factors; it is a number divisible only by itself and unity. Thus, 2, 3, 5, 7, 11, 13, 17, and 19 are prime numbers.

4. A Composite Number is one that may be resolved into factors other than itself and unity. Thus, 6, 12, and 24 are composite numbers.

TABLE OF PRIME NUMBERS FROM 1 TO 1000.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

MENTAL PRACTICE.

1. Name all the even numbers from 9 to 50.
2. Name all the odd numbers from 9 to 50.
3. Name all the prime numbers from 9 to 50.
4. Name all the composite numbers from 9 to 50.

Name two composite factors that will produce each of the following numbers:

- | | | | |
|-------|-------|---------|--------|
| 5. 36 | 7. 81 | 9. 100 | 11. 84 |
| 6. 48 | 8. 72 | 10. 625 | 12. 96 |

Give the prime factors of the following:

- | | | | | |
|--------|----|----|----|-----|
| 13. 4 | 12 | 15 | 25 | 35 |
| 14. 9 | 18 | 16 | 27 | 125 |
| 15. 75 | 60 | 20 | 30 | 45 |

Give two factors, one prime and one composite, that will produce the following:

- | | | | |
|---------|--------|---------|---------|
| 16. 125 | 18. 27 | 20. 126 | 22. 320 |
| 17. 240 | 19. 45 | 21. 875 | 23. 154 |

WRITTEN PRACTICE.

When the numbers are large, use the following operation:
What are the prime factors of 3780?

SOLUTION.

$$\begin{array}{r}
 5 \overline{)3780} \\
 \underline{2)756} \\
 \underline{2)378} \\
 \underline{3)189} \\
 \underline{3)63} \\
 \underline{3)21} \\
 \underline{7)7}
 \end{array}$$

EXPLANATION.—Divide by any prime number that will divide the number; continue dividing by prime numbers until there is no remainder left. The several divisors are the prime factors required.

The product of these prime factors will produce the first dividend.

Give the prime factors of the following:

1. 1000	6. 2048	11. 3465	16. 3003
2. 775	7. 1675	12. 7007	17. 8927
3. 6425	8. 4410	13. 5005	18. 3840
4. 8375	9. 7335	14. 6006	19. 17017
5. 5625	10. 5740	15. 5346	20. 19019

COMMON DIVISORS.

5. A **Common Divisor** of two or more numbers is a number that is an exact divisor of each of them. Thus, 2 is a common divisor of all even numbers.

6. The **Greatest Common Divisor** of two or more numbers is the greatest number that is contained in each of them without a remainder. Thus, 4 is the greatest common divisor of 8, 12, 16, and 20.

PRINCIPLES.

1. The product of the prime factors common to all the given numbers is the greatest common divisor of those numbers.

2. A divisor of a number divides any number of times the number.

3. A common divisor of two numbers divides their difference.

4. A common divisor of **two or more** numbers divides their sum.

5. The greatest common divisor **cannot** be greater than the least number.

Therefore, if any factors of the smallest number are not found in the other numbers, they must be dropped in finding the greatest common divisor, and the product of the remaining factors will be the greatest common divisor.

EXAMPLES.

1. What is the greatest common divisor of 30, 45, 60, and 75?

SOLUTION.

$$30 = 3 \times 2 \times 5$$

$$45 = 3 \times 3 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$75 = 3 \times 5 \times 5$$

EXPLANATION.—According to Prin. 1, select those factors that are common to all the numbers. By inspection 3 and 5 are the only factors found that are common to all. Consequently, 3×5 , or 15, is the greatest common divisor of the numbers.

2. What is the greatest common divisor of 30, 45, 60, and 75?

THOUGHT METHOD.—The g. c. d. of these numbers cannot be greater than 30; if 30 is an exact divisor of the rest of the numbers it is the g. c. d. By inspection it is found not to be a divisor of all the numbers. Dropping the smallest factor, 2, out of 30, and using the other one, 15, as a divisor, we find by inspection that 15 is an exact divisor of all the numbers; therefore, it is the g. c. d. of 30, 45, 60, and 75.

When the numbers are large and factoring difficult, the following method is one in general use:

3. What is the greatest common divisor of 203 and 638?

SOLUTION.

$$\begin{array}{r} 638 \\ 203 \overline{) 3609} \\ 203 \overline{) 729} \end{array}$$

EXPLANATION.—Since 203 is the g. c. d. of itself, if it also divides 638 it is the g. c. d. of the numbers. By inspection 203 is found not to be an exact divisor of 638, but according to (Prin. 2) it divides any number of times itself; 3 is the greatest number of times that it can be found in 638, leaving a remainder of 29.

By (Prin. 3) it should divide 29; 29 is the g. c. d. of itself. By (Prin. 2) if it also divides 203, it is the g. c. d. of the numbers. By inspection it is found to be an exact divisor of 203 and must, therefore, be the g. c. d. of the numbers

WRITTEN PRACTICE.

Find the greatest common divisor of the following:

- | | |
|-------------------|-------------------|
| 1. 484 and 1980. | 6. 917 and 1703. |
| 2. 598 and 1416. | 7. 280 and 692. |
| 3. 540 and 584. | 8. 2002 and 3003. |
| 4. 2490 and 1800. | 9. 2200 and 3825. |
| 5. 780 and 2864. | 10. 1332 and 1887 |

When more than two numbers are given, find the g. c. d. of two of the numbers, then find the g. c. d. of the first g. c. d. and the next given number, etc.

- | | |
|------------------------|---------------------------------|
| 11. 75, 100, and 175. | 14. 423, 564, 705, and 846. |
| 12. 364, 400, and 324. | 15. 525, 600, 675, and 750. |
| 13. 417, 695, and 973. | 16. 1308, 1635, 1962, and 2289. |

17. A carpenter is directed to make the widest possible sidewalk, no wastage, out of the following planks each 1 foot in width and 2 inches in thickness: 9 planks 20 feet long, 8 planks 16 feet long, and 6 planks 12 feet long. What will be the length and width of the walk?

MULTIPLES.

7. A **Multiple** is an exact dividend of a number. Thus, 16 is a multiple of 8, 4, and 2.

8. A **Common Multiple** of two or more numbers is a dividend exactly divisible by each of them. Thus, 84 is a common multiple of 7 and 12.

9. The **Least Common Multiple** of two or more numbers is the least number exactly divisible by each of them. Thus, 24 is the least common multiple of 2, 3, 4, 6, 8, and 12.

PRINCIPLES.

1. Two or more numbers may have any number of common multiples.

2. Two or more numbers can have but one least common multiple.

3. The least common multiple cannot be less than the greatest of the numbers.

Therefore, the least common multiple must contain the largest number and all the prime factors of the other numbers not contained in the largest.

MENTAL PRACTICE.

1. What is the least common multiple of 5, 6, 10, 12, 18, and 24?

THOUGHT METHOD.—Since 24 is the l. c. m. of itself, if it is an exact dividend of the other numbers it is the l. c. m. of the numbers. By inspection it is found to be an exact dividend of but two numbers, 6 and 12. By comparison it is found that 5 is not an exact divisor of 24, and since it is a prime factor no part of it can be found in 24; consequently, 5 as a factor must be used, 5×24 is 120. The next number, 10, is found to be an exact divisor of 120. The last number, 18, is not an exact divisor; its factors are 3, 3, and 2. $9 (3 \times 3)$ is not an exact divisor, but $6 (2 \times 3)$ is; consequently, 120 should be increased by 3 as a factor. 3×120 is 360, the required number, the l. c. m.

Find the least common multiple of the following:

- | | |
|---------------------------|---------------------------|
| 2. 3, 4, 5, 6, and 8. | 6. 24, 32, 28, and 18. |
| 3. 2, 6, 8, 12, and 24. | 7. 6, 8, 12, 24, and 32. |
| 4. 3, 9, 12, 6, and 18. | 8. 4, 8, 16, 32, and 128. |
| 5. 12, 18, 24, 36, and 48 | 9. 4, 6, 14, 21, and 42. |

WRITTEN PRACTICE.

When the numbers are large and factoring difficult, the following method is one in general use:

1. What is the least common multiple of 75, 120, 150, and 180?

SOLUTION.

$$\begin{array}{r}
 5) 75 \quad 120 \quad 150 \quad 180 \\
 3) 15 \quad 24 \quad 30 \quad 36 \\
 2) 5 \quad 8 \quad 10 \quad 12 \\
 5) 5 \quad 4 \quad 5 \quad 6 \\
 2) 1 \quad 4 \quad 1 \quad 6 \\
 \hline
 \quad \quad 2 \quad \quad 3
 \end{array}$$

$$5 \times 3 \times 2 \times 5 \times 2 \times 2 \times 3 = 1800, \text{ l. c. m.}$$

EXPLANATION.—Arrange the numbers as dividends and divide by some divisor that is prime to two or more of the numbers. Continue dividing until the remaining quotients are prime to each other. The product of the several divisors and last quotient, is the least common multiple.

Find the least common multiple of the following:

- | | |
|-----------------------------|-----------------------------|
| 2. 20, 25, 30, and 40. | 7. 25, 75, 225, and 125. |
| 3. 60, 120, 240, and 360. | 8. 4, 8, 10, 12, and 14. |
| 4. 21, 24, 72, and 30. | 9. 3, 5, 7, 9, and 11. |
| 5. 3, 4, 5, 6, 7, 8, and 9. | 10. 8, 12, 16, 24, and 36. |
| 6. 13, 39, 65, and 104. | 11. 9, 32, 21, 35, and 105. |

12. What is the least number that 20, 24, 30, 36, and 40 will each divide without a remainder?

13. What is the shortest distance that may be measured in full lengths by each of 4 tape measures 12, 18, 36, and 50 feet respectively?

14. What is the smallest sum of money with which I may buy colts at \$25, mules at \$40 and horses at \$75, spending an equal sum of money on each, and how many animals will I buy?

15. The front wheel of a bicycle is 18 feet in circumference and the hind wheel is 5 feet in circumference. What is the shortest distance traveled when both wheels will be in the same relative position as when starting?

FRACTIONS.

10. The **Ratio** of numbers is the relation of one number to another. Thus, the ratio of 1 to 2 is $\frac{1}{2}$, since the 1 is one-half of the 2.

What is the ratio of 1 to 3? 1 to 4? 2 to 3? 3 to 4? 4 to 5? 3 to 1? 4 to 1? 4 to 3?

11. A **Fraction** is one or more of the equal parts of a unit. Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{12}$ are fractions and are read one-half, two-thirds, five-twelfths.

12. A **Fractional Unit** is one of the equal parts into which a unit is divided. Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ are fractional units and are read one-half, one-third, one-fourth, one-fifth.

Two numbers are necessary to notate a fraction and are called denominator and numerator.

13. The **Denominator** is written below the line, and shows the number of parts into which the unit is divided.

14. The **Numerator** is written above the line, and shows the number of fractional units taken.

The analysis of a fraction is the separation and naming of its parts. Thus, in the fraction $\frac{7}{8}$, the 8 is the denominator and shows that a unit is divided into 8 parts; the 7 is the numerator and shows the number of parts taken.

Analyze the following fractions:

$\frac{3}{4}, \frac{5}{16}, \frac{9}{24}, \frac{7}{27}, \frac{10}{4}, \frac{9}{7}, \frac{18}{12}, \frac{11}{9}, \frac{10}{3}, \frac{8}{7}.$

With relation to the unit one, fractions are classified as proper and improper.

15. A **Proper Fraction** is one whose value is less than the unit 1. Thus $\frac{1}{2}, \frac{3}{8}, \frac{5}{8},$ and $\frac{9}{16}$ are proper fractions.

16. An **Improper Fraction** is one whose value equals or exceeds the unit 1. Thus, $\frac{4}{3}, \frac{16}{4}, \frac{25}{6}$ are improper fractions.

The value of a fraction is the quotient produced by dividing the numerator by the denominator. Thus, the value of $\frac{15}{3}$ is $15 \div 3$, or 5.

What is the value of $\frac{8}{2}, \frac{16}{4}, \frac{18}{9}, \frac{25}{5}$?

17. The **Reciprocal** of a number is the unit 1 divided by that number. Thus the reciprocal of 2 is $1 \div 2$, or $\frac{1}{2}$; of 3 is $\frac{1}{3}$; of 4 is $\frac{1}{4}$.

What is the reciprocal of 6, 7, 8, 9, 10?

The reciprocal of a fraction is the unit 1 divided by that fraction. Thus the reciprocal of $\frac{5}{6}$ is $1 \div \frac{5}{6}$, or $\frac{6}{5}$; of $\frac{3}{4}$ is $\frac{4}{3}$.

What is the reciprocal of $\frac{7}{8}, \frac{9}{16}, \frac{8}{25}$?

18. The **Terms** of a fraction are the numerator and the denominator of the fraction. Thus, in the fraction $\frac{5}{12}$, 5 and 12 are the terms.

Fractions, in regard to form, are classified as simple, compound, and complex.

19. A **Simple Fraction** is a fraction having integers for its terms. Thus, $\frac{8}{9}, \frac{7}{16}, \frac{2}{5}$ are simple fractions.

20. A **Compound Fraction** is a fraction of a fraction. Thus, $\frac{3}{4}$ of $\frac{7}{8}$ is a compound fraction.

21. A **Complex Fraction** is a fraction having a fraction for one or both terms. Thus, $\frac{\frac{13}{4}}{\frac{7}{8}}$ is a complex fraction and is read $\frac{13}{4} \div \frac{7}{8}$.

Compound and complex fractions may be reduced to simple fractions.

GENERAL PRINCIPLES OF FRACTIONS.

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

Multiplying the denominator of a fraction by any number divides the fraction by that number.

Dividing the numerator of a fraction by any number divides the fraction by that number.

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

Increasing both terms of a fraction by the same ratio does not alter the value of the fraction.

Decreasing both terms of a fraction by the same ratio does not alter the value of the fraction.

22. Reduction of Fractions is the process of changing from one denomination to another without changing the value. Thus, $\frac{2}{3} = \frac{4}{6}$, and $\frac{1}{2} = \frac{4}{8}$.

Reduction is classified as ascending and descending.

23. Reduction Ascending is changing from a lower to a higher denomination. Thus, from halves to fourths, from fourths to eighths, is reduction ascending.

Change $\frac{1}{2}$ to fourths, to eighths, to tenths, to twelfths.

Change $\frac{1}{3}$ to sixths, to ninths, to twelfths, to fifteenths.

Change $\frac{2}{3}$ to sixths, to ninths, to twelfths, to fifteenths.

Change $\frac{3}{4}$ to eighths, to twelfths, to twenty-fourths, to thirty-sixths.

24. Reduction Descending is changing from a higher to a lower denomination. Thus, from eighths to fourths, from sixths to thirds, is reduction descending.

Change $\frac{8}{16}$ to eighths, to fourths, to halves.

Change $\frac{12}{24}$ to twelfths, to eighths, to sixths, to fourths.

There is no limit to the higher reduction of a fraction. When the numerator and denominator are prime to each other, the fraction is reduced to its lowest terms.

25. A Common Denominator of two or more fractions is a denominator to which the several denominators may be

changed. Thus fourths, sixths, eighths, twelfths may be changed to forty-eighths. Any number in which each of these denominators is contained without a remainder is a common denominator of these fractions.

26. The Least Common Denominator of two or more fractions is the lowest denominator to which the several denominators may be changed. Thus, the l. c. d. of fourths, sixths, eighths, and twelfths is twenty-fourths.

What is the l. c. d. of fourths, eighths, twelfths and sixteenths? Of fourths, fourteenths, and twenty-eighths? of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{9}{16}$, and $\frac{7}{24}$? of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, $1\frac{1}{2}$, and $\frac{7}{15}$?

MENTAL PRACTICE.

Two numbers given to find the ratio:

1. 1 bushel is what part of 3 bushels? of 4 bu.? of 7 bu.?
2. What is the ratio of 1 to 3? of 1 to 4? of 1 to 7?
3. What is the ratio of 1 to 5? of 2 to 5? of 4 to 5?
4. What is the ratio of 2 ft. to 8 ft.? to 10 ft.? to 12 ft.?
5. $2\frac{1}{2}$ in. = what part of 5 in.? of 10 in.? of 15 in.? of 20 in.?
6. $3\frac{1}{2}$ miles = what part of 10 miles? of 20 miles? of 30 miles?
7. What is the ratio of 3 miles to 2 miles? to 4 miles? to 3 miles?
8. What is the ratio of 1 dime to \$1? of \$1 to 1 dime?
9. What is the ratio of 6 in. to 1 ft.? to 2 ft.? to 1 yd.?
10. What is the ratio of $\frac{1}{2}$ T. to $\frac{3}{4}$ T.? of $\frac{3}{4}$ T. to $\frac{1}{2}$ T.?
11. What is the ratio of $\frac{3}{4}$ to $\frac{1}{2}$? $\frac{5}{8}$ to $\frac{7}{8}$? $\frac{5}{12}$ to $\frac{7}{12}$?
12. Between what numbers may we have ratios?
13. What is the ratio of \$2.50 to \$7.50? to \$12.50? to \$22.50?
14. What is the ratio of $6\frac{1}{4}$ to $12\frac{1}{2}$? to $18\frac{3}{4}$? to $31\frac{1}{4}$?
15. What is the ratio of $12\frac{1}{2}$ to $18\frac{3}{4}$? to $37\frac{1}{2}$? to $87\frac{1}{2}$?
16. What is the ratio of a field 2 miles square to one 3 miles square?
17. What is the ratio of a field 20 rods square to one 40 rods square?

18. What is the ratio of the cost of $12\frac{1}{2}$ bushels of wheat to the cost of 50 bushels?

19. A man does $\frac{2}{3}$ of a piece of work in a day. What is the ratio of the time required to do all the work, to the part he does in 1 day?

REDUCTION ASCENDING.

MENTAL PRACTICE.

1. 1 equals how many halves? thirds? fourths? fifths?
2. $\frac{1}{2}$ equals how many fourths? sixths? eighths?
3. $\frac{1}{3}$ equals how many sixths? ninths? twelfths?
4. $\frac{2}{3}$ equals how many sixths? ninths? twelfths?
5. $\frac{3}{4}$ equals how many sixths? ninths? twelfths?
6. $\frac{1}{4}$ equals how many twelfths? twenty-fourths?
7. $\frac{3}{4}$ equals how many twelfths? twenty-fourths?
8. $\frac{1}{6}$ of a bushel equals how many thirty-sixths of a bushel?
9. $\frac{1}{8}$ of a peck equals how many forty-eighths of a peck?
10. $1\frac{1}{2}$ of an hour equals how many 48ths of an hour?
11. $1\frac{1}{2}$ of a bushel equals how many 64ths of a bu.?
12. $\frac{1}{16}$ of an ounce equals how many 96ths of an ounce?
13. $1\frac{1}{4}$ of a day equals how many 144ths of a day?
14. $1\frac{1}{2}$ of a bushel equals how many 64ths of a bu.?
15. $2\frac{3}{4}$ of a gallon equals how many 320ths of a gallon?

WRITTEN PRACTICE.

The least common denominator of several fractions is the least common multiple of the denominators. Art. 9.

PRINCIPLE.—Multiplying both terms of a fraction by the same number does not alter the value.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths.
 2. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to twenty-fourths.
 3. Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $1\frac{1}{2}$, $1\frac{1}{3}$, and $2\frac{5}{6}$ to 36ths.
 4. Change $1\frac{1}{2}$, $\frac{3}{4}$, $2\frac{1}{2}$, $1\frac{1}{4}$, and $3\frac{1}{6}$ to 72nds.
 5. Reduce $\frac{2}{3}$, $\frac{7}{8}$, and $1\frac{1}{2}$ to their least common denominator (24ths).
 6. Reduce $2\frac{1}{3}$, $3\frac{1}{4}$, and $2\frac{5}{6}$ to their least common denominator.
- Reduce the following fractions to their least common denominator:

7. $\frac{3}{8}, \frac{9}{16}, \frac{37}{32}, \frac{11}{8}$
 8. $\frac{1}{4}, \frac{4}{7}, \frac{15}{8}, \frac{11}{4}$
 9. $\frac{4}{85}, \frac{8}{8}, \frac{11}{4}$
 10. $\frac{17}{20}, \frac{19}{80}, \frac{7}{10}, \frac{8}{5}$
 11. $\frac{8}{7}, \frac{8}{85}, \frac{2}{8}, \frac{13}{4}$

12. $\frac{8}{80}, \frac{7}{40}, \frac{11}{80}, \frac{11}{40}$
 13. $5\frac{1}{2}, 7\frac{9}{16}, 3\frac{3}{8}$
 14. $\frac{9}{81}, \frac{7}{18}, \frac{11}{24}$
 15. $\frac{25}{24}, \frac{7}{48}, \frac{13}{64}, \frac{25}{84}$
 16. $\frac{111}{128}, \frac{39}{144}, \frac{7}{12}, \frac{13}{1728}$

REDUCTION DESCENDING.

PRINCIPLE.—Dividing both terms of a fraction by the same number does not alter its value.

MENTAL PRACTICE.

1. $\frac{16}{72}$ = how many eighths? fourths? halves?
 2. $\frac{8}{72}$ = how many eighths? fourths? halves?
 3. Reduce $\frac{48}{72}$ to thirds.

SOLUTION.—Since 72 is to be reduced to thirds, 3 must be one of the divisors and 24 the other; therefore, divide each term by 24, and the result is $\frac{2}{3}$.

4. Reduce $\frac{25}{60}$ of an hour to 12ths of an hour.
 5. Reduce $\frac{120}{144}$ of a square foot to 6ths of a square foot.
 6. Reduce $\frac{96}{128}$ of a cord to 4ths of a cord.
 7. Reduce $\frac{720}{1728}$ of a cubic foot to 12ths of a cubic foot.
 8. Reduce $\frac{84}{80}$ of a minute to 5ths of a minute.
 9. Reduce $\frac{12}{7}$ of a cubic yard to its lowest terms.
 10. Change $\frac{54}{80}$ of an hour to its lowest terms.

WRITTEN PRACTICE.

Reduce the following fractions to their lowest terms:

17. Reduce $\frac{48}{72}$ to its lowest terms.

SOLUTION.

$$\frac{48 \div 8}{72 \div 8} = \frac{6}{9}; \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

RULE.—Cancel all factors common to both numerator and denominator; or divide both terms by their greatest common divisor.

18. $\frac{125}{375}$

25. $\frac{55}{100}$

32. $\frac{572}{1578}$

19. $\frac{42}{189}$

26. $\frac{168}{216}$

33. $\frac{9375}{10000}$

20. $\frac{104}{196}$

27. $\frac{296}{488}$

34. $\frac{2685}{8080}$

21. $\frac{125}{1000}$

28. $\frac{888}{1880}$

35. $\frac{3002}{4004}$

22. $\frac{225}{850}$

29. $\frac{448}{896}$

36. $\frac{525}{12918}$

23. $\frac{825}{1875}$

30. $\frac{127}{8080}$

37. $\frac{7189}{11648}$

24. $\frac{206}{1018}$

31. $\frac{385}{880}$

38. $\frac{25211}{87348}$

CHANGING A WHOLE OR MIXED NUMBER TO AN IMPROPER FRACTION.

MENTAL PRACTICE.

1. How many halves in 1? 2? 3? 4? 5?
2. How many thirds in 1? 2? 3? 4? 5?
3. How many halves in $1\frac{1}{2}$? $2\frac{1}{2}$? $3\frac{1}{2}$?
4. How many fourths in 1? 2? $2\frac{1}{4}$? $3\frac{1}{4}$? $7\frac{3}{4}$?
5. How many eighths in 1? 2? $3\frac{3}{8}$? $4\frac{7}{8}$? $6\frac{5}{8}$? $7\frac{1}{8}$?
6. How many twelfths in 3? $4\frac{5}{12}$? $5\frac{7}{12}$? $7\frac{11}{12}$? $5\frac{1}{12}$?
7. How many fifteenths of a bu. in $10\frac{7}{15}$ bu.? in $20\frac{14}{15}$ bu.?
8. How many sixtieths of an hr. in $5\frac{1}{6}$ hr.? in $16\frac{25}{60}$ hr.?
9. Reduce to improper fractions: $7\frac{3}{4}$ oz.; $12\frac{1}{4}$ gi.; $24\frac{5}{12}$ yd.
10. Change to improper fractions: $10\frac{7}{8}$ bu; $25\frac{7}{8}$ mo.; $2\frac{1}{2}$ pecks.

WRITTEN PRACTICE

39. Change $28\frac{7}{15}$ to 15ths.

SOLUTION.

$$28 \times 15 = 420$$

$$28 = \frac{420}{15}$$

$$\frac{420}{15} + \frac{7}{15} = \frac{427}{15}$$

RULE.—Multiply the whole number by the required denominator, and to this product add the numerator of the fraction, writing the required denominator under the result

40. Change 324 to twenty-sevenths.
 41. Reduce $18\frac{9}{16}$ to sixteenths.
 42. Reduce $124\frac{2}{3}$ days to eighths of a day.
 43. Reduce $127\frac{5}{16}$ to an improper fraction.
 44. Change $375\frac{1}{3}$ to an improper fraction.
 45. What improper fraction is equal to $246\frac{5}{8}$?
 46. What improper fraction is equal to $34\frac{9}{125}$?
 47. Reduce $124\frac{1}{2}$ weeks to sevenths of a week.
 48. Change 128 to a fraction whose denominator is 125.
- Reduce the following to improper fractions:
- | | | |
|----------------------------|------------------------------|---------------------------------|
| 49. $27\frac{2}{3}$ gills. | 54. $245\frac{3}{10}$ tons. | 59. $537\frac{5}{1728}$ cu. ft. |
| 50. $54\frac{1}{2}$ gal. | 55. $396\frac{7}{16}$ lb. | 60. $245\frac{3}{8}$ gal. |
| 51. $86\frac{5}{12}$ hr. | 56. $935\frac{1}{2}$ cu. yd. | 61. $96\frac{11}{144}$ sq. ft. |
| 52. $72\frac{7}{10}$ mo. | 57. $842\frac{2}{3}$ min. | 62. $87\frac{1}{128}$ cd. |
| 53. $185\frac{1}{16}$ rd. | 58. $376\frac{2}{15}$. | 63. $92\frac{1}{37}$ bu. |

CHANGING AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

MENTAL PRACTICE.

1. What is the ratio of $\frac{4}{3}$ to 1? $\frac{5}{2}$ to 1? $\frac{8}{3}$ to 1?
SOLUTION.—The ratio of $\frac{4}{3}$ to 1 is $\frac{2}{3}$; since $1 = \frac{2}{3}$, and $\frac{4}{3}$ is 2 times $\frac{2}{3}$.
2. What is the ratio of $\frac{9}{8}$ to 1? $\frac{12}{8}$ to 1? $\frac{15}{8}$ to 1?
3. How many units in $\frac{8}{4}$? $\frac{12}{4}$? $\frac{16}{4}$? $\frac{20}{4}$?
4. How many units in $\frac{10}{5}$? $\frac{15}{5}$? $\frac{24}{5}$? $\frac{37}{5}$?
5. What is the value of $\frac{24}{7}$? $\frac{85}{12}$? $\frac{96}{14}$? $\frac{125}{7}$?
6. What is the value of $\frac{124}{8}$? $\frac{246}{5}$? $\frac{427}{10}$?
7. Find the value of $\frac{365}{8}$, $\frac{379}{16}$, $\frac{3260}{20}$?
8. Find the value of $\frac{72}{10}$, $\frac{64}{12}$, $\frac{100}{16}$.
9. How many bu. in $\frac{324}{8}$ bu? $\frac{256}{4}$ bu?
10. How many miles in $\frac{260}{40}$ mi? $\frac{645}{320}$ mi?

WRITTEN PRACTICE.

64. What is the value of $\frac{120}{7}$?

SOLUTION.

$\frac{120}{7}$ or $120 \div 7 = 17\frac{1}{7}$ RULE.—Divide the numerator by the denominator.

65. What is the value of $\frac{185}{16}$ of a pound?
66. What is the value of $\frac{328}{8}$ of a bushel?
67. What is the value of $\frac{428}{32}$ of a rod?
68. What is the value of $\frac{1728}{24}$ of a cubic foot?

Reduce the following to whole or mixed numbers:

- | | | |
|-----------------------|------------------------|-------------------------|
| 69. $\frac{428}{16}$ | 74. $\frac{3265}{48}$ | 79. $\frac{12654}{144}$ |
| 70. $\frac{965}{24}$ | 75. $\frac{27642}{8}$ | 80. $\frac{9365}{1728}$ |
| 71. $\frac{848}{75}$ | 76. $\frac{9327}{125}$ | 81. $\frac{8324}{12}$ |
| 72. $\frac{1278}{64}$ | 77. $\frac{275}{150}$ | 82. $\frac{86542}{625}$ |
| 73. $\frac{2876}{84}$ | 78. $\frac{2643}{16}$ | 83. $\frac{4527}{160}$ |

RATIO OF FRACTIONS.

MENTAL PRACTICE.

1. $\frac{1}{4}$ is what part of $\frac{3}{4}$? $\frac{3}{4}$? $\frac{5}{4}$?
2. $\frac{3}{4}$ is what part of $\frac{5}{4}$? $\frac{7}{4}$? $\frac{10}{4}$?
3. $\frac{1}{8}$ is what part of $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{8}{16}$?

SUGGESTION.—Reduce to a common denominator:

4. What is the ratio of $\frac{3}{8}$ to $\frac{3}{4}$? $\frac{3}{4}$ to $\frac{5}{8}$?
 5. What is the ratio of $\frac{7}{8}$ to $\frac{1}{8}$? $\frac{2}{8}$? $\frac{3}{8}$?
 6. What is the ratio of 1 to $\frac{3}{4}$? $\frac{7}{8}$? $\frac{5}{8}$? $\frac{4}{8}$? $\frac{7}{12}$?
 7. What is the ratio of $6\frac{1}{4}$ to $8\frac{1}{8}$? $8\frac{1}{8}$ to $6\frac{1}{4}$?
 8. What is the reciprocal of the ratio of 2 to 5? to 7? to 8?
 9. What is the ratio of the cost of $18\frac{3}{4}$ bu. to $31\frac{1}{4}$ bu.?
 10. What is the ratio of the cost of $87\frac{1}{2}$ acres to the cost of $66\frac{2}{3}$ acres?
 11. What is $\frac{1}{2}$ of 2? 6? 12? 7? 21? 75?
 12. What is $\frac{2}{3}$ of 15? 30? 10? 20? 27? 100?
 13. What is $\frac{1}{4}$ of 12? 16? 24? 30? 50? 75?
 14. What is $\frac{3}{4}$ of 12? 16? 24? 30? 50? 75?
 15. What is $\frac{1}{6}$ of 12? 15? 18? 21? 24? 27?
 16. What is $\frac{5}{6}$ of 12? 15? 18? 21? 24? 27?
 17. What is $\frac{2}{3}$ of 24? $\frac{7}{8}$ of 54? $\frac{3}{4}$ of 36?
 18. What is $\frac{1}{2}$ of 4 bushels? 4 fourths? 8 eighths?
 19. What is $\frac{2}{3}$ of 25? $\frac{4}{5}$ of 50? $\frac{8}{9}$ of $62\frac{1}{2}$?
 20. A man owned $\frac{2}{3}$ of a section of 640 acres of land. How many acres did he own?
 21. What is $\frac{2}{3}$ of the value of a factory, the whole factory being worth \$10,000?
 22. What is the cost of $\frac{7}{8}$ of 120 bushels of wheat at 50 cents per bushel?
 23. What is the cost of $\frac{3}{4}$ of $\frac{3}{4}$ of 200 quarts of berries at $16\frac{2}{3}$ cents per quart?
 24. $16 = \frac{1}{2}$ of what number? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{10}$?
- SOLUTION BY RATIO.—The ratio of 1 to $\frac{1}{2}$ is $\frac{2}{1}$; $2 \times 16 = 32$.
- SOLUTION BY ANALYSIS.—Since 16 is $\frac{1}{2}$, $\frac{2}{2}$ is 2 times 16, or 32.
25. $12\frac{1}{2} = \frac{1}{2}$ of what number? $\frac{1}{8}$? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$?
 26. $37\frac{1}{2} = \frac{3}{8}$ of what number? $50 = \frac{4}{5}$ of what number?
 27. $\frac{2}{3}$ of what number is 8? 12? 14? 18? 25? $33\frac{1}{3}$?
 28. $\frac{1}{4}$ of what number is 2? 3? $2\frac{1}{2}$? $3\frac{1}{3}$? $6\frac{1}{4}$? $12\frac{1}{2}$?
 29. $\frac{3}{4}$ of what number is 6? 9? 10? 15? $18\frac{3}{4}$? 25?
 30. $\frac{1}{6}$ of what number is 5? $3\frac{1}{3}$? $6\frac{1}{4}$? $8\frac{1}{3}$? $12\frac{1}{2}$?
 31. 24 is $\frac{2}{3}$ of what number? $\frac{4}{5}$? $\frac{6}{5}$? $\frac{8}{5}$? $1\frac{2}{5}$?

32. What is the cost of a yard of cloth, if $\frac{5}{8}$ of a yard cost \$.62 $\frac{1}{2}$?

33. What is the cost of 3 horses, if $\frac{3}{8}$ of the cost of 1 horse is \$60?

34. What cost 6 bushels of potatoes, if $\frac{5}{8}$ of a bushel cost \$.62 $\frac{1}{2}$?

35. $\frac{1}{2}$ of 4 is $\frac{1}{3}$ of what number?

SOLUTION BY RATIO.—The ratio of $\frac{1}{2}$ to $\frac{1}{3}$ is $\frac{3}{2}$; $\frac{3}{2}$ of 4 is 6, the required number.

SOLUTION BY ANALYSIS.— $\frac{1}{2}$ of 4 is 2; since 2 is $\frac{1}{3}$, $\frac{3}{2}$ is 3 times 2, or 6, the required number.

36. $\frac{3}{8}$ of 12 is $\frac{1}{4}$ of what number?

37. $\frac{3}{4}$ of 16 is $\frac{1}{5}$ of what number?

38. $\frac{1}{5}$ of 60 is $\frac{2}{3}$ of what number?

39. $\frac{1}{6}$ of 96 is $\frac{4}{15}$ of what number?

40. $\frac{3}{8}$ of 24 is $\frac{1}{3}$ of what number?

41. $\frac{7}{8}$ of 64 is $\frac{4}{7}$ of what number?

42. $\frac{3}{5}$ of 62 $\frac{1}{2}$ is $\frac{3}{4}$ of what number?

43. A house costs $\frac{1}{3}$ of \$12,000, and the cost of the house is $\frac{1}{4}$ of the cost of the lot. How much did the lot cost?

44. A carriage costs $\frac{3}{8}$ of \$180, and the cost of the carriage is $\frac{3}{4}$ of the cost of a horse. What is the cost of both?

45. If $\frac{1}{2}$ of a number is 12, what is $\frac{1}{3}$ of the number?

SOLUTION BY RATIO.—The ratio of $\frac{1}{2}$ to $\frac{1}{3}$ is $\frac{3}{2}$; $\frac{3}{2}$ of 12 is 8.

SOLUTION BY ANALYSIS.—Since 12 is $\frac{1}{2}$, $\frac{2}{3}$ is 2 times 12 or 24; $\frac{1}{3}$ of 24 is 8, the required number.

46. If $\frac{1}{3}$ of a number is 15, what is $\frac{2}{3}$ of the number?

47. If $\frac{3}{8}$ of a number is 8, what is $\frac{3}{4}$ of the number?

48. If $\frac{3}{4}$ of a number is 27, what is $\frac{2}{3}$ of the number?

49. If $\frac{3}{2}$ of a number is 36, what is $\frac{4}{3}$ of the number?

50. If $\frac{3}{4}$ of a barrel of flour cost \$9, what is the cost of $\frac{3}{8}$ of a barrel?

51. If $\frac{5}{8}$ of a yard of silk cost \$.62 $\frac{1}{2}$, what is the cost of $\frac{3}{8}$ of a yard?

52. If 2 $\frac{1}{2}$ bushels of oats cost \$.84, what is the cost of 7 $\frac{1}{2}$ bushels?

53. If 4 bushels of wheat cost \$1.50, what is the cost of $\frac{3}{8}$ of a bushel?

54. If $\frac{3}{4}$ of a gallon of molasses cost \$.66 $\frac{2}{3}$, what is the cost of 4 gallons?

55. If 6 $\frac{1}{4}$ gallons of wine cost \$5.25, what is the cost of 10 gallons?

56. 20 is $\frac{1}{4}$ more than what number?

SUGGESTION.—20 is $\frac{5}{4}$ of the number.

57. 24 is $\frac{1}{7}$ more than what number?

58. 36 is $\frac{1}{8}$ more than what number?

59. 37 $\frac{1}{2}$ is $\frac{1}{2}$ more than what number?

60. 87 $\frac{1}{2}$ bushels is $\frac{1}{8}$ more than what quantity?

61. 30 is $\frac{3}{8}$ more than what number?

62. 49 is $\frac{3}{4}$ more than what number?

63. Sold butter at \$.25 per pound which was $\frac{1}{4}$ more than it cost. How much did it cost?

64. Sold a horse for \$80, thereby gaining $\frac{3}{8}$ of the cost price. How much did the horse cost?

65. There are 30 sheep in a pasture, this being $\frac{1}{4}$ more than the sheep in another pasture. How many in the second pasture?

66. I paid \$120 for a horse, which is $\frac{1}{3}$ more than I paid for a carriage. How much did I pay for both?

67. There are 48 bushels of wheat in a load, this being $\frac{1}{5}$ more than the bushels in another load. How many more bushels in the first load than in the second?

68. 6 is $\frac{1}{3}$ less than what number?

SUGGESTION.—6 is $\frac{2}{3}$ of the number.

69. 24 is $\frac{1}{5}$ less than what number?

70. 42 is $\frac{4}{7}$ less than what number?

71. 87 $\frac{1}{2}$ is $\frac{3}{8}$ less than what number?

72. $\frac{3}{8}$ of 60 is $\frac{1}{6}$ less than what number?

73. $\frac{3}{4}$ of 96 is $\frac{2}{5}$ less than what number?

74. A pair of gloves cost \$2.50, which is $\frac{1}{6}$ less than the cost of a hat. How much did the hat cost?

75. I paid \$120 for a carriage, and this sum is $\frac{1}{4}$ less than what I paid for a horse. How much did I pay for both?

ADDITION.

PRINCIPLE.—Only like numbers can be added.

MENTAL PRACTICE.

1. 1 bu. plus 3 bu. plus 5 bu. = how many bushels?
2. 1 eighth, plus 2 eighths, plus 3 eighths = how many eighths?
3. $\frac{9}{16} + \frac{7}{16}$ = how many 16ths? How many units?
4. $\frac{3}{4} + \frac{1}{4} + \frac{5}{4} + \frac{7}{4}$ = how many fourths? How many units?
5. $\frac{7}{12} + \frac{5}{12} + \frac{3}{12} + \frac{6}{12}$ = how many 12ths? How many units?
6. 2 ft. plus 8 in. = how many inches?
7. 3 yds. plus 2 ft. = how many feet?
8. 1 half plus 1 third = how many 6ths?
9. $\frac{1}{3} + \frac{1}{4}$ = how many 12ths?

Find the sum of

- | | | |
|---------------------------------------|--|-------------------------------------|
| 10. $\frac{1}{4}$ and $\frac{1}{5}$. | 14. $\frac{1}{6}$ plus $\frac{1}{7}$ = ? | 18. $\frac{1}{7} + \frac{1}{8}$ = ? |
| 11. $\frac{1}{5}$ and $\frac{1}{6}$. | 15. $\frac{2}{6}$ plus $\frac{2}{7}$ = ? | 19. $\frac{2}{7} + \frac{3}{8}$ = ? |
| 12. $\frac{2}{5}$ and $\frac{1}{6}$. | 16. $\frac{3}{6}$ plus $\frac{2}{7}$ = ? | 20. $\frac{3}{7} + \frac{3}{8}$ = ? |
| 13. $\frac{3}{5}$ and $\frac{1}{6}$. | 17. $\frac{5}{6}$ plus $\frac{5}{7}$ = ? | 21. $\frac{4}{7} + \frac{5}{8}$ = ? |
22. $\frac{1}{2}$ plus $\frac{1}{3}$ plus $\frac{1}{4}$ = how many 12ths?
 23. $\frac{1}{2}$ plus $\frac{1}{4}$ plus $\frac{1}{6}$ = ?
 24. $\frac{2}{3}$ plus $\frac{2}{4}$ plus $\frac{5}{6}$ = how many 12ths?
 25. A man owning $\frac{3}{8}$ of a farm bought $\frac{3}{16}$ of it at another time. What part of the whole farm did he then own?
 26. A farmer sold $\frac{1}{3}$ of his grain at one time, $\frac{1}{4}$ at another, and $\frac{1}{5}$ at another. What part of the whole did he sell?
 27. How many are \$4.50 and \$3.75?
 28. How many are $3\frac{3}{4}$ yd. and $4\frac{2}{3}$ yd.?
 29. What is the sum of $2\frac{1}{4}$ and $3\frac{1}{5}$?
 30. What is the sum of $2\frac{3}{4}$ and $3\frac{4}{5}$?
 31. Find the amount of $3\frac{2}{3}$ hr. and $5\frac{7}{12}$ hr.
 32. Find the sum of \$2.25, \$3.50, and \$5.75.
 33. What is the sum of $4\frac{1}{5}$, $3\frac{1}{4}$, and $2\frac{1}{6}$?
 34. What is the sum of $3\frac{2}{3}$ bu., $3\frac{1}{4}$ bu., and $4\frac{5}{6}$ bu.?
 35. A gardener has $1\frac{1}{2}$ A. in turnips, $2\frac{3}{4}$ A. in cabbages, and $4\frac{1}{4}$ A. in melons. How many acres in all?
 36. A can do a piece of work in 5 days, and B can do it in 6 days. How much can both do in one day?

37. A can do a piece of work in 4 days, B can do it in 5 days, and C can do it in 6 days. How much can they all do in 1 day?

WRITTEN PRACTICE.

84. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, and $\frac{11}{24}$?

SOLUTION.

$$\begin{aligned}\frac{3}{4} + \frac{7}{8} + \frac{9}{16} + \frac{11}{24} &= ? \\ \frac{36}{48} + \frac{42}{48} + \frac{27}{48} + \frac{22}{48} &= \frac{127}{48} \\ \frac{127}{48} &= 2\frac{31}{48}.\end{aligned}$$

85. What is the sum of $14\frac{1}{2}$, $25\frac{3}{8}$, $9\frac{5}{8}$, and $11\frac{5}{12}$?

SOLUTION.

OR

$$14\frac{1}{2} + 25\frac{3}{8} + 9\frac{5}{8} + 11\frac{5}{12} = ?$$

24 L. C. D.

$$14 + 25 + 9 + 11 = 59$$

$$\frac{1}{2} + \frac{3}{8} + \frac{5}{8} + \frac{5}{12} = ?$$

$$\frac{12}{24} + \frac{9}{24} + \frac{15}{24} + \frac{10}{24} = \frac{51}{24}$$

$$\frac{51}{24} = 2\frac{3}{8}, \text{ or } 2\frac{1}{8}$$

$$59 + 2\frac{1}{8} = 61\frac{1}{8}$$

$14\frac{1}{2}$	12
$25\frac{3}{8}$	9
$9\frac{5}{8}$	20
$11\frac{5}{12}$	10
59	51 = 21
$2\frac{1}{8}$	24
61 $\frac{1}{8}$	

86. What is the sum of $16\frac{1}{4}$, $18\frac{7}{8}$, $24\frac{5}{16}$, $19\frac{7}{4}$, $36\frac{5}{8}$, and $42\frac{3}{4}$?

87. A farmer has $36\frac{3}{4}$ A. in wheat, $44\frac{7}{8}$ A. in oats, $58\frac{3}{8}$ A. in corn, and $96\frac{1}{4}$ A. in pasture. How many acres has he?

88. A grocer packed in a box 38 lbs. of sugar, $6\frac{7}{8}$ lbs. of coffee, $2\frac{1}{2}$ lbs. of tea, $16\frac{3}{4}$ lbs. of beef, $12\frac{1}{4}$ lbs. of bacon, and $16\frac{5}{8}$ lbs. of butter; what was the total weight, the box weighing $56\frac{3}{4}$ lbs.?

89. A farmer sold grain as follows: $624\frac{3}{4}$ bu. of oats for \$124 $\frac{7}{8}$, $473\frac{1}{4}$ bu. of wheat for \$296 $\frac{5}{16}$, $324\frac{7}{8}$ bu. of barley for \$96 $\frac{1}{4}$, $306\frac{7}{16}$ bu. of beans for \$142 $\frac{7}{12}$, and $224\frac{1}{2}$ bu. of corn for \$175 $\frac{1}{2}$. How many bushels did he sell and how much did he get for it all?

90. A merchant bought 30 bolts of dry goods containing the following number of yards: $40\frac{1}{4}$, $41\frac{1}{4}$, $42\frac{3}{4}$, 41, $43\frac{3}{4}$, $43\frac{1}{4}$, $44\frac{1}{4}$, $36\frac{3}{4}$, $37\frac{3}{4}$, $39\frac{3}{4}$, 37, 36, $35\frac{1}{4}$, $37\frac{1}{4}$, 34, $33\frac{3}{4}$, 37, $35\frac{1}{4}$, $36\frac{1}{4}$, $34\frac{3}{4}$, $34\frac{1}{4}$, $32\frac{3}{4}$, $35\frac{1}{4}$, $24\frac{3}{4}$, $27\frac{1}{4}$, $28\frac{1}{4}$, 29, $26\frac{3}{4}$, $22\frac{3}{4}$, 24. How many yards did he buy?

Find the total of the sums:

91. $\frac{1}{4} + \frac{3}{8} + \frac{7}{8} + \frac{9}{16} + \frac{5}{12} = ?$

$1\frac{1}{2} + 2\frac{2}{8} + 3\frac{3}{4} + 4\frac{4}{8} + 5\frac{5}{8} = ?$

$6\frac{7}{8} + 8\frac{3}{4} + 9\frac{5}{16} + 12\frac{3}{8} = ?$

$18\frac{1}{4} + 16\frac{1}{16} + 9\frac{7}{24} + 8\frac{7}{48} = ?$

92. $364\frac{7}{8} + 243\frac{5}{6} + 327\frac{7}{24} + 162\frac{9}{16} = ?$

$123\frac{5}{16} + 247\frac{3}{8} + 842\frac{1}{8} + 375\frac{7}{24} = ?$

$325\frac{5}{8} + 426\frac{5}{12} + 342\frac{5}{24} + 136\frac{5}{48} = ?$

$243\frac{7}{16} + 327\frac{7}{24} + 436\frac{7}{48} + 244\frac{7}{96} = ?$

93. $7 + 8\frac{3}{4} + 9\frac{1}{4} + 6\frac{5}{8} + 8 = ?$

$3\frac{1}{2} + 9\frac{5}{12} + 8 + 7\frac{5}{6} + 6\frac{1}{4} = ?$

$\frac{3}{4} + \frac{7}{8} + \frac{9}{16} + \frac{1}{2} + \frac{7}{8} + \frac{5}{6} = ?$

$51\frac{1}{4} + 91\frac{1}{12} + 8\frac{5}{6} + 1\frac{7}{16} + 1\frac{5}{14} = ?$

94. $\frac{4}{5} + \frac{7}{8} + \frac{9}{10} + \frac{7}{20} = ?$

$4\frac{1}{2} + 2\frac{1}{8} + 5\frac{3}{4} + 6\frac{7}{8} = ?$

$\frac{7}{8} + \frac{3}{4} + \frac{5}{6} + \frac{7}{16} + \frac{1}{24} = ?$

$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = ?$

$1\frac{1}{2} + 2\frac{2}{3} + 3\frac{3}{4} + 4\frac{4}{5} = ?$

$4\frac{1}{2} + \frac{3}{16} + 5\frac{3}{8} + \frac{1}{4} = ?$

SUBTRACTION.

PRINCIPLE.—Only like numbers can be subtracted.

MENTAL PRACTICE.

1. 7 hours less 4 hours = how many hours?

2. 7 eighths less 4 eighths = how many eighths?

3. 11 twelfths of an hour less 5 twelfths of an hour = ?

4. 15 sixteenths of a mile less 4 sixteenths of a mile = ?

5. $\frac{7}{8}$ less $\frac{5}{8} = ?$

7. $\frac{5}{6} - \frac{4}{6} = ?$

9. $\frac{7}{8} - \frac{3}{8} = ?$

6. $\frac{5}{7}$ less $\frac{2}{7} = ?$

8. $\frac{7}{11} - \frac{3}{11} = ?$

10. $\frac{19}{24} - \frac{15}{24} = ?$

11. A man owning a farm sold $\frac{3}{4}$ of it. What part had he left?

12. A merchant owns $\frac{7}{9}$ of a building and sells $\frac{5}{9}$ of the building. What part has he left?

13. From 2 hours subtract $\frac{1}{2}$ hour?

14. From $\frac{1}{2}$ subtract $\frac{1}{3}$.

15. $\frac{1}{3}$ less $\frac{1}{4} = ?$

16. $\frac{1}{4} - \frac{1}{5} = ?$

17. $\frac{3}{4} - \frac{2}{5} = ?$

18. $\frac{7}{8} - \frac{1}{4} = ?$

19. $\frac{7}{8} - \frac{3}{4} = ?$

20. $\frac{9}{10} - \frac{3}{5} = ?$

21. $\frac{5}{6} - \frac{1}{3} = ?$

22. $\frac{4}{5} - \frac{2}{3} = ?$

23. $\frac{11}{12} - \frac{2}{3} = ?$

24. $\frac{11}{16} - \frac{1}{4} = ?$

25. A boy having $\$2\frac{1}{2}$ spends $\$2$. How much has he left?

26. $\$5\frac{1}{2} - \$3 = ?$

27. $\$7\frac{1}{2} - \$4 = ?$

28. $3\frac{1}{2}$ bu. $- 2$ bu. $= ?$

29. $8\frac{1}{2}$ qt. $- 5$ qt. $= ?$

30. A boy having $5\frac{3}{4}$ oranges, ate $2\frac{1}{2}$ oranges. How many had he left?

31. $4\frac{7}{8} - 3\frac{3}{8} = ?$

32. $8\frac{5}{8} - 5\frac{3}{8} = ?$

33. $8\frac{7}{11}$ bu. less $5\frac{4}{11}$ bu. $= ?$

34. $9\frac{8}{13}$ pt. less $6\frac{5}{13}$ pt. $= ?$

35. From a piece of cloth containing $15\frac{7}{8}$ yds., $8\frac{3}{4}$ yds. were sold. How many yards remained?

36. $2\frac{3}{8} - 1\frac{1}{8} = ?$

37. $3\frac{3}{4} - 2\frac{1}{4} = ?$

38. $\$8\frac{7}{8} - \$5\frac{1}{4} = ?$

39. $6\frac{3}{8}$ bu. $- 3\frac{1}{8}$ bu. $= ?$

40. A merchant sold $7\frac{3}{4}$ gallons from a keg containing 10 gallons. How many gallons remained?

41. $12 - 8\frac{1}{2} = ?$

42. $8 - 5\frac{7}{8} = ?$

43. 24 ft. less $12\frac{7}{12}$ ft. $= ?$

44. 16 rd. less $11\frac{1}{3}$ rd. $= ?$

45. $8\frac{3}{8}$ bu. $- 6\frac{7}{8} = ?$

46. $5\frac{1}{8}$ hr. $- 3\frac{1}{2}$ hr. $= ?$

47. $6\frac{3}{8}$ bu. $- 3\frac{5}{8}$ bu. $= ?$

48. $3\frac{1}{4}$ pt. $- 2\frac{1}{2}$ pt. $= ?$

49. $8\frac{1}{2}$ mi. $- 5\frac{5}{8}$ mi. $= ?$

50. $4\frac{3}{8}$ min. $- 2\frac{3}{4}$ min. $= ?$

51. A lady bought a hat for $\$6\frac{3}{4}$, a pair of shoes for $\$4\frac{1}{2}$, a parasol for $\$3\frac{5}{8}$, and a dress pattern for $\$4\frac{1}{8}$. How much change should she receive, if she gave the merchant two ten-dollar bills?

52. A can do a piece of work in 5 days, and B can do it in 6 days. How much more can A do in 1 day than B?

53. A and B can do a piece of work in $3\frac{1}{2}$ days, and C can do the same amount of work in 6 days. How much more can A and B do in one day than C?

WRITTEN PRACTICE.

Find the difference between the following:

95.
$$\begin{array}{r} 24\frac{3}{4} \\ 16 \\ \hline 8\frac{3}{4} \end{array}$$

96.
$$\begin{array}{r} 75\frac{7}{8} \\ 26 \\ \hline \end{array}$$

97.
$$\begin{array}{r} 83\frac{7}{8} \\ 74 \\ \hline \end{array}$$

98.
$$\begin{array}{r} 93\frac{5}{8} \\ 28 \\ \hline \end{array}$$

99. From $82\frac{1}{4}$ take 16.

100. From $79\frac{3}{8}$ take 63.

101. Subtract 18 from $75\frac{3}{4}$.

102. Subtract 24 from $96\frac{1}{2}$.

Find the difference between the following:

$$103. \begin{array}{r} 75\frac{3}{4} \\ 24\frac{1}{7} \\ \hline 51\frac{3}{7} \end{array} \quad 104. \begin{array}{r} 87\frac{1}{3} \\ 24\frac{2}{3} \\ \hline \end{array} \quad 105. \begin{array}{r} 175\frac{11}{16} \\ 9\frac{4}{16} \\ \hline \end{array} \quad 106. \begin{array}{r} 287\frac{31}{32} \\ 981\frac{3}{32} \\ \hline \end{array}$$

107. From $125\frac{9}{32}$ bu. take $89\frac{5}{32}$ bu.

108. From $37\frac{9}{16}$ lb. take $24\frac{2}{16}$ lb.

109. $124\frac{11}{21} - 29\frac{7}{21} = ?$ 110. $247\frac{5}{17} - 193\frac{2}{17} = ?$

Find the difference between the following:

$$111. \begin{array}{r} 25\frac{5}{8} = 25\frac{10}{16} \\ 18\frac{3}{4} = 18\frac{9}{12} \\ \hline 7\frac{1}{12} \end{array} \quad 112. \begin{array}{r} 74\frac{2}{3} \\ 29\frac{2}{3} \\ \hline \end{array} \quad 113. \begin{array}{r} 64\frac{7}{16} \\ 49\frac{5}{16} \\ \hline \end{array} \quad 114. \begin{array}{r} 83\frac{31}{32} \\ 69\frac{5}{16} \\ \hline \end{array}$$

115. From $29\frac{5}{8}$ qt. take $16\frac{5}{12}$ qt.

116. From $32\frac{17}{24}$ bu. take $18\frac{7}{24}$ bu.

117. $63\frac{9}{14} - 28\frac{3}{8} = ?$

118. $82\frac{5}{8} - 37\frac{5}{8} = ?$

Find the difference between the following:

$$119. \begin{array}{r} 24 = 23\frac{7}{8} \\ 14\frac{3}{4} = 14\frac{6}{8} \\ \hline 9\frac{5}{8} \end{array} \quad 120. \begin{array}{r} 371 \\ 245\frac{8}{9} \\ \hline \end{array} \quad 121. \begin{array}{r} 237 \\ 99\frac{3}{32} \\ \hline \end{array} \quad 122. \begin{array}{r} 178 \\ 25\frac{7}{16} \\ \hline \end{array}$$

123. From 85 take $14\frac{1}{8}$. 124. From 69 take $63\frac{5}{24}$.

125. Subtract 84 from $125\frac{5}{14}$. 126. Subtract 92 from $320\frac{1}{17}$.

Find the difference between the following:

$$127. \begin{array}{r} 27\frac{3}{8} = 26\frac{6}{16} \\ 18\frac{3}{4} = 18\frac{9}{12} \\ \hline 8\frac{11}{12} \end{array} \quad 128. \begin{array}{r} 85\frac{9}{16} \\ 72\frac{3}{4} \\ \hline \end{array}$$

$$129. \begin{array}{r} 97\frac{5}{14} \\ 36\frac{5}{8} \\ \hline \end{array} \quad 130. \begin{array}{r} 33\frac{1}{16} \\ 28\frac{7}{12} \\ \hline \end{array}$$

131. From $84\frac{1}{8}$ take $27\frac{1}{4}$. 132. Take $37\frac{5}{8}$ from $89\frac{3}{4}$.

133. Subtract $75\frac{9}{16}$ from $126\frac{1}{16}$.

134. From $97\frac{1}{16}$ take $8\frac{9}{14}$.

135. What is the difference between $245\frac{7}{12}$ and $124\frac{9}{16}$?

136. From a cask of wine containing $63\frac{1}{2}$ gallons, $24\frac{3}{4}$ gallons were sold at one time, and $16\frac{5}{8}$ gallons at another time. How many gallons remained?

137. A farmer bought $124\frac{1}{8}$ acres of land at one time, and $246\frac{9}{16}$ acres at another time. He afterward sold $184\frac{3}{4}$ acres. How many had he left?

138. From the sum of $16\frac{3}{4}$ and $19\frac{1}{8}$, take the sum of $8\frac{1}{8}$ and $16\frac{5}{12}$.

139. From the sum of $75\frac{7}{8}$ and $94\frac{1}{4}$, take the sum of $36\frac{5}{16}$ and $23\frac{3}{8}$.

140. From the sum of $84\frac{3}{8}$ and $76\frac{5}{8}$, take the sum of $27\frac{3}{4}$ and $18\frac{5}{8}$.

141. Take the sum of $13\frac{9}{16}$ and $29\frac{1}{2}$, from the sum of $42\frac{7}{8}$ and $19\frac{1}{4}$.

142. Take the difference between $84\frac{9}{16}$ and $37\frac{5}{12}$, from the sum of $48\frac{3}{8}$ and $15\frac{7}{8}$.

143. Take the sum of $29\frac{5}{7}$ and $36\frac{7}{9}$, from the difference between $84\frac{1}{4}$ and $365\frac{3}{8}$.

144. Find the total of the results:

$$34\frac{1}{2} + 16\frac{7}{8} + 24\frac{3}{4} - 75\frac{3}{8} =$$

$$24\frac{3}{4} - 16\frac{7}{8} + 25\frac{5}{12} - 14\frac{3}{8} =$$

$$124\frac{5}{8} + 327\frac{7}{12} - 27\frac{3}{4} - 129\frac{7}{8} =$$

$$243\frac{7}{8} + 345\frac{9}{16} - 588\frac{3}{8} + 46\frac{1}{4} =$$

145. Find the total of the results:

$$24\frac{3}{4} + 16\frac{3}{8} - 40\frac{1}{2} + 63\frac{7}{8} =$$

$$36\frac{7}{8} - 14\frac{5}{8} + 125\frac{1}{16} + 3\frac{5}{8} =$$

$$26\frac{5}{14} + 29\frac{3}{32} + 84\frac{7}{8} - 36\frac{3}{8} =$$

146. A speculator bought 480 acres of land. To A he sold $56\frac{1}{4}$ acres, to B $75\frac{1}{2}$ acres, to C as much as to A and B, and to D as much as to A and C. How many acres had he left?

147. From a barrel of molasses containing $58\frac{1}{2}$ gallons, there were drawn at one time $18\frac{3}{4}$ gallons, at a second time $16\frac{1}{2}$ gallons, at a third time $14\frac{7}{8}$ gallons, and at a fourth time $5\frac{5}{16}$ gallons. How many gallons are left in the barrel?

MULTIPLICATION.

MENTAL PRACTICE.

I.

1. If a bushel of wheat costs $\$1\frac{1}{2}$, what cost 12 bu.? 20 bu.? 24 bu.?

2. A boy earns $\$2\frac{3}{4}$ per day. What does he earn in 6 days?

3. A laborer digs $\frac{3}{4}$ rods of ditch in 1 hour. How many rods will he dig in 6 days, working 10 hours per day?

4. If I spend $\$2\frac{3}{8}$ per day, how much will I spend in 31 days?

5. At $12\frac{1}{2}$ cents per yard, what will 3 yards of cloth cost? 4 yards?

6. At $7\frac{1}{2}$ cents per pound, what will 6 pounds of rice cost? 7 pounds? 8 pounds? 10 pounds?

7. If a jar of butter holds $6\frac{1}{4}$ pounds, how many pounds will 2 jars hold? 3? 4? 5? 6? 7? 8? 9? 10?

8. At $14\frac{1}{2}$ cents per yard, what will 2 yards cost? 3 yds.? 4 yds.? 5 yds.? 6 yds.? 7 yds.?

9. A can do a piece of work in 5 days, and B can do it in 6 days. How much can both do in 2 days?

Find the value of the following:

10. $3 \times \frac{3}{4}$

14. $12 \times \frac{5}{7}$

18. $5 \times 6\frac{1}{4}$

11. $4 \times \frac{5}{6}$

15. $15 \times \frac{4}{5}$

19. $6 \times 12\frac{1}{2}$

12. $5 \times \frac{6}{7}$

16. $18 \times \frac{3}{7}$

20. $4 \times 16\frac{3}{4}$

13. $8 \times \frac{3}{8}$

17. $24 \times \frac{4}{9}$

21. $8 \times 8\frac{1}{3}$

II.

1. A house rents for \$40 per month. What is the rent for $\frac{1}{4}$ of a month? $\frac{1}{2}$ mo.? $\frac{2}{3}$ mo.? $\frac{3}{4}$ mo.? $\frac{5}{8}$ mo.?

2. A farm sold for \$12000. What is $\frac{2}{3}$ of it worth? $\frac{3}{4}$ of it? $\frac{5}{8}$ of it? $\frac{7}{8}$ of it? $\frac{5}{12}$ of it?

3. What is the cost of $\frac{3}{4}$ T. of coal, if 1 T. cost \$8? \$4? \$2?

4. What is the cost of 3 months board if 4 months cost \$100?

5. If A can do a piece of work in 24 days, in what time can he do $\frac{1}{4}$ of it? $\frac{3}{4}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{5}{12}$?

6. A certain sum of money will buy 60 bushels of wheat, how much will $\frac{2}{3}$ of this sum buy? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{1}{3}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{10}$? $\frac{5}{12}$?

7. A hogshead of molasses is worth \$75. What is $\frac{1}{2}$ of it worth? $\frac{1}{4}$ of it? $\frac{3}{4}$ of it? $\frac{5}{8}$ of it? $\frac{5}{12}$ of it?

8. If a man can ride 200 miles on a bicycle in 2 days, how far can he ride in $\frac{1}{2}$ of a day? in $\frac{3}{4}$? in $\frac{5}{8}$? in $\frac{3}{10}$?

Find the value of the following:

9. $\frac{1}{2}$ of 14

14. $\frac{4}{7}$ of 100

19. $\frac{2}{3} \times 8$

10. $\frac{2}{3}$ of 16

15. $\frac{3}{16}$ of 100

20. $\frac{5}{6} \times 7$

11. $\frac{3}{4}$ of 15

16. $\frac{5}{12}$ of 150

21. $\frac{3}{8} \times 15$

12. $\frac{3}{8}$ of 50

17. $\frac{2}{20}$ of 75

22. $\frac{7}{12} \times 11$

13. $\frac{5}{8}$ of 75

18. $\frac{7}{15}$ of 50

23. $\frac{9}{16} \times 7$

III.

1. What is the value of $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{7}$ of $\frac{7}{10}$?
2. What is the cost of $\frac{1}{4}$ lb. of tea, if 1 lb. cost $\$4$?
3. A boy has $\frac{1}{2}$ of a dollar, and gives $\frac{1}{2}$ of it to his friend. What part does he give away?
4. A farmer has $\frac{7}{8}$ of a bushel of cloverseed, and sows $\frac{1}{8}$ of it. What part of a bushel does he sow, and what part of a bushel does he have left?
5. What is the value of $\frac{3}{8}$ A., if $1\frac{1}{2}$ A. cost $\$75$?
6. If $\frac{3}{4}$ of a bushel of barley costs $18\frac{3}{4}$ cents, what will $2\frac{1}{4}$ bu. cost? $1\frac{1}{2}$ bu.? 3 bu.? $2\frac{1}{4}$ bu.? $\frac{1}{2}$ bu.?
7. What will $\frac{3}{8}$ of a barrel of flour cost at $\$6\frac{1}{4}$ per barrel? $\frac{3}{4}$ of a barrel? $\frac{5}{8}$ of a barrel? $\frac{3}{8}$ of a barrel? $\frac{5}{12}$ of a barrel?

SUGGESTION.— $6\frac{1}{4} = \frac{25}{4}$.

Find the value of the following:

- | | | |
|------------------------------------|---------------------------------------|--|
| 8. $\frac{1}{2}$ of $\frac{3}{8}$ | 13. $\frac{3}{4} \times \frac{2}{3}$ | 18. $\frac{1}{2}$ of $3\frac{1}{2}$ |
| 9. $\frac{1}{3}$ of $\frac{7}{8}$ | 14. $\frac{3}{5} \times \frac{5}{8}$ | 19. $\frac{3}{8}$ of $6\frac{1}{4}$ |
| 10. $\frac{1}{4}$ of $\frac{4}{7}$ | 15. $\frac{5}{6} \times \frac{6}{7}$ | 20. $\frac{3}{4}$ of $8\frac{1}{2}$ |
| 11. $\frac{3}{4}$ of $\frac{7}{8}$ | 16. $\frac{7}{8} \times \frac{5}{12}$ | 21. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{3}{8}$ |
| 12. $\frac{3}{4}$ of $\frac{4}{7}$ | 17. $\frac{3}{8} \times \frac{8}{5}$ | 22. $\frac{3}{8}$ of $\frac{3}{4}$ of $\frac{5}{16}$ |

WRITTEN PRACTICE.

PRINCIPLE.—Multiplying the numerator multiplies the fraction, or dividing the denominator multiplies the fraction.

GENERAL RULE:

1. To multiply a fraction by a whole number.
 2. To multiply a whole number by a fraction.
 3. To multiply a fraction by a fraction.
 4. To multiply mixed numbers.
- I. Change all whole or mixed numbers to fractions.*
- II. Eject all factors common to both the numerator and denominator.*
- III. Multiply the numerators together for a new numerator; multiply the denominators together for a new denominator.*
- IV. If the new numerator is equal to, or greater than, the new denominator, reduce the fraction to a whole or mixed number.*

Illustrations:

1. $7 \times \frac{3}{4}$ is $\frac{7}{1} \times \frac{3}{4}$ or $\frac{21}{4}$ or $5\frac{1}{4}$.
2. $\frac{5}{6} \times 11$ is $\frac{5}{6} \times \frac{11}{1}$ or $\frac{55}{6}$ or $9\frac{1}{6}$.
3. $\frac{3}{4} \times \frac{5}{7}$ is $\frac{15}{28}$.
4. $2\frac{1}{4} \times 3\frac{1}{2}$ is $\frac{9}{4} \times \frac{7}{2}$ or $\frac{63}{8}$ or $7\frac{7}{8}$.

I.

Find the value of the following:

148. $12 \times \frac{4}{5}$

SOLUTION.

$\frac{12}{1} \times \frac{4}{5} = \frac{48}{5}$ or $9\frac{3}{5}$

149. $18 \times \frac{5}{6}$

155. $1280 \times \frac{2}{15}$

161. $36 \times \frac{1}{6}$

150. $23 \times \frac{3}{8}$

156. $36 \times \frac{1}{12}$

162. $93 \times 3\frac{3}{4}$

151. $125 \times \frac{2}{3}$

157. $48 \times \frac{3}{8}$

163. $125 \times 4\frac{3}{8}$

152. $240 \times \frac{9}{16}$

158. $96 \times \frac{5}{6}$

164. $625 \times 3\frac{7}{16}$

153. $84 \times \frac{3}{25}$

159. $397 \times \frac{7}{16}$

165. $1825 \times 2\frac{3}{8}$

154. $25 \times \frac{7}{8}$

160. $3125 \times \frac{3}{8}$

166. $124 \times 7\frac{5}{16}$

167. What cost 27 bushels of apples at $\$ \frac{3}{4}$ per bushel?168. If you earn $\$ \frac{7}{8}$ per day, how much will you earn in 31 days?169. If a man rides $18\frac{3}{4}$ miles per hour on a bicycle, how far will he go in 9 hours?170. I bought 20 pounds of sugar at $\$ \frac{1}{18}$ per pound, 6 pounds of tea at $\$ \frac{5}{8}$ per pound, 12 pounds of coffee at $\$ \frac{3}{16}$ per pound, and 14 gallons of molasses at $\$ \frac{3}{8}$ per gallon. What was the amount of my purchases?

II.

Find the value of the following:

171. $\frac{3}{4} \times 27$

SOLUTION.

$\frac{3}{4} \times \frac{27}{1} = \frac{81}{4}$ or $20\frac{1}{4}$

172. $\frac{7}{8} \times 21$

178. $\frac{5}{16} \times 35$

184. $\frac{18}{5} \times 36$

173. $\frac{3}{16} \times 125$

179. $\frac{7}{8} \times 36$

185. $2\frac{1}{2} \times 47$

174. $\frac{4}{15} \times 128$

180. $\frac{19}{64} \times 84$

186. $3\frac{1}{8} \times 64$

175. $\frac{3}{25} \times 24$

181. $\frac{1}{128}$ of 25

187. $5\frac{7}{8} \times 23$

176. $\frac{2}{175} \times 80$

182. $\frac{27}{120} \times 65$

188. $14\frac{3}{7} \times 85$

177. $\frac{3}{24}$ of 16

183. $\frac{3}{1728} \times 184$

189. $6\frac{3}{8} \times 27$

190. A mill is valued at \$9400. What is the value of $\frac{7}{16}$ of it?

191. What is the value of $\frac{3}{4}$ of a farm of 84 acres at \$95 per acre?

192. A man owns $\frac{7}{8}$ of a mill valued at \$7500, and sells $\frac{3}{4}$ of his share. What is the value of the part sold? What part has he remaining?

III.

Find the value of the following:

193. $\frac{3}{4} \times \frac{7}{8}$. SOLUTION.— $\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$.

194. $\frac{2}{3} \times \frac{7}{16}$ 197. $1\frac{1}{2} \times \frac{3}{4}$ 200. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

195. $\frac{5}{6} \times \frac{7}{8}$ 198. $2\frac{1}{2} \times 1\frac{1}{2}$ 201. $1\frac{1}{2} \times 2\frac{3}{4} \times \frac{3}{4}$

196. $\frac{9}{16}$ of $\frac{2}{3}$ 199. $4\frac{3}{4} \times 3\frac{3}{8}$ 202. $2\frac{1}{2} \times 2\frac{1}{5} \times 3\frac{1}{4}$

203. At $\frac{7}{8}$ of a dollar per pound, what will $\frac{3}{4}$ of a pound of tea cost?

204. A man owns $\frac{3}{4}$ of a store and sells $\frac{3}{4}$ of his share. What part does he sell, and what part has he left?

205. What is $\frac{2}{3}$ of the cost of $3\frac{1}{2}$ cords of wood at \$21 per cord?

IV.

To multiply, when mixed numbers are large.

206. $24\frac{3}{4} \times 86$; $15 \times 316\frac{3}{4}$; $15\frac{2}{3} \times 24\frac{3}{4}$.

SOLUTION.

86	$316\frac{3}{4}$	$24\frac{3}{4}$
24 $\frac{3}{4}$	15	15 $\frac{2}{3}$
344	1580	120
172	316	24
641 $\frac{1}{2}$	111 $\frac{1}{4}$	$16 = 24 \times \frac{2}{3}$
2128 $\frac{1}{2}$	4751 $\frac{1}{4}$	$11\frac{1}{4} = 15 \times \frac{3}{4}$
		$\frac{1}{2} = \frac{3}{4} \times \frac{2}{3}$
		387 $\frac{3}{4}$

Find the value of the following:

207. $36\frac{1}{2} \times 15$ 212. $75 \times 24\frac{3}{14}$ 217. $24\frac{3}{4} \times 16\frac{3}{4}$

208. $84\frac{3}{4} \times 16$ 213. $84 \times 25\frac{5}{8}$ 218. $36\frac{2}{3} \times 18\frac{3}{4}$

209. $37\frac{5}{8} \times 24$ 214. $12\frac{1}{2} \times 12\frac{1}{2}$ 219. $75\frac{3}{4} \times 36\frac{3}{8}$

210. $96\frac{3}{2} \times 64$ 215. $25\frac{1}{2} \times 25\frac{1}{2}$ 220. $25\frac{3}{8} \times 48\frac{3}{4}$

211. $25\frac{1}{4} \times 9$ 216. $35\frac{1}{4} \times 36\frac{3}{8}$ 221. $96\frac{3}{8} \times 28\frac{5}{8}$

Find the cost of

222. $24\frac{3}{4}$ bu. of wheat at \$1 $\frac{1}{8}$ per bu.

223. $18\frac{3}{8}$ cords of wood at $\$6\frac{3}{4}$ per cord.

224. $16\frac{3}{4}$ tons of coal at $\$3\frac{1}{8}$ per ton.

225. $36\frac{7}{8}$ bu. of clover seed at $\$2\frac{1}{2}$ per bu.

226. $25\frac{3}{8}$ M feet of lumber at $\$12\frac{1}{4}$ per M.

227. A commission merchant receives 4 hogs weighing respectively $246\frac{1}{4}$, $275\frac{3}{8}$, $321\frac{7}{16}$, and $302\frac{5}{8}$ pounds to be sold on commission. He sells at $7\frac{1}{2}$ cents per pound. How much does he remit, if he deducts $\$16.50$ as his commission?

228. A merchant sold 20 pieces of denim containing respectively 40^2 , 41^1 , 41^1 , 43 , 42^2 , 44 , 43^3 , 42 , 45 , 42^1 , 40^1 , 41^1 , 42^2 , 44 , 41^3 , 43 , 47^2 , 43^1 , 42 , 43^1 yards, at $12\frac{1}{2}$ cents per yard. How much did he receive? (The small figures represent numerators with the denominator 4 understood.)

229. A merchant sold 20 yards of silk at $\$2\frac{1}{2}$ per yard, $26\frac{1}{4}$ yards of satin at $\$1\frac{1}{2}$ per yard, $36\frac{3}{4}$ yards of denim at $\$1\frac{1}{8}$ per yard, $24\frac{3}{4}$ yards of carpet at $\$3\frac{3}{4}$ per yard. What was the amount of the sale?

230. A wood and coal dealer sold $16\frac{3}{4}$ cords of hard wood at $\$6\frac{1}{2}$ per cord, $20\frac{1}{2}$ cords of soft wood at $\$3\frac{1}{4}$ per cord, $18\frac{3}{4}$ tons of hard coal at $\$7\frac{1}{2}$ per ton, $23\frac{1}{8}$ tons of soft coal at $\$2\frac{1}{4}$ per ton. Find the amount of the sales.

Find the value of:

$$231. \frac{3}{4} \times \frac{7}{8} \times \frac{9}{16} \times \frac{8}{9} \times \frac{5}{8}$$

$$235. 5\frac{1}{2} \times 5\frac{1}{2} \times 7\frac{1}{2} \times \frac{5}{8}$$

$$232. 2\frac{1}{2} \times 4\frac{1}{2} \times 6\frac{3}{4} \times \frac{3}{8}$$

$$236. 3\frac{7}{8} \times 4\frac{1}{2} \times 18\frac{3}{4}$$

$$233. 16\frac{3}{8} \times 9 \times 24\frac{3}{4} \times 12\frac{1}{2}$$

$$237. 88\frac{1}{2} \times 9\frac{3}{4} \times 7\frac{1}{2}$$

$$234. 12\frac{1}{2} \times 12\frac{1}{2} \times 8 \times 4\frac{3}{4}$$

$$238. 16\frac{3}{8} \times 30\frac{7}{8} \times 1\frac{1}{16}$$

239. At $\frac{7}{8}$ of $\$16\frac{3}{4}$ per ton, what will $\frac{5}{8}$ of a ton of clover hay cost?

240. Find the total of the results:

$$75\frac{3}{4} + 18\frac{7}{8} + 62\frac{1}{2} - 46\frac{2}{3}, \times 6\frac{1}{4} =$$

$$25\frac{1}{8} + 63\frac{5}{8} - 33\frac{7}{16} - 6\frac{5}{8}, \times 5\frac{1}{2} =$$

$$14\frac{7}{4} + 1264\frac{3}{8} - 325\frac{7}{8}, \times 2 =$$

241. Find the total of the results:

$$42\frac{1}{2} + 18\frac{3}{4} + 62\frac{7}{8} - (25\frac{5}{16} \times 3\frac{1}{4}) =$$

$$(\frac{3}{4} + \frac{7}{8} - \frac{5}{8} - \frac{5}{4}) \times (\frac{3}{4} + 6\frac{7}{8}) =$$

$$7\frac{1}{2} + 8\frac{1}{4} + 9\frac{1}{8} - 12\frac{1}{16} \times (2\frac{1}{2} + 3\frac{1}{4}) =$$

NOTE.—The value of the numbers in parenthesis should be taken as a single number.

DIVISION.

MENTAL PRACTICE.

I.

1. If a pound of coffee cost $\$1\frac{2}{10}$, how many pounds can be had for $\$4$? $\$1\frac{6}{10}$? $\$1$? $\$2$? $\$5$? $\$10$?

SOLUTION BY RATIO.—The ratio of $\frac{4}{10}$ to $\frac{2}{10}$ is $\frac{4}{2}$, or 2, the number of pounds that can be bought for $\$1\frac{6}{10}$.

SOLUTION BY ANALYSIS.—Since $\$1\frac{2}{10}$ buys 1 pound, $\frac{4}{10}$ dollars will buy as many pounds as $4 \div 2$, or 2 pounds.

2. If a boy earns $\$3$ per day, in what time can he earn $\$4$? $\$3$? $\$3$? $\$7$? $\$2$? $\$5$? $\$10$? $\$30$?

3. How many bushels of wheat, at $\$4$ per bushel, can be bought for $\$3$? $\$4$? $\$6$? $\$8$? $\$10$? $\$12$?

4. If a horse eats $\frac{3}{8}$ of a bushel of corn per day, in what time will he eat 2 bu.? 4 bu.? 6 bu.? 8 bu.? 12 bu.?

5. Divide 12 by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$.

6. Divide 15 by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$.

7. How many bushels of potatoes can be bought for $\$12$ at $\$1$ per bushel? at $\$2$? at $\$3$? at $\$4$? at $\$5$? at $\$6$?

8. 8 is how many times $\frac{1}{2}$? $\frac{3}{4}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{9}{8}$? $\frac{10}{8}$?

9. How many oranges at $\frac{3}{4}$ of a cent each, can be bought for 6 cents? 8 cents? $7\frac{1}{2}$ cents?

10. How many times is $\frac{5}{8}$ contained in 2? in 4? in 8? in $10\frac{1}{2}$?

Find the value of

11. $8 \div \frac{2}{3}$, or $\frac{3}{2}$ of 8

16. $15 \div 2\frac{1}{2}$

12. $\frac{1}{2} \div \frac{3}{4}$, or $\frac{4}{3}$ of $\frac{1}{2}$

17. $21 \div 2\frac{1}{3}$

13. $12 \div \frac{5}{8}$, or $\frac{8}{5}$ of 12

18. $25 \div 6\frac{1}{2}$

14. $10 \div 3\frac{1}{2}$, or $\frac{2}{3}$ of 10

19. $50 \div 8\frac{1}{2}$

15. $12 \div 1\frac{1}{2}$, or $\frac{2}{3}$ of 12

20. $75 \div 2\frac{1}{2}$

II.

1. If 6 bushels of potatoes cost $\$1\frac{1}{2}$, what is the cost of one bushel?

SOLUTION.—The ratio of the cost of 1 bu. to the cost of 6 bu. is $\frac{1}{6}$; $\frac{1}{6}$ of $\$1\frac{1}{2}$ is $\$1\frac{1}{2} \times \frac{1}{6}$, the cost of 1 bu.

2. If 5 pounds of sugar cost $\$1$, what is the cost of 1 pound?

3. If you earn $\$6\frac{3}{4}$ in 4 days, how much do you earn in 1 day?

4. If 11 bushels of clover seed cost $\$16\frac{1}{2}$, how much does 1 bushel cost? $2\frac{1}{2}$ bushels?

5. If you pay $\$3\frac{1}{2}$ for 5 days' board, how much is that per day?

Find the value of:

- | | |
|--|----------------------------|
| 6. $\frac{3}{4} \div 6$, or $\frac{1}{6}$ of $\frac{3}{4}$ | 13. $3\frac{1}{3} \div 4$ |
| 7. $\frac{3}{8} \div 2$, or $\frac{1}{2}$ of $\frac{3}{8}$ | 14. $8\frac{1}{3} \div 5$ |
| 8. $1\frac{2}{3} \div 3$, or $\frac{1}{3}$ of $1\frac{2}{3}$ | 15. $14\frac{2}{7} \div 7$ |
| 9. $\frac{3}{8} \div 4$, or $\frac{1}{4}$ of $\frac{3}{8}$ | 16. $\frac{1}{8} \div 5$ |
| 10. $\frac{5}{6} \div 7$, or $\frac{1}{7}$ of $\frac{5}{6}$ | 17. $3\frac{2}{3} \div 6$ |
| 11. $1\frac{1}{2} \div 3$, or $\frac{1}{3}$ of $1\frac{1}{2}$ | 18. $8\frac{1}{4} \div 2$ |
| 12. $7\frac{1}{2} \div 5$, or $\frac{1}{5}$ of $7\frac{1}{2}$ | 19. $16\frac{2}{3} \div 9$ |

III.

1. If a yard of cloth cost $\$1\frac{2}{3}$, how many yards can be bought for $\$1$? for $\$1\frac{1}{3}$? for $\$1\frac{1}{2}$? for $\$2\frac{1}{2}$?

SOLUTION.— $\frac{3}{4} = 1\frac{2}{3}$. The ratio of $1\frac{2}{3}$ to $\frac{3}{4}$ is $1\frac{2}{3}$ or 4, the number of yards that can be bought for $\$1$.

2. If coffee can be bought at the rate of $\$1$ per lb., how many lb. can be bought for $\$1\frac{1}{2}$? $\$2$? $\$2\frac{1}{2}$? $\$3$? $\$3\frac{1}{2}$? $\$4$? $\$4\frac{1}{2}$? $\$5$? $\$5\frac{1}{2}$? $\$6$? $\$6\frac{1}{2}$?

3. At $\$3\frac{1}{4}$ per bushel for potatoes, how many bushels can be bought for $\$1\frac{1}{2}$? $\$2\frac{1}{4}$? $\$5\frac{1}{2}$? $\$7\frac{1}{4}$?

4. How many times $\frac{2}{3}$ is $\frac{4}{3}$? $\frac{5}{3}$? $3\frac{1}{3}$? $4\frac{2}{3}$? $5\frac{5}{3}$?

5. In what time can a boy earn $\$7\frac{1}{2}$ at $\$1\frac{1}{2}$ per day? a. $\$1$ at $\$1\frac{1}{2}$? at $\$1\frac{1}{3}$? at $\$1\frac{1}{4}$?

6. How many times $\frac{3}{4}$ is $\frac{7}{4}$? $1\frac{1}{4}$? $2\frac{1}{4}$? $2\frac{5}{4}$? $3\frac{1}{4}$?

Find the value of:

- | | |
|--|---------------------------------------|
| 7. $\frac{5}{8} \div \frac{3}{8}$, or $\frac{3}{8}$ of $\frac{5}{8}$ | 12. $1\frac{1}{2} \div \frac{5}{8}$ |
| 8. $\frac{3}{4} \div \frac{1}{2}$, or $\frac{1}{2}$ of $\frac{3}{4}$ | 13. $4\frac{1}{2} \div \frac{3}{4}$ |
| 9. $\frac{7}{8} \div \frac{3}{4}$, or $\frac{3}{4}$ of $\frac{7}{8}$ | 14. $3\frac{1}{3} \div \frac{2}{3}$ |
| 10. $\frac{9}{7} \div \frac{2}{3}$, or $\frac{3}{2}$ of $\frac{9}{7}$ | 15. $6\frac{1}{4} \div \frac{1}{2}$ |
| 11. $3\frac{1}{2} \div \frac{1}{4}$, or $\frac{1}{4}$ of $3\frac{1}{2}$ | 16. $12\frac{1}{2} \div 2\frac{1}{2}$ |

WRITTEN PRACTICE.

PRINCIPLE.—Dividing the numerator divides the fraction, or multiplying the denominator divides the fraction.

1. To divide a whole number by a fraction.
2. To divide a fraction by a whole number.
3. To divide a fraction by a fraction.
4. To divide mixed numbers.

GENERAL RULE:

- I. Reduce all whole and mixed numbers to fractions.*
- II. Change the division sign to a multiplication sign and invert the terms of the divisor.*
- III. Cancel all factors common to both terms.*
- IV. Multiply the remaining numerators for a new numerator; multiply the remaining denominators for a new denominator.*
- V. If the new numerator is equal to, or greater than, the new denominator, divide the numerator by the denominator, and the result is the required answer.*

I

To divide a whole number by a fraction.

242. $7 \div \frac{2}{3} =$

SOLUTION.— $7 \div \frac{2}{3} = 7 \div \frac{2}{3} = 7 \times \frac{3}{2} = 10\frac{1}{2}$

Find the value of the following:

243. $\frac{1}{2} \div \frac{3}{4}$

250. $7 \div 1\frac{1}{2}$

257. $640 \div \frac{3}{8}$

244. $\frac{2}{3} \div \frac{7}{8}$

251. $11 \div 1\frac{3}{8}$

258. $560 \div 2\frac{1}{4}$

245. $36 \div \frac{5}{8}$

252. $15 \div 2\frac{5}{8}$

259. $420 \div 3\frac{1}{2}$

246. $120 \div 1\frac{5}{8}$

253. $62 \div 3\frac{7}{8}$

260. $144 \div 5\frac{1}{4}$

247. $18 \div 3\frac{1}{8}$

254. $300 \div 6\frac{2}{3}$

261. $28 \div 2\frac{3}{8}$

248. $11 \div 2\frac{5}{4}$

255. $105 \div 7\frac{1}{2}$

262. $120 \div 12\frac{1}{2}$

249. $12 \div \frac{6}{5}$

256. $195 \div 9\frac{3}{4}$

263. $1720 \div 18\frac{3}{4}$

II.

To divide a fraction by a whole number.

264. $\frac{3}{4} \div 7 = ?$

SOLUTION.— $\frac{3}{4} \div 7 = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$

Find the value of:

265. $\frac{7}{8} \div \frac{5}{1}$

272. $\frac{144}{95} \div 72$

279. $\$66\frac{2}{3} \div 9$

266. $\frac{15}{16} \div \frac{3}{1}$

273. $\frac{125}{144} \div 25$

280. $\$18\frac{3}{4} \div 15$

267. $\frac{3}{2} \div 3$

274. $\frac{156}{23} \div 4$

281. $\$112\frac{1}{2} \div 9$

268. $\frac{5}{4} \div 9$

275. $\frac{320}{17} \div 8$

282. $133\frac{1}{3} \div 25$

269. $\frac{11}{96} \div 24$

276. $7\frac{1}{2} \div 3$

283. $85\frac{5}{7} \div 30$

270. $\frac{2}{3} \div 12$

277. $16\frac{1}{4} \div 4$

284. $93\frac{3}{4} \div 124$

271. $\frac{7}{8} \div 24$

278. $21\frac{7}{8} \div 2$

285. $202\frac{1}{4} \div 125$

286. At $\$ \frac{3}{4}$ for 12 pounds of sugar, what is the cost of 1 pound?

287. When $5\frac{1}{4}$ bushels of potatoes cost 126 cents, what is the cost per bushel?

288. If $\frac{7}{8}$ of an estate is equally divided among 12 heirs, what part of the estate does each receive?

289. What is the cost of 12 pounds of beef, if 5 pounds cost $23\frac{3}{4}$ cents?

III.

To divide a fraction by a fraction.

290. $\frac{7}{8} \div \frac{4}{5} = ?$

SOLUTION.— $\frac{7}{8} \div \frac{4}{5} = \frac{7}{8} \times \frac{5}{4} = \frac{35}{32} = 1\frac{3}{32}$

Find the value of:

291. $\frac{5}{6} \div \frac{3}{4}$

298. $1\frac{1}{2} \div \frac{2}{3}$

305. $2\frac{1}{2} \div 2\frac{1}{3}$

292. $\frac{9}{16} \div \frac{4}{3}$

299. $2 \div \frac{3}{4}$

306. $3 \div 2\frac{1}{2}$

293. $\frac{1}{4} \div \frac{1}{8}$

300. $3\frac{1}{3} \div \frac{5}{6}$

307. $6\frac{2}{3} \div 3\frac{1}{3}$

294. $\frac{9}{24} \div \frac{3}{4}$

301. $6\frac{1}{4} \div \frac{5}{7}$

308. $8\frac{1}{3} \div 2\frac{1}{2}$

295. $\frac{15}{16} \div \frac{5}{6}$

302. $12\frac{1}{2} \div \frac{3}{8}$

309. $16\frac{2}{3} \div 6\frac{2}{3}$

296. $\frac{3}{16} \div \frac{5}{6}$

303. $16\frac{2}{3} \div \frac{5}{6}$

310. $18\frac{3}{4} \div 6\frac{1}{4}$

297. $\frac{1}{28} \div \frac{1}{12}$

304. $21\frac{7}{8} \div 2$

311. $202\frac{1}{4} \div 125$

312. What number multiplied by $32\frac{3}{4}$ will produce $283\frac{5}{8}$?

313. If coffee is worth $\$ \frac{2}{3}$ per pound, how many pounds can be bought for $\$6\frac{3}{4}$?

314. I bought $\frac{7}{8}$ of a ton of coal for $\$6\frac{9}{16}$. How much was that a ton?

315. Paid $\frac{3}{4}$ of \$10 for $\frac{3}{4}$ of 25 bushels of corn. How much was that per bushel?

316. If a man can travel $23\frac{1}{2}$ miles in 2 hours, how far can he travel in $1\frac{1}{4}$ hours?

IV.

To divide when the mixed numbers are large.

317. $96\frac{3}{8} \div 24\frac{3}{4} = ?$

SOLUTION.

$$\begin{array}{r} 24\frac{3}{4} \overline{) 96\frac{3}{8}} \\ 297 \overline{) 1160} (3\frac{2}{3} \frac{6}{9} \frac{9}{9} \\ \underline{891} \\ 269 \\ \underline{297} \end{array}$$

EXPLANATION.—Fractions are to each other as their numerators. Reduce to common denominator and divide the numerator of the dividend by the numerator of the divisor.

Find the value of:

318. $24\frac{3}{4} \div 8\frac{1}{8}$

319. $87\frac{1}{2} \div 22\frac{1}{2}$

320. $114\frac{3}{7} \div 28\frac{4}{7}$

321. $3264\frac{3}{4} \div 124\frac{1}{2}$

322. $863\frac{3}{8} \div 20\frac{1}{4}$

323. $362\frac{1}{8} \div 21\frac{1}{6}$

324. $1263\frac{4}{9} \div 124\frac{3}{8}$

325. $3614\frac{5}{12} \div 144\frac{1}{4}$

326. A mechanic receives $\$363\frac{3}{8}$ for $42\frac{3}{4}$ days' work. How much does he receive per day?

327. My farm contains $371\frac{1}{4}$ acres. How many fields can be formed from it, if each field contains $24\frac{3}{4}$ acres?

328. If $18\frac{3}{4}$ acres produce $568\frac{3}{4}$ bushels of corn, how many bushels will $131\frac{1}{4}$ acres produce?

329. How many cords of wood can be bought for $\$265\frac{1}{2}$ at $\$7\frac{3}{8}$ per cord?

Find the value of

330. $\frac{7\frac{1}{2}}{4}$ or $7\frac{1}{2} \div 4$

331. $\frac{31\frac{3}{4}}{6\frac{1}{2}}$ or $31\frac{3}{4} \div 6\frac{1}{2}$

332. $\frac{36\frac{3}{4} + 6\frac{1}{2}}{6\frac{1}{8}}$

333. $\frac{4\frac{1}{2} + 3\frac{3}{4} + 2\frac{1}{2}}{4 + 3\frac{1}{2}}$

334. $\frac{6\frac{3}{4} + 18\frac{3}{8} - 6\frac{1}{2}}{3\frac{1}{2} \times 2\frac{1}{3}}$

335. $\frac{\frac{3}{4} \text{ of } 18\frac{3}{4} + 8\frac{1}{8}}{6\frac{3}{4} - 2\frac{1}{3}}$

336. $\frac{\frac{7}{8} \text{ of } (8\frac{3}{4} + 16\frac{3}{8})}{\frac{3}{4} \text{ of } (8\frac{1}{8} - 5\frac{1}{4})}$

337. $\frac{\frac{3}{4} \times 7\frac{1}{2} + 24\frac{7}{8}}{12\frac{3}{4} + 16\frac{3}{8} + 96\frac{3}{8}}$

338. $\frac{3\frac{1}{4} + 27\frac{5}{6} - 6\frac{1}{2} \times 3\frac{1}{4}}{(\frac{3}{8} \text{ of } 25\frac{1}{4}) + (\frac{3}{4} \text{ of } 27)}$

339. $\frac{75\frac{1}{2} + 8\frac{1}{6} + 7\frac{3}{4} - 1\frac{7}{8}}{31\frac{1}{4} \div 6\frac{1}{4}}$

MENTAL PRACTICE.

1. What is the ratio of 15 bu. to 36 bu.?
2. $\$4 = \frac{3}{4}$ = what part of \$4? \$6? $\$7\frac{1}{2}$?
3. What is the ratio of $\frac{3}{8}$ of a quart to 6 quarts?
4. $\frac{3}{8}$ of an apple = what part of $\frac{3}{4}$ of an apple?
5. $\frac{3}{4}$ = what part of $\frac{7}{8}$?
6. What is the ratio of $\$6\frac{1}{4}$ to $\$31\frac{1}{4}$?
7. $\$18\frac{3}{4}$ = what part of $\$56\frac{1}{4}$?
8. $\frac{7}{8}$ = what part of $\frac{5}{12}$?
9. The cost of $62\frac{1}{2}$ acres of land is what part of the cost of $87\frac{1}{2}$ acres of land?
10. If I pay $\$33\frac{1}{3}$ for $66\frac{2}{3}$ bu. of wheat, how many bushels should I receive for $\$83\frac{1}{3}$?
11. $\frac{5}{8}$ of 48 is $\frac{4}{5}$ of what number?
12. $\frac{7}{8}$ of 96 is $\frac{1}{6}$ of what number?
13. How many pounds of butter in 7 jars holding respectively, $6\frac{1}{2}$, $7\frac{3}{4}$, $5\frac{3}{8}$, $8\frac{1}{4}$, $6\frac{5}{8}$, $5\frac{9}{16}$, and $8\frac{7}{16}$ pounds?
14. What cost 15 oranges at 2 oranges for 3 cents?
15. How many gallons of water will it take to fill 5 tubs, each holding $6\frac{1}{2}$ gallons?
16. At $8\frac{1}{2}$ cents per pound, how many pounds of sugar can be bought for $\$1\frac{1}{2}$?
17. What is the sum of $\frac{1}{2} + \frac{3}{8} + \frac{3}{4} + \frac{5}{8}$?
18. What is the value of $6 \times 5\frac{2}{3}$? $8 \times 7\frac{3}{4}$?
19. What is the value of $6 \times 5\frac{2}{3}$? $8 \times 7\frac{3}{4}$? $9\frac{1}{2} \times 7$?
20. What is the cost of $\frac{3}{4}$ of 24 bu. of wheat at \$.75 per bushel?
21. A can do a piece of work in 4 days, B can do the same work in 5 days. How much can each do in one day? Both?
22. A can do a piece of work in 6 days, and B can do the same amount of work in 8 days. What part of the work more can A do in one day than B?
23. A can do a piece of work in 8 days, and B can do the same amount of work in 10 days. How long will it take both to do the work?
24. A can do $\frac{3}{8}$ of a piece of work in one day, and B can do $\frac{4}{8}$ of an equal piece in one day. How long will it take both to do the work?

25. A can do a piece of work in $\frac{3}{4}$ of 4 days, and B can do the same amount of work in $\frac{3}{4}$ of 5 days. How long will it take both to do the work?

26. A, B, and C can do a piece of work in 6 days; B and C can do it in 9 days. How long will it take A to do it?

27. What cost $\frac{7}{8}$ of a barrel of flour at \$6.40 per barrel?

28. What cost 3 barrels of pork, if $\frac{3}{4}$ of a barrel cost \$12 $\frac{3}{4}$?

29. By selling a cow for \$36, a gain of $\frac{1}{3}$ of the cost was made. What was the cost?

30. I sold a horse for \$75, thereby losing $\frac{1}{4}$ of the cost. What was the cost?

31. The selling price of a house and lot was \$2400, thereby gaining $\frac{1}{5}$ of the cost price. What was the gain?

32. If you sell an article for \$39, and lose $\frac{1}{14}$ of the cost, what is the loss?

33. A man spent $\frac{3}{5}$ of his fortune and has \$2400 left. How much had he at first?

34. If you spend $\frac{1}{2}$ of your weekly salary for board, $\frac{1}{4}$ for books, and $\frac{1}{8}$ for incidentals, and have \$2 left, what is your weekly salary?

35. A clerk spends $\frac{1}{2}$ of his salary for board and room rent, $\frac{1}{3}$ of the remainder for incidentals, and has \$9 left. How much does he receive in 4 weeks?

36. A farmer sold $\frac{3}{4}$ of his sheep, and bought $\frac{1}{2}$ as many as he sold. He now has 90 sheep. How many had he at first?

37. When Mary is $\frac{3}{4}$ older than she is now, she will be 30 years old. How old is she now?

38. A boy sold oranges at the rate of 6 for 10 cents. How much did he receive for 3 oranges? 12 oranges? 15 oranges?

39. If $\frac{3}{4}$ of a yard of satin cost \$ $\frac{8}{9}$, what will $\frac{3}{4}$ of a yard cost?

40. If $\frac{3}{4}$ of a yard of broadcloth cost \$4, what will 3 yards cost?

41. If you divide 12 apples equally among 9 boys, how many will each receive?

42. By giving each girl $\frac{3}{8}$ of an orange, it takes 12 oranges to go around. How many girls are there?

43. How many yards of silk will $\$66\frac{2}{3}$ buy at $\$3\frac{1}{3}$ per yard?
44. $\frac{4}{7} = \frac{2}{5}$ of what number?
45. What is the ratio of $\frac{3}{7}$ to $\frac{4}{5}$?
46. $\frac{3}{4}$ of a number $+ 6 = 42$, what is the number?
47. $\frac{3}{8}$ of a number $- 7 = 38$, what is the number?
48. A and B together have $\$24\frac{1}{4}$; B has $\$3\frac{1}{2}$ more than A. How many have each?
49. What will $6\frac{1}{4}$ acres of land cost, if $2\frac{1}{2}$ acres cost $\$112\frac{1}{2}$?
50. The difference between $\frac{3}{4}$ of a number and $\frac{2}{5}$ of it is 9. What is the number?
51. $\frac{3}{8}$ of a number plus $\frac{3}{4}$ of it is $42\frac{1}{2}$. What is the number?
52. $24 = \frac{3}{4}$ of $\frac{1}{10}$ of the number of sheep in a pasture. How many sheep in the pasture?
53. If 8 pounds of butter cost $\$3\frac{1}{4}$, what will 5 pounds cost?
54. What is the cost of 1 bushel of rye, if 7 bushels cost $\$2\frac{3}{8}$?
55. What cost $8\frac{1}{2}$ cords of wood at $\$6\frac{3}{4}$ per cord?
56. If you sell $\frac{3}{4}$ of an acre of land for $\$27$, what will you get for 6 acres at the same rate?
57. A clerk pays $\$6\frac{2}{3}$ per week for board. How long will $\$83\frac{1}{3}$ pay his board?
58. If 3 pounds of butter cost $37\frac{1}{2}$ cents, how much change should you receive from a two-dollar bill, when buying 13 pounds?
59. What part of a ton of hay can be bought for $\$6\frac{1}{4}$, when hay sells at $\$8\frac{1}{8}$ per ton?
60. A farmer has $\frac{1}{4}$ of his farm in wheat, $\frac{1}{5}$ in oats, $\frac{1}{6}$ in corn, and the balance, which is 72 acres, in pasture. How many acres in all, and how many in each kind of grain?
61. Four tables that cost $\$16$ each were sold at the following prices, $\$18$, $\$20$, $\$21$, and $\$24$. What part of the cost price was the gain in each case?
62. How many pounds of coffee at $\$1\frac{1}{8}$ a pound can be had for 8 bushels of potatoes at $\$1\frac{5}{8}$ a bushel?
63. If 3 be added to both terms of the fraction $\frac{7}{8}$, will its value be increased or diminished, and how much?
64. If 3 be subtracted from both terms of the fraction $\frac{7}{8}$, will its value be increased or diminished, and how much?
65. A man spent $\frac{1}{4}$ of his money for a suit of clothes, and

$\frac{1}{2}$ of the remainder for a rifle, and has left \$80. How much had he at first?

66. A farmer has $\frac{1}{3}$ of his sheep in one pasture, $\frac{2}{5}$ in another, and 56 in the third. How many has he in all?

67. A has \$32, B has $\frac{1}{4}$ more than A, and C has $\frac{1}{3}$ more than A and B together. How many dollars have they together?

68. $\frac{3}{4}$ of \$25 will buy how many barrels of apples, each holding 3 bushels at 25 cts. per bushel?

69. Two men own one-half of a farm valued at \$9600. One has one-half more than the other. What is the value of each one's share?

70. A and B together have \$8500. $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's. How much has each?

71. An old lady bought 60 eggs at the rate of 5 for 2 cents, and sold them at the rate of 2 for 5 cents. How much did she gain?

72. Two men bought a barrel of flour; one paying \$4 $\frac{1}{2}$ and the other \$6. What part of it should each have?

73. A had \$180. He paid \$40 for a buggy, and $\frac{3}{4}$ of the remainder for a horse. What part of \$180 has he left?

74. A man owns $\frac{3}{4}$ of 1100 acres of land, and sells $\frac{2}{3}$ of his share. How many acres has he left?

WRITTEN PRACTICE.

340. Add $3\frac{3}{4}$, $4\frac{1}{5}$, $9\frac{2}{3}$, $12\frac{5}{8}$, and $7\frac{1}{2}$.

341. What is the sum of $9\frac{3}{8}$, $18\frac{7}{8}$, $24\frac{3}{4}$, $21\frac{5}{8}$, $34\frac{3}{8}$, $14\frac{7}{4}$, and $18\frac{1}{2}$?

342. From the sum of $24\frac{3}{4}$ and $36\frac{7}{8}$ take the sum of $25\frac{3}{8}$ and $16\frac{5}{8}$.

343. What is the total of the sum, difference, product, and quotient of $8\frac{3}{4}$ and $5\frac{1}{2}$?

344. At $52\frac{1}{4}$ miles a day, how far can you travel in $16\frac{1}{2}$ days?

345. The sum of three numbers is $175\frac{1}{6}$, two of the numbers are $26\frac{1}{4}$ and $120\frac{7}{8}$ respectively. What is the other number?

346. The difference between two numbers is $87\frac{1}{2}$, the larger is $256\frac{1}{4}$. What is the less number?

347. The product of two numbers is $364\frac{3}{4}$, one of the numbers is $36\frac{1}{4}$. What is the other number?

348. The quotient is $12\frac{2}{3}$, the divisor is $8\frac{1}{4}$. What is the dividend?

349. The dividend is $316\frac{1}{3}$, the quotient is 15, the remainder is $3\frac{5}{6}$. What is the divisor?

350. If a man earns $\$3\frac{1}{4}$ per day, how much will he earn in $25\frac{1}{2}$ days?

351. If a man earns $\$2\frac{3}{4}$ per day, how long will it take him to earn $\$23\frac{3}{8}$?

352. There are $268\frac{4}{5}$ cubic inches in one gallon, dry measure. How many cubic inches in $\frac{3}{4}$ of a gallon?

353. At $\$2\frac{1}{2}$ per day, how much will 11 masons earn in $7\frac{1}{2}$ days?

354. I gave $\frac{1}{4}$ of my money to A, $\frac{1}{3}$ to B, and $\frac{1}{5}$ to C. What part of my money have I left?

355. If 15 cords of wood cost $\$71\frac{1}{4}$, what will $16\frac{3}{4}$ cords cost?

356. If $7\frac{1}{2}$ cords of wood cost $\$35\frac{5}{8}$, how many cords can be bought for $\$83\frac{1}{4}$?

357. I bought a horse for $\$125$ and found that I had $\frac{3}{5}$ of my money left. How much had I at first?

358. I paid $\$7500$ for a house and lot. The house cost $1\frac{1}{2}$ times as much as the lot. Find the cost of each.

359. A man invested $\frac{1}{3}$ of his money in real estate, $\frac{1}{4}$ in bonds, $\frac{1}{8}$ in railroad stock, and put the balance, $\$2400$, in a bank. How much did he have in all, and how much in each investment?

360. I sold two horses for $\$140$ each. On one I lost $\frac{1}{5}$ of the cost price, and on the other I gained $\frac{1}{5}$. Did I gain or lose by the sale of the horses, and how much?

361. A merchant sold 16 pieces of flannel containing 29^1 , 28 , 28^1 , 26^2 , 27^1 , 33^3 , 36^1 , 34 , 37^1 , 32^2 , 30^2 , 31 , 32^1 , 34^3 , 35 , and 36^2 yards respectively, at $6\frac{1}{4}$ cents per yard. How much did he receive? (The small figures represent quarter yards.)

362. A farm consists of 6 fields as follows: The first contains $24\frac{3}{4}$ acres, the second $43\frac{1}{3}$ acres, the third $37\frac{1}{2}$ acres, the fourth $14\frac{1}{2}$ acres, the fifth $23\frac{2}{3}$ acres, and the sixth $19\frac{1}{5}$ acres. How much was received for the farm, it being sold at $\$75\frac{1}{4}$ per acre?

363. A has $7\frac{1}{2}$ times \$25 $\frac{1}{4}$, B has $12\frac{1}{2}$ times \$124 $\frac{1}{2}$, and C has $25\frac{1}{2}$ times \$25 $\frac{1}{2}$. How many dollars have all?

364. A has $\frac{7}{8}$ of \$475; B has $\frac{3}{4}$ of $\frac{2}{3}$ of \$1250; C has $\frac{4}{5}$ of \$750; D has $\frac{1}{2}$ as much as A, B, and C. How many dollars have all?

365. What will $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{2}{3}$ of $12\frac{1}{2}$ yards of silk cost at $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{1}{8}$ of \$18 a yard?

366. B and C have \$7200; B has $2\frac{1}{2}$ times as much as C. How many dollars has each?

367. A speculator bought 250 acres of land at \$24 $\frac{3}{4}$ per acre, and 175 $\frac{3}{8}$ acres at \$18 $\frac{3}{4}$ per acre. He sold $\frac{3}{4}$ of the first piece at a profit of \$2 $\frac{1}{4}$ per acre, and $\frac{2}{3}$ of the second piece at \$20 per acre. What would be the gain on the whole by selling the balance of the land at \$25 per acre?

368. What will 62 $\frac{1}{2}$ yards of cloth cost, if 25 yards cost \$6 $\frac{1}{4}$?

369. Bought 125 $\frac{3}{4}$ barrels of apples at \$1 $\frac{2}{3}$ per barrel, and sold them at \$1 $\frac{7}{8}$ per barrel. What was the gain?

370. At the rate of 16 $\frac{3}{4}$ cords of wood for \$79 $\frac{9}{16}$, how many cords can be bought for \$198 $\frac{3}{4}$?

371. A husks $7\frac{1}{2}$ bushels of corn in $\frac{3}{4}$ of an hour, and B husks $5\frac{3}{4}$ bushels in $\frac{2}{3}$ of an hour. How many bushels can both husk in 8 hours?

372. Gave 16 $\frac{3}{8}$ pounds of cheese at 12 $\frac{1}{4}$ cents a pound for sugar at 5 cents a pound. How many pounds of sugar was received?

373. Land averaging $3\frac{1}{8}$ tons per acre will yield how many tons on $32\frac{1}{4}$ acres?

374. Bought 14 $\frac{3}{4}$ yards of cashmere for \$42 $\frac{3}{4}$. How many yards can be bought for \$65 $\frac{1}{4}$?

375. A man receives \$4 $\frac{1}{8}$ per day, and his daily expenses are \$2 $\frac{1}{4}$. How long will it take him to save \$156?

376. Find the cost of 11 $\frac{1}{2}$ tons of coal at \$4 $\frac{1}{8}$ per ton with drayage at \$ $\frac{1}{8}$ per ton?

377. A dealer bought 69 $\frac{3}{4}$ bushels of corn at 62 $\frac{1}{2}$ cents per bushel; he sold $\frac{1}{4}$ of it at 67 $\frac{3}{4}$ cents per bushel, and the remainder at 81 $\frac{1}{4}$ cents per bushel. How much did he gain?

378. A can do a piece of work in 8 days, B can do it in 9

days, and C can do it in 12 days. What part of the work can all do in one day?

379. A can do a piece of work in 8 days, B can do it in 9 days, and C can do it in 12 days. How much more can B and C do in one day than A?

380. A can do a piece of work in 6 days, B can do it in 8 days, and C can do it in 10 days. How long will it take them all to do the work?

381. B and C can do a piece of work in 6 days. B can do it in 10 days. How long will it take C to do the work?

382. A can plow a field in $\frac{3}{4}$ of a day, and B can do the same amount of work in $\frac{2}{3}$ of a day. In what time can they together do the work?

383. What will $\frac{3}{8}$ of 7 acres cost, if $\frac{3}{4}$ of 9 acres cost \$577 $\frac{1}{2}$?

384. Bought 48 $\frac{3}{4}$ pounds of tea at 63 $\frac{3}{4}$ cents per pound. How many pounds more would have been received, if the price had been 7 $\frac{1}{2}$ cents per pound less, for the same money?

385. A merchant bought 200 crates of peaches at 62 $\frac{1}{2}$ cents per crate. He sold $\frac{1}{4}$ of them at an advance of 11 cents per crate, $\frac{1}{2}$ of the remainder at 87 $\frac{1}{2}$ cents per crate, and the balance at a loss of 4 $\frac{1}{4}$ cents per crate. How much did he gain?

386. A farmer exchanged 10 $\frac{1}{2}$ tons of hay at \$9 $\frac{3}{4}$ per ton, for coal at \$5 $\frac{1}{4}$ per ton. How many tons of coal did he receive?

387. Bought 48 $\frac{1}{2}$ cords of wood at \$7 per cord; sold $\frac{1}{2}$ of it at \$7 $\frac{1}{2}$ per cord, and the balance for \$175. How much did I gain?

388. A man gave $\frac{3}{7}$ of his inheritance for a house and lot, invested $\frac{1}{2}$ of the remainder in bonds, and had \$6400 left, which he deposited. How much did he inherit?

389. A speculator invested $\frac{1}{2}$ of his money in real estate, $\frac{1}{2}$ of the remainder in city bonds, $\frac{1}{2}$ of the second remainder in railroad stocks, and the balance, \$7500, was deposited. How much had he at first?

390. A grocer bought 3 cheese weighing respectively 124 $\frac{1}{2}$, 113 $\frac{3}{4}$, and 118 $\frac{7}{8}$ pounds, at 9 $\frac{3}{8}$ cents per pound. He gave in payment 20 pounds of coffee at 26 $\frac{1}{2}$ cents per pound, 7 pounds

of tea at $62\frac{1}{2}$ cents per pound, and the balance in money. How much money did the grocer give?

391. A grocer purchased 12 firkins of butter weighing respectively $62\frac{1}{2}$, $60\frac{3}{4}$, $61\frac{1}{4}$, $63\frac{3}{8}$, $58\frac{1}{4}$, $59\frac{9}{16}$, $49\frac{1}{16}$, $63\frac{7}{8}$, $65\frac{1}{16}$, $60\frac{3}{4}$, $62\frac{1}{8}$, and $64\frac{5}{8}$ pounds, at $16\frac{3}{4}$ cents per pound. What is the total cost, allowing $5\frac{1}{4}$ pounds for each tub?

392. A man performed $\frac{1}{4}$ of his journey the first day, $\frac{1}{2}$ of the remainder the second day, and found that he still had $36\frac{3}{8}$ miles to go. How far did he travel the first day?

393. A flagstaff fell, breaking into 3 pieces. The first was $\frac{3}{4}$ as long as the second, and the third was $\frac{1}{2}$ as long as the other pieces. What was the length of each piece, if the total length was 120 feet?

394. Bought $9\frac{1}{2}$ yards of silk at $\$4\frac{3}{4}$ per yard, $4\frac{3}{4}$ yards of cassimere at $\$2\frac{1}{2}$ per yard, $22\frac{1}{2}$ yards of cashmere at $\$1\frac{1}{4}$ per yard, and paid for it all in wood at $\$8$ per cord. How many cords were required?

395. A has $25\frac{1}{2}$ acres of land; B has $24\frac{3}{4}$ acres more than A; C has $16\frac{2}{3}$ acres less than A and B have; D has $5\frac{1}{2}$ acres more than A, B, and C have; E has $2\frac{3}{4}$ acres less than A and C have. How many acres have they all?

396. A wholesale merchant sold 32 pieces of denim as follows: 44^1 , 45^1 , 47^1 , 48^2 , 43^2 , 42^3 , 41^3 , 44^1 , 52^1 , 53^2 , 54 , 52 , 48 , 49^3 , 41 , 42^2 , 50 , 48^1 , 46 , 51^1 , 52 , 52^3 , 53 , 47^2 , 49^3 , 53^2 , 43^1 , 48^2 , 49^1 , 48^3 , 52^1 , 51^2 , at $12\frac{1}{2}$ cts. per yard. How much did he receive?

397. I sold 5 jars of butter weighing $35\frac{1}{2}$, $32\frac{3}{4}$, $31\frac{7}{8}$, $29\frac{5}{8}$, and $36\frac{1}{4}$ pounds respectively, at $18\frac{3}{4}$ cents per pound. The jars weighed $5\frac{1}{2}$, $6\frac{1}{4}$, $5\frac{5}{16}$, $6\frac{1}{4}$, and $7\frac{7}{8}$ pounds respectively. I received in payment 20 pounds of coffee at $33\frac{1}{3}$ cts. per pound, 5 pounds of tea at $62\frac{1}{2}$ cts. per pound, and the balance in sugar at 4 cts. per pound. How many pounds of sugar did I receive?

398. A speculator invested $\frac{1}{4}$ of his money and $\$200$ in bonds, $\frac{1}{3}$ of his money and $\$500$ in railroad stocks, $\frac{1}{6}$ of his money and $\$600$ in real estate, and the balance, which was $\$1200$, he deposited in a bank. What was the value of his property?

DECIMALS.

27. A Decimal Fraction is a fraction, whose denominator is ten or some power of ten.

The numerator of the decimal fraction is written the same as in common fractions, but the denominator is expressed by means of a decimal point, or period (.), placed to the left of the decimal, thereby pointing off as many places as are necessary to indicate the lowest order. Thus, $\frac{3}{10}$ is written .3, and is read three tenths. 1st order. $\frac{3}{100}$ is written .03, and is read three hundredths. 2d order. $\frac{3}{1000}$ is written .003, and is read three thousandths. 3d order.

DECIMAL NOTATION SCALE.

Etc., etc.	9.	8.	7.	6.	5.	4.	3.	2.	1.	.	1.	2.	3.	4.	5.	6.	7.	8.	9.
Hundred-millions.											Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
Ten-millions.										Units.	Decimal point.								Etc., etc.
Millions.																			
Hundred-thousands.																			
Ten-thousands.																			
Thousands.																			
Hundreds.																			
Tens.																			

In the above table the decimal point (.) separates the whole number from the fraction. Decimal fractions, the same as whole numbers, increase and decrease in a tenfold ratio, decreasing when removed to the right, and increasing when removed to the left. They may be added, subtracted, multiplied, and divided the same as whole numbers.

MENTAL PRACTICE.

What order of decimals occupies

1st place?	2d place?	5th place?
3d place?	4th place?	9th place?
5th place?	6th place?	3d place?
7th place?	8th place?	7th place?
9th place?	10th place?	2d place?
10th place?	3d place?	6th place?

What decimal place is occupied by

Tenths?	Ten-thousandths?
Hundredths?	Hundred-millionths?
Millionths?	Thousandths?
Thousandths?	Tenths?
Hundred-thousandths?	Ten-thousandths?
Ten-millionths?	Millionths?
Billionths?	Hundred-thousandths?

How many places to write

Thousandths?	Billionths?
Millionths?	Ten-millionths?
Tenths?	Hundred-thousandths?
Ten-thousandths?	Hundredths?
Hundredths?	Hundred-millionths?
Ten-millionths?	Ten-billionths?

How many ciphers prefixed to write.

5 hundredths?	125 millionths?
25 thousandths?	16 ten-millionths?
4 millionths?	4 hundred-thousandths?
26 hundred-thousandths?	225 ten-thousandths?
7 ten-thousandths?	30 thousandths?
8 thousandths?	3 $\frac{1}{8}$ millionths?

Analyze the following:

.05, point, zero, five, is read five hundredths; 100 shows that the unit is divided into 100 equal parts, 5 is the numerator and shows that 5 of the 100 equal parts is taken.

.25	.003	.0125	.000001
.125	.025	.0006	.14 $\frac{2}{7}$

In reading decimals it is best to use "and" only when passing the decimal point. Thus, 14.325, is read fourteen, and three hundred-twenty-five thousandths; and not fourteen, and three-hundred *and* twenty-five thousandths. It will from necessity be used in 14 $\frac{2}{7}$, as fourteen and $\frac{2}{7}$ hundredths.

Introduction to reading and writing decimals.

.5, is read five tenths.

.25, is read twenty-five hundredths.

.09, is read nine hundredths.

.325, is read three-hundred-twenty-five thousandths.

.025, is read twenty-five thousandths.

.005, is read five thousandths.

5.9, is read five, and nine tenths.

9.0324, is read nine, and three-hundred-twenty-four ten-thousandths.

.325, is read three-hundred-twenty-five thousandths.

300.025, is read three hundred, and twenty-five thousandths.

$14\frac{2}{7}$, is read, fourteen and $\frac{2}{7}$ hundredths.

$14.00\frac{2}{7}$, is read fourteen, and $\frac{2}{7}$ hundredths.

Read the following:

SUGGESTION.—First read the numerators, then the denominators, thereby showing relation to common fractions.

1. .5	22. 5.5	43. 4000.400400
2. .25	23. 25.25	44. .00004
3. .08	24. 125.325	45. .400
4. .125	25. .365	46. 1000000.000001
5. .025	26. 300.065	47. 1100100.100100
6. .005	27. .3245	48. 8001.8001
7. .7625	28. 3000.0245	49. .12625
8. .0625	29. .101	50. 12000.00625
9. .0055	30. .2002	51. .300000
10. .0005	31. .30003	52. 420.1234567
11. .32467	32. .0586	53. 255000.255
12. .03254	33. 500.0086	54. 4000.40004004
13. .00625	34. .250	55. 1234.023451
14. .00075	35. .2500	56. 123456.06060606
15. .00001	36. .00200	57. 320.0320
16. .325625	37. .33 $\frac{1}{3}$	58. 6000000.000006
17. .025625	38. 33.00 $\frac{1}{3}$	59. .6000006
18. .005625	39. .666 $\frac{2}{3}$	60. 3000.0095
19. .000625	40. 666.000 $\frac{2}{3}$	61. .3095
20. .000025	41. .100001	62. 14700009.1000001
21. .000005	42. .5005	63. 40404040.04040404

Write the following as decimals:

Seven-tenths. Twenty-seven hundredths. Seven hundredths. Three-hundred-twenty-seven-thousandths. Twenty-seven thousandths. Seven thousandths. Three-thousand-two-hundred-twenty-five ten-thousandths. Six-hundred-five ten-thousandths. Seventy-five ten-thousandths. Eight ten-thousandths. Eighty-thousand-eight hundred-thousandths. Eight-thousand-eight hundred-thousandths. Nine-hundred-nine hundred-thousandths. Seventy-five hundred-thousandths. Eight hundred-thousandths. 625 ten-millionths. 324 hundred-thousandths. 4165 millionths. 34 ten-thousandths. Seven, and seven tenths. Eight, and 24 hundredths. 9, and 175 thousandths. Twenty-five, and 3125 ten-thousandths. 225, and 324 millionths. Two-hundred-twenty-six millionths. Two-hundred, and twenty-six millionths. Six-thousand-forty-five hundred-thousandths. Six thousand, and forty-five hundred-thousandths. Five hundred-thousandths. Five-hundred thousandths. Twenty-five tenths. Two-hundred-six hundredths.

$\frac{5}{10}$	$\frac{125}{1000}$	$\frac{204}{1000000}$	$\frac{13}{1000}$	$\frac{2400}{1000000}$
$\frac{24}{100}$	$\frac{524}{10000}$	$\frac{25}{10000}$	$\frac{25}{10000}$	$\frac{6}{100}$
$\frac{4035}{1000}$	$\frac{1200065}{10000}$	$\frac{87}{1000000}$	$\frac{5005}{1000}$	$\frac{120021}{100}$

28. To reduce a common fraction to a decimal fraction.

MENTAL PRACTICE.

- $\frac{1}{2}$ = how many 10ths?
- $\frac{1}{4}$ = how many 100ths?
- $\frac{3}{4}$ = how many 100ths?
- $\frac{1}{8}$ = how many 10ths?
- $\frac{2}{3}$ = how many 10ths?
- $\frac{3}{8}$ = how many 10ths?
- How many thousandths are there in $\frac{1}{8}$? $\frac{2}{8}$? $\frac{3}{8}$? $\frac{4}{8}$? $\frac{5}{8}$? $\frac{6}{8}$? $\frac{7}{8}$?
- How many hundredths are there in $\frac{1}{4}$? in $\frac{1}{8}$? in $\frac{1}{16}$? in $\frac{1}{32}$? in $\frac{1}{64}$? in $\frac{1}{128}$? in $\frac{1}{256}$?

WRITTEN PRACTICE.

RULE.—I. Annex ciphers to the numerator and divide by the denominator.

II. Point off as many places in the result as there are ciphers annexed to the numerator.

1. Reduce $\frac{7}{8}$ to a decimal fraction.

SOLUTION.

$$\begin{array}{r} 8 \overline{)7.000} \\ \underline{.875} \end{array}$$

Reduce the following common fractions to equivalent decimals:

$$\begin{array}{l} 2. \frac{7}{10} \\ 3. \frac{25}{100} \\ 4. \frac{125}{1000} \\ 5. \frac{3}{4} \\ 6. \frac{7}{16} \\ 7. \frac{15}{32} \\ 8. \frac{27}{32} \\ 9. \frac{25}{64} \\ 10. \frac{27}{256} \end{array}$$

$$\begin{array}{l} 11. \frac{1}{16} \\ 12. \frac{3}{32} \\ 13. \frac{5}{64} \\ 14. \frac{3}{128} \\ 15. \frac{13}{80} \\ 16. \frac{24}{256} \\ 17. \frac{9}{800} \\ 18. \frac{12}{125} \\ 19. \frac{21}{64} \\ 20. \frac{1}{1024} \end{array}$$

$$\begin{array}{l} 21. 14\frac{3}{4} \\ 22. 5\frac{1}{8} \\ 23. 9\frac{5}{16} \\ 24. 7\frac{3}{32} \\ 25. 4\frac{11}{16} \\ 26. 3\frac{9}{64} \\ 27. 21\frac{3}{8} \\ 28. 5\frac{8}{125} \\ 29. 4.0\frac{7}{250} \\ 30. 1.40\frac{19}{256} \end{array}$$

Extend the following correct to four places. Annex four ciphers, and if the fifth place in the quotient equals or exceeds five, increase the fourth place one:

$$\begin{array}{llll} 31. \frac{1}{8} & 34. \frac{2}{3} & 37. \frac{8}{13} & 40. 12\frac{5}{8} \\ 32. \frac{2}{9} & 35. \frac{5}{7} & 38. \frac{1}{17} & 41. 8\frac{7}{18} \\ 33. \frac{3}{7} & 36. \frac{7}{11} & 39. \frac{9}{70} & 42. 9\frac{1}{19} \end{array}$$

29. To reduce a decimal fraction to a common fraction.

MENTAL PRACTICE.

1. $\frac{25}{100}$ = how many 4ths? $\frac{75}{100}$ = ?

2. $\frac{125}{1000}$ = how many 8ths? $\frac{375}{1000}$ = ? $\frac{625}{1000}$ = ? $\frac{875}{1000}$ = ?

3. Change to common fractions, reduced to their lowest terms: 5 tenths, 8 tenths, 5 hundredths, 10 hundredths, 20 hundredths, 25 hundredths, 30 hundredths, 40 hundredths, 50 hundredths, 90 hundredths.

WRITTEN PRACTICE.

43. Reduce .125 to a common fraction.

.125 = $\frac{125}{1000} = \frac{1}{8}$, the required fraction.

RULE.—Write the decimal fraction as a common fraction, and reduce to its lowest terms.

Reduce the following decimals to equivalent common fractions in their lowest terms:

- | | | |
|-----------|-------------|-------------------------|
| 44. .32 | 53. .0125 | 62. 18.75 |
| 45. .75 | 54. .1875 | 63. 9.3125 |
| 46. .625 | 55. .3125 | 64. 31.0075 |
| 47. .750 | 56. .4375 | 65. 4.0 $\frac{1}{4}$ |
| 48. .375 | 57. .4725 | 66. 4.09 $\frac{1}{4}$ |
| 49. .5625 | 58. .6875 | 67. 3.7 $\frac{1}{2}$ |
| 50. .9375 | 59. .8325 | 68. 8.18 $\frac{1}{4}$ |
| 51. .3125 | 60. .003125 | 69. 14.9 $\frac{3}{8}$ |
| 52. .0625 | 61. .005625 | 70. 4.003 $\frac{1}{8}$ |
71. Reduce $13\frac{1}{3}$ to a common fraction.

SOLUTION.

$13\frac{1}{3} = \frac{13\frac{1}{3}}{1} = \frac{40}{300} = \frac{2}{15}$, the required fraction.

- | | | |
|-----------------------|-------------------------|---------------------------|
| 72. .06 $\frac{2}{3}$ | 75. 12.98 $\frac{3}{4}$ | 78. 19.14 $\frac{2}{7}$ |
| 73. .83 $\frac{1}{3}$ | 76. 6.166 $\frac{2}{3}$ | 79. 4.22 $\frac{3}{8}$ |
| 74. .66 $\frac{2}{3}$ | 77. 14.23 $\frac{1}{3}$ | 80. 34.181 $\frac{9}{11}$ |
81. Reduce to an improper fraction 3.5625.
82. Reduce to an improper fraction 7.66 $\frac{2}{3}$.

Decimals may be reduced to common denominators by giving each decimal the same number of places.

ADDITION.

30. The process of Adding Decimals is the same as that of whole numbers.

MENTAL PRACTICE.

- What is the sum of $\frac{3}{10}$ and $\frac{6}{10}$? $\frac{7}{10}$ and $\frac{25}{100}$?
- What is the sum of $\frac{15}{100}$, $\frac{25}{100}$, and $\frac{30}{100}$?
- Add $\frac{5}{10}$ and $\frac{4}{10}$. Add .5 and .4. Add .3 and .28.
- Add $2\frac{5}{100}$, $3\frac{20}{100}$ and $5\frac{25}{100}$. (2.05, 3.20, 5.25).
- Add \$.25, \$.15, \$.12, \$.20, and \$.24.
- Add \$1.25, \$1.50, \$2.20, \$1.40 and \$.25.

7. A man paid \$5.50 for a hat, \$1.25 for a tie, \$4.20 for an umbrella, and \$2 for a pair of gloves. Find the amount of his purchases.

WRITTEN PRACTICE.

83. Add .325, .27, .12675, .004, and 25.3125.

SOLUTION.

OR,

.325	.32500
.27	.27000
.12675	.12675
.004	.00400
25.3125	25.31250
<u>26.03825</u>	<u>26.03825</u>

RULE.—I. Write the numbers so that the decimal points stand in the same column.

II. Add as in whole numbers, bringing down the decimal point directly under the points in the numbers added.

84. Add .125, 9.75, 31.5625, 3.6, and 75.00004.

85. Add .325, 300.025, .9375, 32.125462, and 7.5.

86. Add three, and seven tenths; twenty-five, and one-hundred twenty-five thousandths; ninety-five millionths; two-hundred-two ten-thousandths; two-hundred, and two ten-thousandths.

87. What is the sum of 25, and 75 millionths; 4, and 24 hundredths; 700, and 25 thousandths; 925 thousandths; 7, and 8 hundredths; 125 hundred-millionths; 327 ten-thousandths; and 1000, and 625 thousandths?

88. Add $16\frac{3}{4}$, $25\frac{1}{2}$, $13\frac{7}{8}$, $4\frac{7}{16}$, $5\frac{1}{32}$, and $3\frac{5}{8}$.

SUGGESTION.—Change common fractions to decimal fractions before adding.

89. What is the sum of \$16 $\frac{1}{2}$, \$13 $\frac{3}{8}$, \$12 $\frac{3}{4}$, \$8 $\frac{3}{4}$, and \$7?

90. What is the sum of $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{8}{9}$, correct to five decimal places?

91. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, $\frac{5}{32}$, $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{8}$, expressing the result in a decimal fraction?

92. What is the sum of .125, .75, .625, .5625, .9375, .0025, .075, .00875, and .00005, expressing the result in a mixed number?

93. A farmer has 48 $\frac{1}{2}$ acres in oats, 52 $\frac{3}{4}$ acres in wheat, 36 $\frac{1}{2}$ acres in barley, 4 $\frac{4}{16}$ acres in potatoes, and 75 $\frac{5}{32}$ acres in corn. How many acres in all?

94. A man traveled 30.5 miles the first day, 28.25 miles the second day, 40.125 miles the third day, 52 $\frac{7}{8}$ miles the fourth day, 174 $\frac{3}{4}$ miles the fifth day, 5 $\frac{1}{32}$ miles the sixth day, 49 $\frac{1}{8}$

miles the seventh day, $245.01\frac{1}{4}$ miles the eighth day, and .625 miles the ninth day. How far did he travel in the nine days?

95. What is the total of the four totals:
 $5.25 + 16.525 + 3.5625 + 30.003 + .0005 =$
 $327.075 + 3.6006 + 300.056 + .356 + 41.35 =$
 $99.4 + .125000365 + 1245. + 36.0065 =$
 $1.0001 + 2.200002 + 3.30003 + 4.40004 =$

96. What is the total of the three totals:
 $4\frac{3}{4} + 5\frac{7}{8} + 3\frac{1}{16} + 4\frac{7}{32} + 5\frac{1}{4} =$
 $5\frac{1}{2} + 3.125 + 3.75 + .9375 + 4\frac{5}{8} =$
 $4.7\frac{1}{2} + 21.3\frac{3}{4} + 8.9\frac{3}{8} + 11.0\frac{1}{16} + 9. =$

SUBTRACTION.

31. The process of Subtraction of Decimals is the same as that of whole numbers.

MENTAL PRACTICE

1. From $\frac{25}{100}$ take $\frac{15}{100}$ (.25 — .15).
2. From $\frac{9}{10}$ take $\frac{5}{10}$ (.9 — .5).
3. From $\frac{15}{100} + \frac{20}{100} + \frac{25}{100}$ take $\frac{35}{100}$.
4. From .15 + .20 + .25 subtract .35.
5. From $\frac{75}{100}$ take .24.
6. From \$2. take \$.75.
7. From \$1.25 + \$.75 + \$.24 take \$1.75.
8. From \$1 $\frac{1}{2}$ + \$1 $\frac{1}{4}$ + \$ $\frac{3}{4}$ subtract \$2.
9. A lady made the following purchases: Hat \$5.25, gloves \$1.75, shoes \$5.50, and parasol \$7. How much should she receive in return if she hands the merchant a twenty dollar bill?

WRITTEN PRACTICE.

97. From 3.9375 take 1.12546.

SOLUTION.

OR

RULE—I. Write the numbers so that the decimal points stand in the same column.

3.9375

3.93750

1.12546

1.12546

II. Subtract as in whole numbers, bringing down the decimal point as in addition.

2.81204

2.81204

98. From 32.049 take 14.753.
99. From 4.0325 take .943.
100. From 31.042 take .000025.

101. From 25 thousandths subtract 25 millionths.
102. Subtract 14 dollars 4 cents from 25 dollars 16 cents 7 mills.
103. From one-tenth take one millionth.
Find the difference between
104. $25.16\frac{3}{4}$ hr. and $12\frac{7}{8}$ hr.
105. $3.46\frac{5}{8}$ rd. and $.957\frac{5}{16}$ rd.
106. $24\frac{1}{8}$ mi. and $16.8\frac{5}{8}$ mi.
107. $124\frac{3}{16}$ ft. and $113\frac{5}{8}$ ft., correct to 5 places
108. $32\frac{4}{9}$ fur. and 12.75 fur., correct to 4 places.
109. From twenty-five, and nine-tenths, subtract twelve, and one-hundred-two millionths.
110. A merchant having 94.875 yards of silk, sells $36\frac{1}{4}$ yards at one time, and $23\frac{3}{32}$ yards at another time. How many yards has he remaining?
111. Find the total of the four results:
 $14.375 + 1.12045 + 3.174092 - .006493 =$
 $3.64 + 3.17 + .141414 - 3.6524 - .0054 =$
 $126.0\frac{3}{4} + 14\frac{7}{8} + 3\frac{5}{16} - 4\frac{9}{16} - 5.009\frac{3}{4} =$
 $24.0\frac{1}{8} + 3125 - .9375 + 800.008 - 9.9 =$ _____
112. Find the total of the following results, correct to 5 places:
 $3\frac{1}{8} + 4\frac{5}{8} + 9.125 + 14\frac{3}{7} - 18.6\frac{3}{8} =$
 $3.125 + 43.76 - 2\frac{3}{7} - 1\frac{5}{14} + 24\frac{3}{8} =$
 $4.25 + 5.0016 + 2\frac{3}{4} - 8\frac{7}{16} - 3\frac{1}{8} =$ _____

MULTIPLICATION.

32. The process of Multiplication of Decimals is the same as that of whole numbers, with the proper location of the decimal point in the product.

MENTAL PRACTICE.

1. What is the value of 3 times $\frac{3}{10}$? $\frac{25}{100}$?
2. What is the value of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10} \times \frac{2}{10}$?
3. What will 3 books cost at \$.75 each?
4. $3 \times \frac{25}{100} = ?$ $3 \times \$.25 = ?$ $3 \times .25 = ?$
5. What is $\frac{3}{10} \times \frac{7}{10} = ?$ $.3 \times .7 = ?$
6. What is $\frac{6}{10} \times \frac{25}{100} = ?$ $.5 \times .25 = ?$

7. What will be paid for 15 quarts of berries at \$.12 per quart?

8. What will 2000 admission tickets cost at \$.25 each?

WRITTEN PRACTICE.

Principles of the multiplication of decimals.

Units \times Units = Units.

Units \times Tenths = Tenths.

Units \times Hundredths = Hundredths.

Tenths \times Tenths = Hundredths.

Units \times Thousandths = Thousandths.

Tenths \times Hundredths = Thousandths.

Units \times Ten-thousandths = Ten-thousandths.

Tenths \times Thousandths = Ten-thousandths.

Hundredths \times Hundredths = Ten-thousandths, etc., etc.

Therefore the following

RULE.—I. *Multiply as in whole numbers.*

II. *From the right, point off as many places in the result as there are places in both multiplier and multiplicand.*

113. Multiply .24 by .9.

SOLUTION.

$$\begin{array}{r} .24 \\ .9 \\ \hline .216 \end{array}$$

Relationship to common fractions.

$$\frac{24}{100} \times \frac{9}{10} = \frac{216}{1000}$$

114. Multiply 1.04267 by .0046.

SOLUTION.

$$\begin{array}{r} 1.04267 \\ .0046 \\ \hline 625602 \\ 417068 \\ \hline .004796282 \end{array}$$

115. Multiply .325 by .27; .107 by .15.

116. Multiply 343 by .25; 247 by .9.

117. Multiply 14.25 by 1.81; 14.3 by .85.

118. Multiply 300.125 by 8; .375 by 48.

Find the value of the following:

$$119. 25 \times .125 \times .3$$

$$123. .3\frac{1}{8} \times .25 \times .0048$$

$$120. .001 \times 75 \times .07$$

$$124. .675 \times 10 \times 100$$

$$121. 7\frac{1}{2} \times 8\frac{3}{4} \times 12.5625$$

$$125. 30.03 \times .5 \times .0007$$

$$122. .33\frac{1}{3} \times 36 \times 12\frac{1}{8}$$

$$126. \frac{7}{8} \times .62\frac{1}{2} \times .48 \times 325$$

Multiply, expressing the result as a common fraction:

$$127. 3 \times .125$$

$$130. .6\frac{2}{3} \times .75$$

$$128. .25 \times .25$$

$$131. .42\frac{6}{7} \times .333\frac{1}{3}$$

$$129. .125 \times .15$$

$$132. .875 \times .4$$

133. Multiply six-hundred, and seventy-five thousandths by six-hundred-seventy-five thousandths.

134. Multiply one-thousand by one-thousandth.

135. A farmer sold $246\frac{3}{4}$ bushels of corn at \$.5625 per bushel. What did he receive for the corn?

136. What is the cost of $3\frac{3}{4}$ bales of paper, averaging 240.875 pounds each at \$.12 $\frac{3}{8}$ per pound?

137. A grocer's sales for one day were as follows: 125 lb. of sugar at $4\frac{1}{4}$ c. per lb., $24\frac{1}{2}$ lb. of tea at \$.62 $\frac{1}{2}$ per lb., 23 cans of peaches at \$.12 $\frac{1}{2}$ each, 16 bbl. salt at \$1.25 per bbl., 15 bbl. of flour at \$5 $\frac{3}{4}$ per bbl. What was the amount of his sales for the day?

138. What is .125 of 320 bu. of oats?

139. A farmer sold .5625 of 250 bushels of potatoes at \$.18 $\frac{1}{4}$ per bushel. What did he receive?

140. A man has 600 acres of land; he sells .325 of it at one time, and .33 $\frac{1}{3}$ of it at another. What is the value of the remainder at \$65 $\frac{3}{4}$ per acre?

141. Find the total of the following results:

$$.325 \times 1.25 \times 3 \times .75 =$$

$$12\frac{3}{4} \times 1\frac{7}{8} \times .005 \times 100 =$$

$$8\frac{3}{8} + 5\frac{3}{4} - 4.6\frac{3}{8} \times .05 =$$

$$.3 \times .4 \times .5 \times .6 \times .7 \times .8 =$$

DIVISION.

33. The process of Division of Decimals is the same as that of whole numbers, with the proper location of the decimal point in the quotient.

MENTAL PRACTICE.

1. What is $\frac{1}{2}$ of $\frac{25}{100}$? of $\frac{85}{100}$? of $\frac{75}{100}$?
2. What is $\frac{1}{2}$ of .25? of .45? of .95?
3. What is $\frac{1}{4}$ of .96? of .84? of .64?
4. What is $\frac{1}{3}$ of $\frac{18}{100}$? of .24? of 2.4?
5. What is the value of $.8 \div 2$? $.75 \div 3$?
6. What is the value of $.8 \times .4$? $.12 \times .12$?
7. What is the value of $.48 \div .8$? $.0144 \div .12$?
8. At \$.20 each how many balls can be bought for \$.80?
for \$1.20? for \$3.60?
9. At \$.02 each how many apples can be bought for \$.64?
for \$.84? for \$3?

WRITTEN PRACTICE.

Principles of the division of decimals.

Units \div Units = Units.

Tenths \div Tenths = Units.

Hundredths \div Hundredths = Units.

Hundredths \div Tenths = Tenths.

Thousandths \div Thousandths = Units.

Thousandths \div Hundredths = Tenths.

Ten-thousandths \div Ten-thousandths = Units.

Ten-thousandths \div Thousandths = Tenths.

Ten-thousandths \div Hundredths = Hundredths, etc., etc.

Therefore the following:

RULE.—I. Reduce the divisor to a whole number by moving the decimal point the required number of places to the right.

II. Move the decimal point in the dividend the same number of places to the right, annexing ciphers if necessary.

III. The quotient to the decimal point is a whole number, that to the right is a decimal.

The decimal point should be placed in the quotient directly above the one in the dividend, as soon as it is reached in the dividend.

The new decimal point is indicated by a colon (:)

142. Divide 9375 by 25; .9375 by 25; .9375 ÷ .25; 9375 by .025.

SOLUTION.

$\begin{array}{r} 375. \\ 25 \overline{)9375} \\ \underline{75} \\ 187 \\ \underline{175} \\ 125 \\ \underline{125} \\ 0 \end{array}$	$\begin{array}{r} .0375 \\ 25 \overline{).9375} \\ \underline{75} \\ 187 \\ \underline{175} \\ 125 \\ \underline{125} \\ 0 \end{array}$	$\begin{array}{r} 3.75 \\ .25 \overline{).9375} \\ \underline{75} \\ 187 \\ \underline{175} \\ 125 \\ \underline{125} \\ 0 \end{array}$	$\begin{array}{r} 375000. \\ .025 \overline{)9375.000} \\ \underline{75} \\ 187 \\ \underline{175} \\ 125 \\ \underline{125} \\ 000 \end{array}$
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143. Divide .5625 by 25.
 144. Divide .1291136 by 262.
 145. Divide .96351 by 346.
 146. Divide .83094 by 75.
 147. Divide .84375 by .25.
 148. Divide .1291136 by .262.
 149. Divide .763 by .125.
 150. Divide .855 by 2.25.
 151. Divide 375 by .25.
 152. Divide 9.45 by .075.
 153. Divide 94.3 by .0004.
 154. Divide .635 by .000005.
 155. 29 feet by .06 (4 places).
 156. 82.97 mi. by 71 (3 places).
 157. 2.3783 yds. by 43 (4 places).
 158. $75\frac{2}{3}$ cords by 27 (5 places).
 159. $\$15\frac{7}{8}$ by $14\frac{1}{8}$ (4 places).
 160. $\frac{5}{8}$ cu. ft. by 27 (3 places).
 161. $\frac{5}{8}$ mo. by 30 (4 places).
 162. $1.008\frac{3}{4}$ qt. by 9 (4 places).
 163. $14\frac{2}{3}$ bu. by $8\frac{1}{3}$ (2 places).
 164. 35 days by 21 (4 places).
 165. 378.45 miles by 93.4 (6 places).
 166. $63.71\frac{7}{8}$ rods by 25 (3 places).
 167. Divide 975 ten-thousandths by 75 hundredths.
 168. How many bushels of wheat can be bought for \$268.735 at 71 cts. per bushel?

109. A man having \$840 invested .65 of it in corn at \$.37½ per bushel, and the balance in wheat at \$.75 per bushel. How many bushels of grain did he buy?

170. Divide one millionth by one thousandth. Divide one thousandth by one millionth.

Find the sum of the following quotients:

(171.)	(172.)	(173.)
$5 \div 5 = ?$	$33 \div 11 = ?$	$28 \div 7 = ?$
$5 \div .5 = ?$	$33 \div .11 = ?$	$28 \div .007 = ?$
$5 \div .05 = ?$	$33 \div .110 = ?$	$2.8 \div .00007 = ?$
$5 \div .005 = ?$	$3.3 \div .110 = ?$	$.28 \div .0007 = ?$
$.5 \div 5 = ?$	$.33 \div .11 = ?$	$.28 \div 7 = ?$
$.05 \div 5 = ?$	$.330 \div 1.1 = ?$	$280 \div .7 = ?$
$.005 \div 5 = ?$	$3300 \div .011 = ?$	$2800 \div .07 = ?$
$.0005 \div 5 = ?$	$330 \div .00011 = ?$	$28000 \div .0007 = ?$
$5 \div 50 = ?$	$.33 \div 110 = ?$	$280 \div .007 = ?$
$5 \div 500 = ?$	$.33 \div 11000 = ?$	$.0028 \div .07 = ?$
(174.)	(175.)	(176.)
$325 \div 1.25 = ?$	$75 \div 3 = ?$	Correct to five places. $24 \div .037 = ?$
$.75 \div 15 = ?$	$75 \div .03 = ?$	$.03\frac{1}{3} \div .07 = ?$
$374.04 \div .0004 = ?$	$.75 \div .3 = ?$	$8\frac{1}{2} \div .043 = ?$
$9 \div .003 = ?$	$75 \div .003 = ?$	$.375 \div 12.5 = ?$
$.66 \div 15 = ?$	$.075 \div 3 = ?$	$14\frac{2}{3} \div 3.6 = ?$
$6.25 \div .000025 = ?$	$.0000075 \div .3 = ?$	$66\frac{2}{3} \div 3.7\frac{1}{2} = ?$
$.069184 \div .376 = ?$	$.000075 \div .03 = ?$	$8.3\frac{1}{3} \div .05 = ?$
$78.74 \div .13\frac{1}{2} = ?$	$75 \div .3 = ?$	$2.637 \div .436 = ?$
$7.5 \div .000025 = ?$	$0000075 \div .03 = ?$	$74.16\frac{2}{3} \div 83\frac{1}{3} = ?$
$840 \div .66\frac{2}{3} = ?$	$7.5 \div 3 = ?$	$75\frac{3}{4} \div .05 = ?$
$.36 \div 12000 = ?$	$.00000075 \div .3 = ?$	$94.06 \div 11\frac{1}{2} = ?$

MENTAL PRACTICE.

1. What is .25 of 80 bu.? 160 bu.? 240 bu.?
2. What is .125 of \$64? \$84? \$120?
3. What is 1.5 of 20 qt.? $33\frac{1}{3}$ bu.? 14 hr.?
4. What is the cost of .2 of 400 bushels of wheat at \$.62½ per bushel?
5. What will .375 of 120 oxen cost at \$20 per head?

6. 24 is .25 of what number?
7. $33\frac{1}{3}$ is .4 of what number?
8. One hour is .75 of how many minutes?
9. If .6 of the value of a factory is worth \$1440, what is the value of the factory?
10. .25 of 48 is $33\frac{1}{3}$ of what number?
11. $.66\frac{2}{3}$ of 60 is .25 of what number?
12. .6 of $62\frac{1}{2}$ is .75 of what number?
13. A man gave his son .75 of a section of land. What part of a section had he left?
14. A sold B .6 of 120 acres of land at \$25 per acre. How much did he receive?
15. If .625 yards of tapestry cost \$18.75, what will .875 yards cost?
16. If I receive 1 \$3 for $.3\frac{1}{3}$ days' work, what would I receive for $33\frac{1}{3}$ days' work?
17. .75 of \$60 is .4 of the cost of a carriage. What is the cost of the carriage?
18. After spending .4 of my money, I have \$240 left. How much had I at first?
19. If .75 of an acre of land is worth \$36, what are 4 acres worth?

WRITTEN PRACTICE.

177. Add nine, and nine tenths; twenty-five, and twenty-five thousandths; five-thousand, and six-hundred-twenty-five ten-thousandths; six-thousand-six-hundred-twenty-five ten-thousandths; five, and five-hundred-five thousandths; forty-eight tenths; and one-hundred, and one-hundred-six millionths.

178. What is the sum of 6.25, 9.5, 300.00035, $4\frac{1}{8}$, $3\frac{3}{4}$, $9\frac{1}{2}$, 3.005, $2.1403\frac{1}{3}$, 41520.325164, $3.0\frac{1}{8}$, and $400.0014\frac{2}{3}$, correct to six decimal places?

179. Reduce to decimals $\frac{7}{8}$, $\frac{9}{16}$, $\frac{2}{3}$, and $\frac{7}{84}$.

180. Reduce to common fractions .125, .0625, .008 $\frac{1}{4}$, .06 $\frac{3}{8}$, and .011 $\frac{1}{3}$.

181. From one, take one-millionth.

182. Find the product of $.13\frac{1}{3}$ and 304.569.

183. Divide 9 by .01; by .001; by .00001.

184. A speculator has 640 acres of land; he sells .25 of it at \$43 per acre, $16\frac{2}{3}$ of the remainder at \$30 per acre, as many acres as he sold the first and second times at $\$37\frac{1}{2}$ per acre, and the balance at \$24 per acre. How much did he gain if he paid \$25 per acre for the land?

185. If 72.125 bushels of corn cost \$28.85, what is the cost of one bushel?

186. What is the cost of nine barrels of pork, weighing 285, 284, 286³, 283², 276, 281¹, 279³, 291, and 276² pounds respectively, at $\$7\frac{1}{2}$ per hundred pounds?

187. If .375 of a barrel of sugar cost \$4.77, what will .875 of a barrel cost?

188. A man spent .55 of his yearly salary, and had \$540 left. What was his monthly salary?

189. A has a flock of 720 sheep, and sells .625 of the flock. What is the value of the remainder of the flock at $\$3\frac{1}{4}$ per head?

190. My profits in business for two years, were \$5312.50, What were my profits each year, if the profits this year were .125 greater than last year?

191. A man invests .375 of his money in real estate, .2 in bonds, .25 in stocks, and deposits the remainder, \$3500, in a bank. How much does he invest in each?

192. If $.33\frac{1}{3}$ of 75 cords of wood cost \$212.50, what will .75 of a cord cost?

193. A has \$1500 which is .25 more than B has. How much have they together?

194. A has \$1200 which is $.33\frac{1}{3}$ less than B has. How much have they together?

195. A and B together have \$6800, A has .125 more than B. How much has each?

196. A man left .375 of his estate to his wife, .4 of the remainder to his daughter, and the remainder to his son, who received \$7678.50. How much did the wife and daughter each receive?

UNITED STATES MONEY.

34. United States Money is the legal currency of the United States, usually called Federal Money.

The currency of the United States is based on the decimal scale, a uniform increase and decrease.

TABLE.

10 mills	= 1 cent	c, ct., ¢.
10 cents	= 1 dime.....	d.
10 dimes	= 1 dollar.....	\$.
10 dollars	= 1 eagle.....	E.

The gold dollar is the standard unit weighing 25.8 grains.

Problems involving United States money are applications of decimals.

The rules and solutions of decimals will therefore apply.

SUGGESTION.—In the computation of problems in United States money it is customary in final results to count a half cent as a whole cent, and drop the fraction when less than one-half.

CLASS DRILL.

1. Write thirty-six dollars twenty-five cents.
2. Write four dollars nine cents.
3. Write three dollars three and one-half cents.
4. Write 75 dollars $5\frac{3}{4}$ cents.
5. Write $12\frac{1}{2}$ cents.
6. Reduce 12376 cents to dollars.
7. Change \$325 to cents.
8. Reduce 34650 mills to dollars and cents.
9. Read the following:

\$24.65	\$25.045	\$360.60 $\frac{1}{2}$	\$23.006 $\frac{3}{4}$
30.06	140.06 $\frac{2}{3}$	12.634 $\frac{1}{2}$	125.639

WRITTEN PRACTICE.

1. Change \$32.64 to mills.
2. Find the sum of twenty-five dollars twenty-five cents, sixteen dollars thirty-six cents, seventy-five dollars sixteen cents, and forty dollars seven cents.
3. In furnishing a house I paid the following bills: Carpet, \$248.75; curtains, \$75.16; furnace, \$95.60; and furniture, \$264.75. What was the total amount paid?
4. A lady went shopping with \$100 and made the following purchases: Dress pattern, \$36; hat, \$15; gloves, \$2.50; shoes, \$7.75; sundry articles, \$14.60; luncheon, \$.90; and carfare, \$.20. How much had she left?
5. Add \$35.60, \$82.64, \$16.72, \$64.80, \$114.60, \$320.42, \$46.84, \$83.84, \$98.34, \$29.36, \$34.62, and \$93.24.
6. I bought a team for \$360, a carriage for \$125, set of harness for \$124.36, a robe for \$12.50, a whip for \$2.50, and sold the whole for \$750. How much did I gain?
7. Find the total cost of the following purchases: 25 tons hard coal at \$7.50 per ton, 36 tons soft coal at \$4.25 per ton, $46\frac{1}{2}$ tons chop feed at \$8.40 per ton, and freight on the whole at \$2 per ton.
8. Sold grain as follows: 325 bu. oats at \$.32 per bu., 624 bu. wheat at \$.63 per bu., 750 bu. barley at \$.52 per bu., and 340 bu. rye at \$.60 per bu. What were the total sales?
9. Bought a section of land, 640 acres, at \$16 $\frac{1}{2}$ per acre; sold 160 acres at \$20.50 per acre, 80 acres at \$12.50 per acre, 80 acres at an increase of \$2 per acre, and the balance at \$17 per acre. How much did I gain?
10. Paid \$75 for 9 cords of wood. What will 25 $\frac{1}{2}$ cords cost?
11. If I paid \$100 for 24 cords of wood, how many will \$175 buy?
12. My house rents at \$45 per month; the expenses are, taxes, \$76.27; insurance, \$10.40; special assessments, \$75.13; repairs, \$60.20, and interest on mortgage, \$89. What is my average monthly rental?
13. A farmer sold a load of pork weighing 3160 lb. at 8¢ per lb. He received in payment 2 bbl. flour at \$6 $\frac{1}{2}$, 312 lb.

of sugar at $3\frac{3}{4}$ cts., 25 lb. coffee, at \$.32, 6 lb. tea, at \$.64, and the balance in money. How much money did he receive?

14. If $\frac{3}{4}$ of a cord of wood cost \$8.50, how many cords can be bought for \$1088?

15. If $\frac{3}{8}$ bbl. of flour cost \$4.50, how much will 72 bbl. cost?

16. Required the cost of

16. $8\frac{1}{2}$ yards of cloth at $3\frac{1}{4}$ cents a yard.

17. $12\frac{1}{2}$ bushels of corn at $87\frac{1}{2}$ cents a bushel.

18. $8\frac{3}{4}$ bushels of apples at $64\frac{1}{2}$ cents a bushel.

19. $12\frac{3}{4}$ pounds of beef at $12\frac{1}{2}$ cents a pound.

20. $24\frac{3}{4}$ pounds of cheese at $8\frac{1}{2}$ cents a pound.

21. $27\frac{1}{2}$ cords of wood at $44\frac{3}{4}$ a cord.

22. $18\frac{1}{8}$ tons of hay at $55\frac{3}{4}$ a ton.

23. $24\frac{1}{2}$ bales of hemp at $12\frac{1}{2}$ a bale.

24. $20\frac{1}{4}$ yards of silk at $45\frac{5}{8}$ a yard.

25. $36\frac{3}{4}$ pounds of coffee at $16\frac{1}{2}$ cents a pound.

26. $7\frac{1}{2}$ tons of hay, if $2\frac{1}{2}$ tons cost \$11 $\frac{1}{4}$.

27. $12\frac{1}{2}$ bushels of wheat, if $37\frac{1}{2}$ bushels cost $99\frac{3}{8}$.

28. 25 cords of wood, if $18\frac{3}{4}$ cords cost \$112.50.

29. $37\frac{1}{2}$ gallons of wine, if $62\frac{1}{2}$ gallons cost \$140.50.

30. $58\frac{1}{8}$ cords of stone, if $41\frac{3}{8}$ cords cost \$375.

31. $\frac{3}{4}$ of a ton of clover hay, if $\frac{3}{8}$ of a ton cost \$12.

32. $\frac{7}{8}$ of an acre of corn, if $\frac{5}{8}$ cost \$25.50.

33. $1\frac{5}{8}$ of a bushel of seed corn, $\frac{5}{8}$ of a bushel cost \$3.50.

ALIQUOT PARTS.

35. An Aliquot Part of a number is one of its equal parts.

ALIQUOT PARTS OF ONE DOLLAR.

50 cents = $\frac{1}{2}$ of \$1.

$33\frac{1}{3}$ " = $\frac{1}{3}$ " \$1.

25 " = $\frac{1}{4}$ " \$1.

20 " = $\frac{1}{5}$ " \$1.

$16\frac{2}{3}$ " = $\frac{1}{6}$ " \$1.

$12\frac{1}{2}$ " = $\frac{1}{8}$ " \$1.

$8\frac{1}{3}$ cents = $\frac{1}{12}$ of \$1.

$6\frac{1}{4}$ " = $\frac{1}{16}$ " \$1.

5 " = $\frac{1}{20}$ " \$1.

4 " = $\frac{1}{25}$ " \$1.

$3\frac{1}{3}$ " = $\frac{1}{30}$ " \$1.

$2\frac{1}{2}$ " = $\frac{1}{40}$ " \$1.

APPLICATIONS.

$$37\frac{1}{2} \text{ cents} = \frac{3}{8} \text{ of } \$1.$$

$$62\frac{1}{2} \text{ " } = \frac{5}{8} \text{ " } \$1.$$

$$66\frac{2}{3} \text{ " } = \frac{2}{3} \text{ " } \$1.$$

$$75 \text{ " } = \frac{3}{4} \text{ " } \$1.$$

$$87\frac{1}{2} \text{ cents} = \frac{7}{8} \text{ of } \$1.$$

$$112\frac{1}{2} \text{ " } = \frac{9}{8} \text{ " } \$1.$$

$$125 \text{ " } = \frac{5}{4} \text{ " } \$1.$$

$$133\frac{1}{3} \text{ " } = \frac{4}{3} \text{ " } \$1.$$

Count by $2\frac{1}{2}$'s to 100 and return.

Count by $3\frac{1}{3}$'s to 100 and return.

Count by $6\frac{1}{4}$'s to 100 and return by deducting 5's.

Count by $8\frac{1}{3}$'s to 100 and return by deducting $6\frac{1}{4}$'s.

Count by $12\frac{1}{2}$'s to 100 and return by deducting $8\frac{1}{3}$'s.

Count by $16\frac{2}{3}$'s to 100 and return by deducting $12\frac{1}{2}$'s.

GIVEN.—Quantity and price to find cost.

RULE.—*Multiply the quantity by the price.*

If the price is an aliquot part of one dollar, take such a part of the quantity as the price is of one dollar.

WRITTEN PRACTICE.

34. What will 896 bu. of rye cost at 25 cents per bushel?

35. What will 1236 yards of prints cost at $12\frac{1}{2}$ cents per yard?

36. What will 360 bu. of apples cost at $37\frac{1}{2}$ cents per bushel?

37. Find the cost of 124 bales of cotton, each bale containing 250 pounds, at $8\frac{1}{3}$ cents per pound.

38. What is the cost of 25 firkins of butter, each weighing 56 pounds, at $37\frac{1}{2}$ cents per pound; and 3 barrels of sugar, each weighing 250 pounds, at $4\frac{1}{2}$ cents per pound?

GIVEN.—Price and cost to find quantity.

RULE.—*Divide the cost by the price.*

WRITTEN PRACTICE.

39. How many tons of coal can be bought for \$175, at \$5 per ton?

40. Paid \$403.20 for oats at 32 cts. per bushel. How many bushels did I buy?

41. I have \$36.25. How many bushels of corn can I purchase at 25 cents per bushel?

42. If 12 men at $\$2\frac{1}{2}$ per day earn \$750, how many days do they work?

43. A farmer sold to a merchant 75 bushels of oats at 25 cents per bushel; 5 dozen chickens at \$5.50 per dozen. He received \$25 in money, and the balance in sugar at 5 cents per pound. How many pounds did he receive?

GIVEN.—Cost and quantity to find price.

RULE.—*Divide the cost by the quantity.*

WRITTEN PRACTICE.

44. Bought 75 pounds of tea for \$56.25. What was the price of one pound?

45. I pay \$647.50 per year for board. How much is that per day?

46. A butcher bought 320 quarters of beef for \$1920. What did he pay per quarter?

47. Paid \$1196 for 184 barrels of flour. How much did I pay per barrel?

48. Paid \$240 for 640 pounds of tea and coffee, buying the same number of pounds of each, but paying twice as much for tea as for coffee. What was the price per pound of each?

GIVEN.—Quantity and price per 100 or 1000 to find cost.

RULE.—*Reduce quantity to hundredths or thousandths, and multiply by cost.*

WRITTEN PRACTICE.

49. Find the cost of 3260 lb. of pork at \$6.50 per cwt.

50. What is the cost of three loads of beef, each weighing 4250 lbs., at \$12.50 per cwt.?

51. Find the cost of 12264 feet of flooring at \$11 per M.

52. What is the cost of 24560 feet of fencing at \$12.50 per M, 3260 feet of 2x4's at \$16 per M, 2400 posts at \$25 per C, and 5 kegs of nails at \$5 per keg?

53. Find the freight on 6 carloads of grain, weighing respectively 23246, 21320, 32640, 41260, 30260, and 36000 pounds, at $37\frac{1}{2}$ cts. per cwt.?

GIVEN.—Quantity and price per ton to find cost.

RULE.—*Multiply the quantity reduced to thousandths by one-half the cost per ton.*

WRITTEN PRACTICE.

54. What cost 13260 pounds of coal at \$6.50 per ton?
 55. At \$9 per ton, what cost 32600 lb. of baled hay?
 56. At \$12.50 per ton, what cost 250 pounds of plaster?
 57. I bought three loads of hay, averaging 3675 pounds each, at \$8.85 per ton, which I retailed at 65 cts. per cwt. How much did I gain?
 58. Find the cost of the following items: 3265 pounds of hay at \$7.50 per ton; 2684 pounds of coal at \$6.75 per ton; 3260 pounds of ice at \$7.50 per ton.

TABLE OF COMPARATIVE WEIGHTS.

Grain is usually bought and sold by weight; i. e., so many pounds to the bushel.

ARTICLES.	LBS.	ARTICLES.	LBS.
Barley	48	Malt	34
Beans	60	Oats	32
Buckwheat	48	Peas	60
Castor Beans	46	Potatoes	60
Clover Seed	60	Rye	56
Corn	56	Timothy Seed	45
Corn, ear	70	Wheat	60
Flax Seed	56	Wheat Bran	20

WRITTEN PRACTICE.

59. What cost 3260 pounds of oats at 30 cents per bushel?
 60. Find the cost of 4 loads of wheat, weighing respectively 4860, 2420, 3270, and 5274 pounds, at \$.96 per bu.?
 61. A farmer sold as follows during the year: 15260 lb. of wheat at 75 cts. per bu., 26842 lb. of corn at 48 cts. per bu., 24064 lb. of oats at 24 cts. per bu., 25 bu. clover seed at \$12.60 per bu., and 96 bu. timothy seed at \$8 per bu. What did the total sales amount to?
 62. A dealer bought 3 loads of potatoes, weighing respectively 3260, 2550, and 3648 pounds, at $62\frac{1}{2}$ cents per bushel; he

sold the first load at 60 cents per bushel, and the second and third loads at $82\frac{1}{2}$ cents per bushel. How much did he gain?

63. I bought a load of barley weighing 3864 pounds, at 60 cents per bushel, and paid for it in coal at $\$7\frac{1}{2}$ per ton. How many tons did I give?

PROMISCUOUS WRITTEN PROBLEMS.

64. Find the cost of 32165 pounds of railroad iron at \$12.50 per ton.

65. What is the cost of 4 barrels of vinegar at $16\frac{2}{3}$ cents per gallon, each barrel containing 44 gallons?

66. What is the cost of 3265 feet of fencing at \$12 per M, and 75 posts at \$21 per C?

67. Find the cost of 36964 yards of calico at $6\frac{1}{4}$ cents per yard.

68. Find the cost of 32064 pounds of oats at $33\frac{1}{3}$ cents per bushel.

69. What is the cost of 516 pounds of cheese at $12\frac{1}{2}$ cents per pound?

70. Find the freight charges on 24364 pounds of beef from Sioux City to Chicago at \$6 per ton.

71. A farmer brought to market 3750 pounds of clover hay which he sold at \$7.50 per ton; he received in part payment 3125 pounds of barley malt at \$1.25 per 1000 lb. How much was still due him?

72. If $\frac{3}{4}$ of a yard of silk cost $\frac{2}{3}$ as much as a yard of broadcloth, how many yards of silk can be bought for \$36, if one yard of broadcloth costs \$7.50?

73. Bought an equal number of pounds of tea and coffee; I paid 65 cents per pound for tea, and 25 cents per pound for coffee. How many pounds of each did I buy, if I expended \$36?

74. A dealer bought a barrel of wine, containing 45 gallons, at \$1.25 per gallon, losing by leakage 15 gallons. How much must he ask per gallon to net 50 cents per gallon on the amount bought?

75. Find the total cost of

320 lb. R. Coffee	25 ¢	124 lb. Dried Apricots	12½ ¢
120 lb. O. Tea	62½ ¢	180 lb. Cocoa	25 ¢
144 lb. Sugar	5 ¢	64 lb. Java Coffee	33½ ¢
84 lb. Starch	6¼ ¢	136 lb. G. H. Tea	37½ ¢
96 lb. Soap	2½ ¢	280 lb. Rice	6¼ ¢
240 lb. Dried Peaches	16¾ ¢	300 lb. J. Tea	66¾ ¢

Compute by using aliquots of one dollar.

76. Find the total cost of

144 ² yd. Muslin	7 ¢	136 ² yd. C. Jean	5 ¢
124 ³ yd. Delaine	16 ¢	92 yd. N. Sateen	5½ ¢
96 ² yd. Cambric	22 ¢	45 yd. U. Print	5½ ¢
130 yd. G. Flannel	25 ¢	184 ³ yd. L. Plaid	11 ¢
75 yd. E. Lining	3½ ¢	240 yd. B. Jeans	6¾ ¢
84 yd. V. Barege	16 ¢	212 yd. Cashmere	18½ ¢
63 yd. Gingham	8½ ¢	125 ² yd. B. Checks	25 ¢

77. Find the total cost of

36 ¹ lb. @ 25 ¢	360 ¹ lb. @ 16¾ ¢	240 ¹ lb. @ 66¾ ¢
83 ² lb. @ 20 ¢	725 ² lb. @ 33½ ¢	275 ² lb. @ 20 ¢
97 ³ lb. @ 10 ¢	1240 lb. @ 18¼ ¢	1344 ³ lb. @ 12½ ¢
146 ¹ lb. @ 12½ ¢	370 lb. @ 2½ ¢	2145 ³ lb. @ 25 ¢
876 ² lb. @ 6¼ ¢	560 lb. @ 5 ¢	2300 ² lb. @ 37½ ¢

78. Find the total cost of

518 yd. @ 14¾ ¢	1008 ¹ yd. @ 58½ ¢	120 ² yd. @ 83½ ¢
576 yd. @ 11½ ¢	595 ¹ yd. @ 28¼ ¢	82 ³ yd. @ 75 ¢
840 ² yd. @ 3½ ¢	336 ² yd. @ 33½ ¢	95 ¹ yd. @ 31½ ¢
1245 ¹ yd. @ 13½ ¢	720 ² yd. @ 66¾ ¢	127 yd. @ 87½ ¢
756 ² yd. @ 41¾ ¢	220 yd. @ 12½ ¢	460 ¹ yd. @ 62½ ¢
378 ² yd. @ 8½ ¢	123 ¹ yd. @ 33½ ¢	574 ² yd. @ 56¼ ¢

79. Find the total cost of

324 ¹ yd. @ 5¼ ¢	1263 ² yd. @ 2½ ¢	436 ¹ yd. @ 5 ¢
125 ³ yd. @ 3½ ¢	928 ¹ yd. @ 3 ¢	328 ² yd. @ 3½ ¢
320 ¹ yd. @ 4¾ ¢	1246 ³ yd. @ 7½ ¢	465 yd. @ 4¾ ¢
246 ² yd. @ 5½ ¢	3125 ¹ yd. @ 10 ¢	379 ¹ yd. @ 6¼ ¢
174 ¹ yd. @ 6 ¢	1260 yd. @ 12½ ¢	436 ³ yd. @ 7 ¢

BILLS REVIEWING UNITED STATES MONEY.

A Bill is an itemized statement of purchase or sale.

In money columns a double line precedes dollars, and a single line precedes cents.

Characters and terms used in billing:

A1.	First quality.	\$	Dollars.
@	at.	#	Number, or Pounds.
%	Account.	X	by.
%	Per cent.	"	The same.
%	Care of.	B. L.	Bill of Lading.
✓	Check Mark.	C. O. D.	Cash on delivery.
¢	Cents.	F. O. B.	Free on board.

Find the amount of each of the following bills:

80.

CHICAGO, ILL., JAN. 25, 1897.

E. M. LAMSON,
Phoenix, Ariz.

Bought of E. N. SPELLMAN.

Terms Cash.

24	pr. Congress Shoes	\$1.60	38	40		
18	" Men's Heavy Shoes	1.75	31	50		
36	" Ladies' French Kid Shoes	3.50	126			
24	" Boys' School Shoes	1.80	—	—		
12	" Infants' Shoes	.90	10	80		
18	" Misses' Gondola Button	1.75	—	—		
36	" Ladies' Kid Slippers	1.50	54			
25	" Calf Boots	3.50	—	—		
36	" Child's Kid Shoes	1.00	36			
30	" Men's Arctics	1.25	—	—		
60	" Rubbers	1.10	66		562	40

Rec'd. Payment,

STRAUS & SMITH.

Per W.

81.

ST. LOUIS, Mo., January 25, 1897.

TRIPP & TRIPP,
Aptakisic, Ill.

Bought of SAMUEL B. WILLEN

Terms 30 days.

2 doz. Kitchen Tables.....	(each)	\$ 4.20
$\frac{2}{3}$ " Common Lounges.....	(each)	9.00
$1\frac{1}{4}$ " 10 ft. Walnut Extension Tables..	(each)	14.00
4 " Dining Room Chairs.....	(per doz.)	11.50
$\frac{3}{4}$ " Cottage Bedsteads.....	(each)	4.50
7 Walnut Marble Top Center Tables....	(each)	9.50
2 doz. Antique Oak Bedroom Sets.....	(each)	25.00
1 " Cherry Bedroom Sets.....	(each)	30.00
$\frac{1}{2}$ " Bird's Eye Maple Bedroom Sets..	(each)	75.00
7 Office Desks, Oak.....	(each)	16.00
$\frac{3}{4}$ doz. Rockers, Upholstered.....	(per doz.)	32.00
2 " Fancy Baby Cabs.....	(each)	12.00

82. Mr. O. M. Miller, San Francisco, Cal., bought of E. M. Lamson, Phoenix, Ariz. Terms 60 days.

20480 feet Flooring.....	\$16.00 per M
6240 " Joists.....	14.00
2360 " Ship lap.....	12.50
15450 " 2x6-15.....	15.50
7260 " 4x4-12.....	15.00
4896 " 2x4-16.....	14.50
24066 " 2x8-14.....	15.50
8412 " $1\frac{1}{4}$ x14-12.....	21.25
12600 " $1\frac{1}{2}$ x16-14.....	20.50
10458 " 6x6-18.....	22.25
14448 " 6x12-24.....	24.00
14260 " Sheeting.....	12.00
240 M Shingles.....	4.00
220 M Extra Shingles.....	5.25
11680 Posts, round.....	25.00 per C
12600 " split.....	21.00

83. W. I. Staley & Co., Salem, Oregon, bought of N. A. Barrett, Wholesale Dry Goods, Chicago, Ill. Terms, cash.

20 Pieces Calico:

36 ¹	42 ²	43	41 ¹	39 ³	38	39 ²	44	41 ¹
39	39 ¹	41 ²	38 ³	42	43 ²	40	41 ¹	38
39	39 ¹							@ 6 $\frac{1}{4}$ ¢

10 Pieces Delaines:

29 ²	28	27 ²	28	26 ³	29 ³	30	31 ¹	28
27								@ 16¢

84. J. G. Weidner, Aptakisic, Ill., bought of J. V. Farwell, Chicago, August 1, 1896.

Case #120, 36 pieces Challie.

46 ¹	43 ¹	41 ¹	40 ²	43 ²	39 ³	38 ¹	37 ¹	42	43 ²	40 ¹	38
41	47 ¹	43	39	43 ¹	42	41 ¹	41 ³	42 ³	42 ¹	44	43 ³
42	38 ²	37 ¹	40 ³	39 ²	41	42 ²	43	40	44	42 ³	41
										@ 75¢	

Case #63, 24 pieces Henrietta Cloth.

34 ²	30 ¹	33 ²	34 ¹	36 ³	34	32 ³	34	36 ²	33 ³	31	30 ²
31 ³	34 ¹	33 ³	37	31 ¹	31 ³	32 ³	30 ¹	32	31 ²	33 ¹	30
										@ 66 $\frac{2}{3}$ ¢	

Case #143, 20 pieces Serge.

32 ¹	34 ²	33	21 ³	29	28 ¹	26 ³	31 ²	32 ¹	30	29 ¹	27
32 ¹	34 ³	32 ¹	33	31 ²	32 ¹	31 ²	33 ³				
										@ 37 $\frac{1}{2}$ ¢	

85. Bert Clemens bought of F. S. McDaniel for cash, 25 lb. C. Sugar @ 5 $\frac{3}{4}$ ¢; $\frac{1}{2}$ bu. Pears @ \$2.40; 20 lb. Butter @ 21¢; 3 baskets Grapes @ 33 $\frac{1}{3}$ ¢; 2 sk. P. P. Flour @ \$1.75; 8 lb. Japan Tea @ 75¢; 6 lb. Raisins @ 16 $\frac{2}{3}$ ¢; 2 sk. Salt @ 12¢; 1 doz. Eggs @ 18¢; 5 baskets Quinces @ 70¢; 2 lb. dressed Chicken @ 11¢; 14 lb. Graham Flour @ 4¢; 5 lb. Cheese @ 18¢.

86. A. A. Brewer bought of H. Hulman on acct., 30 days, 10 bbl. Kerosene, 440 gallons, @ 11¢; 6 bbl. M. Syrup, 43 gallons each, @ 40¢; 8 sk. Rice, 900-25 lb., @ 8 $\frac{1}{3}$ ¢; 4 boxes ground Coffee, each 64-14 lb., @ 28¢; 10 sk. Rio Coffee, 1500-30 lb., @ 18¢; 4 ch. Tea, each 73-13 lb., @ 45¢; 3 boxes Laundry Starch, 156-30 lb., @ 10¢.

87. Messrs. Zucek & Sharp bought of E. L. Coe & Co. Terms: Acct. Draft 30 days. 12600 ft. 2 x 6-16,* 14750 ft. 2 x 8-16 at \$16.75 per M; 2046 ft. 2 x 6-12, 8260 ft. 4 x 4-14, 3280 ft. 4 x 4-12, 16480 ft. 2 x 4-16, 12840 ft. 2 x 8-14, at \$16.25 per M; 1240 ft. 4 x 10-22, 3260 ft. 8 x 10-24, 4560 ft. 6 x 6-24, 5840 ft. 2 x 6-20, 3492 ft. 2 x 6-22, 4980 ft. 2 x 6-24, 8948 ft. 2 x 12-22, at \$19.75 per M; 10960 ft. 1½ x 12-16, 4260 ft. 6 x 6-20, 5840 ft. 8 x 8-16, 6280 ft. 8 x 8-18, 7250 ft. 8 x 8-20, 4650 ft. 8 x 10-20, at \$24 per M; 1260 Posts, round, at \$24 per C; 1875 Posts, split, at \$21 per C; 75 Doors 3 x 6½ x 1½ at \$2.50; 120 Doors 2½ x 6½ x 2½ at \$3.75; 60 M Laths at \$1.75; 160 M XXX Shingles at \$2.62½.

88. The Chicago Hay and Commission Co., sold for Butler & Jones the following Mdse: 12600 lb. Timothy Hay @ \$9 per T.; 14780 lb. Clover Hay @ \$11.50 per T.; 21400 lb. Red Top, baled, @ \$12.60 per T.; 13580 lb. Prairie Hay @ \$8.25 per T.; 15260 lb. Upland @ \$7.75 per T.; 4 loads of Oats, 3640, 4260, 3125, and 2980 lb., @ 20¢ per bu.; 6 loads of Wheat, 3160, 3165, 4280, 4120, 3284, and 4280 lb., @ 80¢ per bu., 1 car Barley #1, 42680 lb. @ 72¢ per bu.

89. Zuttermeyster Bros. bought of H. B. Claffin & Co.

Case #120, 20 pieces Calico.

30 ¹	32 ¹	34 ²	32 ³	34 ²	33 ¹	34 ¹	36 ¹	40	31
32 ²	34	35	36 ³	33	32 ²	34	33 ¹	34	36 ²

@ 4½¢.

Case #124, 30 pieces Lawn.

34 ¹	36	39 ¹	42	41 ²	42 ²	39	40 ²	36 ²	33 ¹
34 ¹	39 ²	34 ²	36	41	42 ²	40	38 ²	36	33 ²
34	42	41 ¹	40	42	39 ¹	38 ²	36 ²	34	40 ²

@ 12½¢.

Case #124, 22 pieces Organdie.

42	43 ¹	41	36 ¹	41	42 ²	44 ²	43	40	42 ²	39
38 ²	42 ¹	43 ¹	39	42 ²	43	44	41 ²	44	42	36 ²

@ 25¢.

Casing \$4.00. Cartage \$2.00.

* 2 x 6-16=16 ft. long, 6 in. wide, and 2 in. thick.

DENOMINATE NUMBERS.

36. A **Denominate Number** is a concrete number whose units are used to measure a scale, the same being established by law or usage. Thus, 7 hours, 3 bushels, 2 miles, 10 pounds, are denominate numbers.

37. A **Simple Denominate Number** is one that expresses units of one denomination. Thus, 1 week, 7 quarts, 24 rods, are simple denominate numbers.

38. A **Compound Denominate Number** is one that expresses units of two or more denominations that are related to each other. Thus, 1 hr. 7 min. 24 sec., and 2 mi. 4 rd. 1 yd. 7 in., are compound denominate numbers.

The Reduction of Denominate Numbers is the process of changing them from one denomination to another, and is of two kinds—ascending and descending. Thus, 3 yd. 2 ft. 6 in.=138 in., is reduction descending; and reversing the operation is reduction ascending.

MEASURES OF VALUE.

UNITED STATES MONEY.

10 mills (m)	= 1 cent.....¢ ct.
10 cents	= 1 dime.....d.
10 dimes	= 1 dollar.....\$.
10 dollars	= 1 eagle.....E.

The unit of value is the gold dollar—weight 25.8 gr.

Denominations coined:

Gold.—\$20, \$10, and \$5 pieces.

Silver.—\$1, 50¢, 25¢, and 10¢ pieces.

Nickel.—5¢ piece.

Bronze.—1¢ piece.

CANADA MONEY.

The currency of Canada is on a decimal scale and is similar to that of the United States.

Denominations coined:

Silver.—50, 25, 10, and 5 cent pieces.

Bronze.—1 cent piece.

ENGLISH MONEY.

4 farthings (far)	= 1 penny.....d.	= \$.0202
12 pence	= 1 shilling.....s.	= \$.2433
20 shillings	= 1 pound.....£.	= \$4.8665

The unit of value is the pound, or sovereign, valued at \$4.8665 in United States money.

Denominations coined:

Gold.—Sovereign (1 £) and half-sovereign.

Silver.—Crown (5 s.), half-crown, florin (2s.), six-pence, and three-pence.

Bronze.—Penny, half-penny, and farthing.

FRENCH MONEY.

10 millimes (ms.)	= 1 centime.....ct.
10 centimes	= 1 decime.....dc.
10 decimes	= 1 franc.....fr.

The unit of value is the franc, valued at \$.193 in United States money.

Denominations coined:

Gold.—100, 40, 20, 10, and 5 franc pieces

Silver.—5, 2, and 1 franc pieces; and the 50 and 20 centime pieces.

Bronze.—10, 5, 2, and 1 centime pieces.

The French use the decimal point to separate francs from centimes, as: 124.36 is read 124 francs and 36 centimes

GERMAN MONEY.

100 pennies (pfennige) = 1 mark.

The unit of value is the mark (reichmark), valued at \$.2385 in United States money.

Denominations coined:

Gold.—20, 10, and 5 mark pieces.

Silver.—2 and 1 mark, and 20 penny pieces

Nickel.—10 and 5 penny pieces.

Bronze.—2 and 1 penny pieces.

The Germans use the decimal point to separate marks from pennies.

MENTAL PRACTICE.

1. How many dimes in \$4? \$6? \$6.50?
2. How many mills in 50 cents? \$1.50?
3. How many farthings in 2 pence? 1 shilling? 1 pound?
4. What is the value in U. S. money of 2 francs? 2 marks?
5. £2 3s. = how many shillings? pence?
6. £5 10s. 10d. = how many pence?
7. How many farthings in 4d.? 5d. 2 far.? 8d. 3 far.? 4½d.? 8¾d.? 12½d.?
8. How many pounds in 20s.? 30s.? 40s.? 60s.? 75s.? 90s.?
9. How many dollars in 400 cents? 650 cents? 30 dimes? 75 dimes? 2000 mills?
10. How many dollars must be paid for 30 bushels of wheat at 5½ dimes per bushel?

WRITTEN PRACTICE.

Reduction descending.

Reduce 2£ 3s. 4d. 1 far. to arthings.

SOLUTION.

2£ 3s. 4d. 1 far.

20

40

3

43s.

12

516

4

520d.

4

2080

1

2081 far.

2£ 3s. 4d. 1 far. = 2081 far.

Reduction ascending.

Reduce 2081 far. to higher denominations.

SOLUTION.

4)2081

12) 520....1 far.20) 43....4d.2....3s.

2081 far. = 2£ 3s. 4d. 1 far.

Let the student make a rule for reduction ascending, and another for reduction descending.

1. Reduce £12 7s. 9d. 3 far. to far.
2. Reduce 8fr. 9dc. 3ct. 1ms. to ms.
3. Reduce 75 marks 29 pennies to pennies.
4. Reduce 4237 far. to higher denominations.
5. Reduce 12324ms. to higher denominations.
6. Reduce £9 3s. 4d. 2 far. to dollars and cents.
7. Reduce 50fr. 9dc. 8ct. to dollars and cents.
8. Reduce 75 marks 25 pennies to dollars and cents.
9. Reduce £11 12s. 8d. 1 far. to lower denominations
10. Change 24.15 francs to marks.

MEASURES OF EXTENSION.

39. Linear Measure has but one dimension, viz., length.

LINEAR, OR LINE MEASURE.

12 inches (in.)	= 1 foot.....ft.
3 feet	= 1 yard.....yd.
5½ yards	= 1 rod.....rd.
320 rods	= 1 mile.....mi.

SURVEYORS' LINEAR.

7.92 inches (in)	= 1 link.....l.
25 links	= 1 rod.....rd.
4 rods	= 1 chain.....ch.
80 chains	= 1 mile.....mi.

MISCELLANEOUS LINEAR.

$\frac{1}{3}$ inch	= 1 size. In measuring boots and shoes
4 inches	= 1 hand. In measuring horses.
9 inches	= 1 span.
3 feet	= 1 pace. Pacing distance.
6 feet	= 1 fathom. In measuring depths at sea.
1.15 statute mile	= 1 nautical, or geographical mile.
1 nautical mile	= 1 knot. In measuring speed of vessels
3 nautical miles	= 1 league. In measuring distances at sea.
60 nautical miles	} = 1 degree.
69.16 statute miles	
360 degrees	= 1 circumference of the earth.

40. Surface has two dimensions, viz., length and width.
The surface is found by multiplying the two dimensions.

SURFACE, OR SQUARE MEASURE.

144 square inches (sq. in.)	= 1 square foot.....sq. ft.
9 square feet	= 1 square yard.....sq. yd.
$30\frac{1}{4}$ square yards	= 1 square rod..sq. rd. or P.
160 square rods	= 1 acre.....A.
640 acres	= 1 square mile.....sq. mi.

SURVEYORS' SQUARE MEASURE.

625 square links (sq. l.)	= 1 square rod.....sq. rd.
160 square rods	= 1 acre.....A.
640 acres	= 1 section.....Sec.
36 sections	= 1 township.....Tp.

41. A Solid, or Cube has three dimensions, viz., length, width, and height.

The solidity is found by multiplying the three dimensions.

SOLID, OR CUBIC MEASURE.

1728 cubic inches (cu. in.) = 1 cubic foot....cu. ft.

27 cubic feet = 1 cubic yard.. cu. yd.

WOOD MEASURE.

16 cubic feet = 1 cord foot....cd. ft.

128 cubic feet } = 1 cord.....cd.
8 cord feet }

A cord of wood is 8 feet long, 4 feet wide, and 4 feet high
($8 \times 4 \times 4$).

STONE MEASURE.

$24\frac{1}{2}$ cubic feet = 1 perch.

A perch of stone, or masonry, is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide,
and 1 foot high ($16\frac{1}{2} \times 1\frac{1}{2} \times 1$).

MENTAL PRACTICE.

1. How many inches in 5 feet? 8 feet? 6 feet? 4 feet?
2. How many inches in 2 yards? $3\frac{1}{2}$ yards?
3. How many feet in 60 inches? 78 inches? 100 inches?
4. How many yards in 72 inches? 108 inches?
5. What part of a yard is 9 inches? 18 inches? 24 inches?
6. What part of a yard is 1 ft. 6 in.? 2 ft. 6 in.?
7. How many feet in 2 rods and 2 yards?
8. How many paces in 24 ft? How many fathoms?
9. What part of a mile is 80 rods? 160 rods? 4 chains?
20 chains?
10. How many feet high is a pony that measures 12 hands
high?
11. How many square inches in a rectangle 8 inches long
and 5 inches wide? One 6 by 7? One 8 by 10?
12. How many square feet in an 8 foot square? In a 12
foot square?
13. How many square yards in the ceiling of a room 24
feet long and 12 feet wide?
14. What part of 8 feet square is 8 square feet? 4 square
feet?

15. At \$1.75 a square yard, what will it cost to carpet a room 12 feet long and 9 feet wide?
16. What part of a square foot is 72 sq. in.? 24 sq. in.? 16 sq. in.?
17. How many cubic feet in 3 cu. yd.? $2\frac{1}{2}$ cu. yd.? 2 cu. yd.? 20 cu. yd.?
18. Give the dimensions of a cord of wood. A perch of stone.
19. How many cubic feet in a solid 6 feet long, 4 feet wide, and 2 feet high?
20. How many cubic feet in a 5 foot cube? In a 10 foot cube?
21. How many square feet of surface on a 5 foot cube? On a 10 foot cube?
22. How many cubic feet in 2 cords of wood? In 2 perches of stone?
23. What part of 2 yards is 12 inches? 24 inches? 4 feet?
24. What part of an acre is a field 10 rods long and 8 rods wide? One 8 rods by 5 rods? One 12 rods by 10 rods?

WRITTEN PRACTICE.

11. Reduce 5 mi. 41 rd. 3 yd. 1 ft. 6 in. to in.
12. Reduce 7 A. 20 sq. rd. 1 sq. yd. 2 sq. ft. 4 sq. in. to sq. in.
13. Reduce 7 sq. ch. 8 P. 420 sq. l. to square links.
14. Reduce 20 cu. ft. 927 cu. in. to cubic inches.
15. Reduce 124635 inches to miles.
16. Reduce 427863 sq. ft. to A.
17. Reduce 4325163 cu. in. to higher denominations.
18. Reduce 325146 sq. ch. to Tp.
19. Reduce 12460 cu. ft. to cords.
20. A warship makes 22.5 knots per hour. How many miles does she go in 5 hours?

MEASURES OF CAPACITY.**LIQUID MEASURE.**

4 gills (gi.)	= 1 pint.....pt.
2 pints	= 1 quart.....qt.
4 quarts	= 1 gallon..gal.
31½ gallons	= 1 barrel.....bbl.
63 gallons }	= 1 hogshead...hhd.
2 barrels }	

42. Liquid Measure is used in measuring all liquids, except liquid medicines sold in prescriptions.

The denominations, barrels and hogsheads, are used in determining the contents of cisterns and reservoirs. The barrels and casks used in commerce vary in size and have the capacity stamped on them. The capacity is found by gauging.

DRY MEASURE.

2 pints (pt.)	= 1 quart.....qt.
8 quarts	= 1 peck.....pk.
4 pecks	= 1 bushel....bu.

43. Dry Measure is used in measuring dry articles; as, grain, fruit, vegetables, etc.

APOTHECARIES LIQUID MEASURE.

60 minims (M)	= 1 fluidrachm	f 3
8 fluidrachms	= 1 fluidounce	f 3
16 fluidounces	= 1 pint	O.
8 pints	= 1 gallon	Cong.

44. Apothecaries Liquid Measure is used in prescribing and compounding liquid medicines.

The Apothecaries gallon contains 231 cubic inches.

COMPARATIVE TABLE OF MEASURES.

Liquid measure,	231 cu. in. in 1 gal.	57½ cu. in. in 1 qt.
Dry measure,	268½ " " " ½ pk.	67½ " " " 1 qt.
Apothecaries measure,	231 " " 1 gal.	

MENTAL PRACTICE.

1. How many gills in 4 pints? 3 quarts? 5 gallons?
2. How many pints in 3 quarts? 2 pecks? 5 bushels?
3. How many pints in 128 gills? quarts? gallons?
4. How many quarts in 64 pints? pecks? bushels?
5. How many gills in 1 gal. 2 qt. 1 pt. 2 gills?
6. How many quarts in 1 bu. 3 pk. 6 qt.?
7. How many pints in a cask that contains 10 gallons?
8. How many quarts in a bin that holds 15 bushels?
9. What part of a bushel is 16 quarts? 8 quarts? 24 quarts?
7 pecks? 32 pints? 48 pints?
10. What part of a gallon is 3 quarts? 5 pints? 16 gills?
8 gills? 24 gills?
11. What will $\frac{1}{2}$ bushel of clover seed cost at 20 cents a quart?
12. How many gallons of vinegar can be bought for \$2.50 at $2\frac{1}{2}$ cents a pint?
13. From a barrel containing 45 gallons of molasses, were drawn at one time 12 gallons, at another time 10 gallons and 2 quarts. What is the value of the remainder at \$.12 $\frac{1}{2}$ a quart?
14. How many fluidrachms in 3 fluidounces? 5 fluidounces? 1 pint? $2\frac{1}{2}$ pints?
15. Bought a cask of cranberries containing $2\frac{5}{16}$ bushels for \$5.25 and sold them at 10 cents a quart. What was the gain?
16. .75 of a gallon equals how many gills?
17. Sold .25 of a bushel of plums at 5 cents a pint. How much was received?
18. What will be the cost of $\frac{3}{4}$ of a gallon of wine at 5 cents a gill?
19. At 10 cents a quart, what will 25 gallons of molasses cost?
20. A merchant bought 25 bushels of oats at $12\frac{1}{2}$ cts. per bushel, and sold them at $6\frac{1}{4}$ cts. per peck. How much did he gain?

WRITTEN PRACTICE.

21. Reduce 9632 gi. to higher denominations.
22. Reduce 8431 pt. to bushels.
23. Reduce 6342 pt. to gallons.
24. Reduce 3204 qt. to bushels.
25. Reduce 45 bu. 2 pk. 6 qt. 1 pt. to pints.
26. Reduce 15 gal. 2 qt. 1 pt. 2 gi. to gills.

MEASURES OF WEIGHT.

TROY WEIGHT.

24 grains (gr.)	= 1 pennyweight....pwt.
20 pennyweights	= 1 ounce.....oz.
12 ounces	= 1 pound.....lb.

45. Troy Weight is used in weighing gold, silver, and jewels.

APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple.....sc. or ℥
3 scruples	= 1 dram.....dr. or ʒ
8 drams	= 1 ounce.....oz. or ʒ
12 ounces	= 1 pound.....lb. or ℔

46. Apothecaries' Weight is used in prescribing and mixing dry medicines.

Medicines are wholesaled by avoirdupois weight.

AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	= 1 pound.....lb.
100 pounds	= 1 hundredweight...cwt.
20 hundredweight	= 1 ton.....T.

47. Avoirdupois Weight is used in weighing all ordinary articles; as, grain, meats, groceries, etc.

COMPARATIVE TABLE OF WEIGHTS.

	Troy.	Apothecaries.	Avoirdupois.
1 pound	= 5760 grains,	= 5760 grains,	= 7000 grains.
1 ounce	= 480 "	480 "	437.5 "

MENTAL PRACTICE.

1. How many grains in 5 pwt.? 2 oz.? 3 oz. 2 pwt.?
2. How many ounces in 60 pwt.? 75 pwt.? 3 lb. 5 oz.?
3. How many grains in 3 sc.? $5\frac{1}{2}$ sc.? 2 dr.? 5 dr.?
4. How many scruples in 480 grains? drams? ounces?
5. What part of a pound Troy is 8 oz.? 10 oz.?
6. What part of a pound Avoir. is 12 oz.? 14 oz.? 6 oz.?
7. How many powders each containing 8 grains can be made from 1 oz. 6 dr. 2 sc. of quinine?
8. How many cwt. in 650 lb.? 375 lb.? 1250 lb.?
9. What part of 2 tons is 15 cwt.? 10 cwt.? 8 cwt.? $7\frac{1}{2}$ cwt.?
10. If 16 rings are made from 2 lb. 4 oz. of gold, what will each ring weigh?
11. What part of $\frac{3}{4}$ of a ton is 10 cwt.? 900 lb.? 750 lb.?

WRITTEN PRACTICE.

27. Reduce 5 lb. 6 oz. 14 pwt. 14 gr. to grains.
28. Reduce 15 lb. 2 dr. 3 sc. 16 gr. to grains.
29. Reduce 5 T. 10 cwt. 90 lb. to ounces.
30. Reduce 8 lb. 12 pwt. to grains.
31. Reduce 9 lb. 2 sc. to grains.
32. Reduce 2 T. 50 lb. to ounces.
33. Reduce 24360 grains to lb., Apoth.
34. Reduce 243160 oz. to tons.
35. Reduce 4264 pwt. to higher denominations.

MEASURE OF DURATION.

TIME.

60 seconds (sec.)	= 1 minute.....min.
60 minutes	= 1 hour.....hr.
24 hours	= 1 day.....d.
7 days	= 1 week.....wk.
{ 365 days	= 1 common year..yr.
{ 12 months	
366 days	= 1 leap year.....yr.
100 years	= 1 century.....C.

48. One Year marks the time of a complete revolution of the earth around the sun. It requires 365 days, 5 hours, 48 minutes, and 49.7 seconds to complete this revolution.

This amount being very nearly $365\frac{1}{4}$ days, three successive years are estimated at 365 days each, and the fourth year at 366 days. This correction of one day in every four years makes a gain in excess of the true amount of 11 minutes and 10.3 seconds per year, or about 3 days every 400 years. In order to correct this gain, the extra day is not added to three out of every four centennial years. All years divisible by 4 are leap years, except centennial years, which must be divisible by 400.

In ordinary business calculations, the year consists of twelve months of thirty days each.

The common year contains 52 weeks and 1 day, and every year following a common year begins one day later in the week. The leap year contains 52 weeks and 2 days, and the year following leap year begins two days later in the week.

MONTHS IN THE YEAR.

Months	Abbrev.	No.	Days	Months	Abbrev.	No.	Days
January	Jan.	1	31	July	July	7	31
February	Feb.	2	28	August	Aug.	8	31
March	Mar.	3	31	September	Sept.	9	30
April	Apr.	4	30	October	Oct.	10	31
May	May	5	31	November	Nov.	11	30
June	June	6	30	December	Dec.	12	31

MISCELLANEOUS.

CIRCULAR MEASURE.

60 seconds of arc (")	= 1 minute of arc....'
60 minutes	= 1 degree.....°
30 degrees	= 1 sign.....S.
12 signs	= 1 circle.....Cir.
60 degrees	= 1 sextant
90 degrees	= 1 right angle.

49. Circular Measure is used principally in surveying, navigation, etc.

STATIONERY.

24 sheets	= 1 quire.....qr.
20 quires	= 1 ream.....rm.
2 reams	= 1 bundle.....bdl.
5 bundles	= 1 bale.

COUNTING.

11 12 units	= 1 dozen..... doz.
12 dozen	= 1 gross..... gro.
12 gross	= 1 great gross..... g. gro.
20 units	= 1 score..... sc.

BOOKS.

The size of the page is indicated by the number of times the sheet is folded

A sheet of paper folded into—	{ 2 leaves is called Folio, making 4 pages.
	{ 4 leaves is called Quarto, making 8 pages.
	{ 8 leaves is called Octavo, making 16 pages.
	{ 12 leaves is called Duodecimo, making 24 pages.
	{ 16 leaves is called 16 mo., making 32 pages.
	{ 18 leaves is called 18 mo., making 36 pages.

MENTAL PRACTICE.

1. What part of a day is 12 hr.? 9 hr.? 8 hr.? 16 hr.?
2. Reduce 2 hr. 30 min. to seconds.
3. How many hours in the month of March?
4. How many seconds in $\frac{3}{4}$ of an hour? $\frac{1}{4}$?
5. How many hours from 3 A. M. to 5 P. M.?
6. How many minutes from 20 minutes before 6 A. M. to 25 minutes after 5 P. M.?
7. How many minutes from 10:45 P. M. to 1:30 A. M.?
8. How many units in 10 doz.? 12 doz.? $2\frac{1}{2}$ gross?
9. How many dozen in 1728 units? Gross? Great gross?
10. How many degrees in a quadrant? $\frac{1}{2}$ of a quadrant? $\frac{3}{4}$ of a quadrant?
11. How many degrees in a sextant? $\frac{1}{3}$ of a sextant? $\frac{2}{3}$ of a sextant?
12. 120° is what part of a circumference? 60° ? 180° ?

WRITTEN PRACTICE.

36. Reduce 6 da. 11 hr. 30 min. 20 sec. to sec.
37. Reduce $20^\circ 20' 20''$ to seconds.
38. Reduce 4 bdl. 20 sheets to sheets.
39. Reduce 5 gro. 6 doz. 7 units to units.

40. Reduce 1 leap year to hours.
41. Reduce the month of January to seconds.
42. Reduce 4365264 min. to common years.
43. Reduce 946342" to signs.

DENOMINATE FRACTIONS.

50. A **Denominate Fraction** is one whose integral unit is a denominate number.

The principles and processes are practically the same as those of denominate integers.

REDUCTION DESCENDING.

To reduce denominate fractions (common or decimal) to integers or fractions of lower denominations.

EXAMPLES.

1. Reduce $\frac{5}{9}$ acres to integers of lower denominations.
2. Reduce .7385 of a gal. to integers of lower denominations.

SOLUTION 1.

$$\begin{aligned}\frac{5}{9} \times 160 &= 88\frac{8}{9} \text{ sq. rd.} \\ \frac{8}{9} \times \frac{1}{4} &= 26\frac{8}{9} \text{ sq. yd.} \\ \frac{8}{9} \times 9 &= 8 \text{ sq. ft.} \\ 88 \text{ sq. rd. } 26 \text{ sq. yd. } 8 \text{ sq. ft.}\end{aligned}$$

EXPLANATION. — Multiply the fraction by the number of units in the next lower denomination. Leave the units as a part of the result; proceed with the remaining fraction as before. The several integers together with the last fraction, if any, is the required result.

SOLUTION 2.

$$\begin{array}{r} .7385 \text{ gal.} \\ 4 \\ \hline 2.9540 \text{ qt.} \\ 2 \\ \hline 1.908 \text{ pt.} \\ 4 \\ \hline 3.632 \text{ gi.} \end{array}$$

EXPLANATION.—Same as above.

2 qt. 1. pt. 3.632 gi.

WRITTEN PRACTICE.

Reduce to lower denominations:

- | | |
|---------------------------------|--------------------------------------|
| 44. $\frac{3}{4}$ of a day. | 51. $\frac{1}{160}$ of a rod. |
| 45. $\frac{7}{12}$ of a mi. | 52. .625 of a mile. |
| 46. $\frac{4}{7}$ of a mo. | 53. .727 of a ton. |
| 47. $\frac{7}{11}$ of an acre. | 54. .625 of a hhd. |
| 48. $\frac{5}{12}$ of a cu. yd. | 55. .4225 of a cord. |
| 49. $\frac{8}{9}$ of a gal. | 56. .5375 of a gal. |
| 50. $\frac{1}{320}$ of a bu. | 57. $\frac{3}{4}$ of a pound, Apoth. |

REDUCTION ASCENDING.

To reduce denominate fractions (common or decimal) to fractions of higher denominations.

EXAMPLES.

1. What part of a pound is $\frac{3}{4}$ of a pwt.?
2. What decimal of a day is .624 min.?

SOLUTION 1.

$$\frac{3}{4} \times \frac{1}{20} \times \frac{1}{12} = \frac{1}{800} \text{ lb.}$$

EXPLANATION.—Divide by the numbers successively, necessary to reduce pwt. to lb.

SOLUTION 2.

$$60 \overline{) .624} \text{ min.}$$

$$24 \overline{) .0104} \text{ hr.}$$

$$.0004\frac{1}{3} \text{ da.}$$

EXPLANATION.—Divide by the numbers successively, necessary to reduce min. to da.

WRITTEN PRACTICE.

58. What fraction of a bu. is $\frac{7}{8}$ of a pint?
59. What decimal of a bushel is .24 of a pint?
60. What fraction of a sq. yd. is $\frac{3}{4}$ of a sq. in.?
61. What decimal of a £ is $\frac{3}{4}$ of a shilling?
62. What fraction of an acre is .725 sq. rd.?
63. Reduce $\frac{3}{4}$ of a cwt. to the decimal of a ton.
64. Reduce .75 of a gill to the fraction of a gallon.

To reduce a compound number to a fraction of a higher denomination.

EXAMPLES.

1. What fractional part of a common year is 5 mo. 15 da.?
2. What decimal part of a day is 4 hr. 30 min.?

OPERATION 1.

5 mo. 15 da. = 165 da. EXPLANATION.—Reduce the number
 1 yr. = 360 da. given to its lowest denomination. 165 days
 are $\frac{165}{360}$ of 1 year, or $\frac{11}{24}$ yr.

OPERATION 2.

4 hr. 30 min. = 270 min. EXPLANATION.—270 min. are $\frac{270}{1440}$ of
 1 da. = 1440 min. a day. Reduced to a decimal equals .1875.
 $\frac{270}{1440} = \frac{3}{18} = .1875$ da.

WRITTEN PRACTICE.

65. What fractional part of a bu. is 3 pk. 4 qt. 2 pt.?
66. What decimal part of a pound Troy is 8 oz. 10 pwt.?
67. What fractional part of 2 hhd. 20 gal. is 30 gal.?
68. What decimal part of $2\frac{1}{2}$ mi. is 10 rd. 4 ft.?
69. Reduce 3 d. 3 far. to the fraction of a shilling.
70. Reduce 8 oz. 2 dr. 2 sc. 10 gr. to the decimal of a lb.
71. Reduce 20 rd. 12 ft. 10 in. to the fraction of a mile.
72. Reduce 2 sq. yd. 6 sq. ft. 100 sq. in. to the fraction of a sq. rd.

ADDITION OF COMPOUND NUMBERS.

EXAMPLE.

What is the sum of £24 6s. 7d. 2 far.; £36 14s. 10d. 3 far.;
 £45 9s.; 15s. 11d. 1 far.; £60 14s. 7d. 2 far.; £4 3 far.; £7 8s.
 5 far.; and £10 4s. 7d. 2 far.?

OPERATION.

	20	12	4
£	s.	d.	far.
24	6	7	2
36	14	10	3
45	9	0	0
	15	11	1
60	14	7	2
4	0	0	3
7	8	0	5
10	4	7	2
186	70	42	18
189	13	10	2

EXPLANATION.—Write the numbers so that units of the same denomination stand in the same column.

Add as in whole numbers; if the total of any column equals one or more of the next denomination it should be reduced to the higher denomination.

WRITTEN PRACTICE.

73. What is the sum of 46 lb. 4 oz. 4 pwt. 20 gr.; 25 lb. 6 oz. 14 pwt. 14 gr.; 35 lb. 11 oz. 2 pwt. 18 gr.; 20 lb. 14 gr.; 9 oz. 14 pwt. 15 gr.; 45 lb. 9 oz. 13 pwt. 12 gr.; 25 lb. 11 pwt.; and 28 lb. 10 oz. 8. pwt. 12 gr.?

74. Add 3 bbl. 20 gal. 3 qt. 1 pt. 3 gi.; 12 bbl. 14 gal. 1 qt.; 30 gal. 2 qt. 1 pt. 2 gi.; 26 gal. 3 qt. 1 pt. 2 gi.; 5 bbl. 28 gal. 1 qt. 2 gi.; and 3 qt. 1 pt. 1 gi.

75. Find the sum of 14 A. 120 sq. rd. 20 sq. yd. 8 sq. ft. 100 sq. in.; 36 A. 140 sq. rd. 25 sq. yd. 4 sq. ft. 24 sq. in.; 96 sq. rd. 8 sq. ft. 36 sq. in.; 48 A. 8 sq. yd., and 42 A. 20 sq. rd. 24 sq. yd. 4 sq. ft. 90 sq. in.

76. Add $\frac{7}{8}$ of a rod, $\frac{3}{8}$ of a foot, and $7\frac{1}{4}$ in.

SUGGESTION.—Reduce fractions to integers and fractions of lower denominations.

77. What is the sum of $\frac{3}{8}$ of a mile, $1\frac{5}{16}$ of a rod, $\frac{3}{4}$ of a rod, and $\frac{7}{8}$ of a foot?

78. Find the sum of .125 of a barrel, .75 of a gal., 8.5 quarts, .375 of a pint.

SUBTRACTION OF COMPOUND NUMBERS.

EXAMPLE.

1. From 160 A. take 56 A. 110 sq. rd. 6 sq. yd.

OPERATION.				
	160	30 $\frac{1}{4}$	9	144
A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
160	—	—	—	—
56	110	6	—	—
103	49	24 $\frac{1}{4}$	—	—
103	49	24	2	36

EXPLANATION.—Write the numbers so that units of the same denomination stand in the same column, subtract as in whole numbers.

If a fraction occurs in an intermediate number, it may be reduced to integers of lower denominations.

WRITTEN PRACTICE.

	79.				80.				
	gal.	qt.	pt.	gi.	yr.	da.	hr.	min.	sec.
From	20	1	1	1	40	185	20	30	32
Take	14	3	1	3	16	200	21	36	20

81. A surveyed distance of 350 miles is made up of 3 lines. The first is 120 mi. 300 rd. 1 yd. 2 ft., the second is 140 mi. 160 rd. 2 yd. 1 ft. 8 in. What is the third?

82. Four men own a section of land. The first owns 120 A. 20 P., the second 240 A. 96 P., the third 154 A. 36 P., and the fourth the remainder. How many acres has the fourth?

83. From $\frac{3}{4}$ of a £, take $\frac{1}{2}$ of a shilling.

84. From the sum of $\frac{1}{2}$ of a mile and $\frac{1}{4}$ of a rod, take the sum of $\frac{1}{4}$ of a mile and $\frac{1}{2}$ of a rod.

85. From 138 sq. rd. 6 sq. ft. 64 sq. in., take 137 sq. rd. 30 sq. yd. 8 sq. ft. 100 sq. in.

MULTIPLICATION OF COMPOUND NUMBERS.

EXAMPLE.

1. Multiply 36 cu. yd. 8 cu. ft. 750 cu. in. by 6.

OPERATION.		
	27	1728
cu. yd.	cu. ft.	cu. in.
36	8	750
		6
<hr/>		
216	48	4500
<hr/>		
217	23	1044

EXPLANATION.—Write the multiplier under the right hand denomination, and multiply as in whole numbers.

If any of the lower denominations will make one or more of the next higher denomination, it may be so changed.

WRITTEN PRACTICE.

86. Multiply 24 bu. 2 pk. 7 qt. 1 pt. by 12.

87. Multiply 4 da. 14 hr. 45 min. 24 sec. by 8.

88. A half section of land is divided into four fields. The first contains 45 A. 120 P. 96 sq. ft., the second is 3 times the size of the first, and the third equals the difference between the first and second. What is the area of the fourth field?

89. What is the freight on 24 carloads of coal averaging 12 T. 7 cwt. 20 lb. at \$1.75 per ton?

90. A farmer sold 45 bu. 3 pk. 1 qt. 1 pt. of timothy seed at \$3.20 per bu. How much did he receive?

91. Find the cost of 3 T. 12 cwt. 20 lb. ore at \$9.40 per cwt.

DIVISION OF COMPOUND NUMBERS.

EXAMPLE.

7. Divide 34 bu. 3 pk. 5 qt. 1 pt. by 6.

OPERATION.			
	4	8	2
6)34 bu.	3 pk.	5 qt.	1 pt.
5	3	2	$\frac{1}{2}$

EXPLANATION.—Divide as in whole numbers, reducing the remainder, if any, to the next lower denomination and continue the division.

WRITTEN PRACTICE.

92. Divide 2 mi. 82 rd. 2 ft. 2 in. by 10.
 93. Divide $12\frac{1}{2}$ bu. $1\frac{1}{2}$ qt. 2 pt. by 8.
 94. Divide 24 yr. 14 da. 25 hr. 30 sec. by 30.
 95. Divide $4\frac{3}{4}$ hhd. $4\frac{3}{4}$ gal. by $6\frac{1}{4}$.
 96. The total weight of 25 hhd. of sugar is 10 T. 14 cwt. 30 lb. What is the average weight of each hhd.?
 97. An estate consisting of two sections of land is divided among a wife and four children as follows: The wife receives $\frac{1}{3}$, and the children each receive $\frac{1}{4}$ of the remainder. What is the area each receives?

PROMISCUOUS WRITTEN PROBLEMS.

98. How many rings of 18 pwt. each can be made from 9 oz. of gold.
 99. How much grain in 10 bags, each containing 2 bu. 1 pk.?
 100. What is the cost of $2\frac{1}{2}$ bu. of nuts, at 5 cts. per quart?
 101. A man sold .8 of a bushel of berries in equal parts to 8 families. How much did each pay him, at the rate of $12\frac{1}{2}$ cts. per quart?
 102. What is the difference in price between two loads of grain, one of oats weighing net 1600 lb. at 25 cts. per bushel, and one of wheat weighing net 1500 lb. at 80 cts. per bushel?
 103. Paid \$3.60 for a ream of paper, and sold it at 30 cts. per quire. How much did I gain per sheet?
 104. What will be the avoirdupois weight of 800 standard silver dollars ($412\frac{1}{2}$ grains)?

PRACTICAL MEASUREMENTS.

SURFACE.

Applications of square measure. Review Article 40.

51. The Area is the number of square units in a given surface.

GIVEN.—Length and breadth.

RULE.—*Multiply the length by the breadth.*

GIVEN.—Area and one dimension to find the other.

RULE.—*Divide the area by the given dimension.*



In the above figure the measuring unit is the sq. yd. The dimensions are in linear measure.

WRITTEN PRACTICE.

1. How many sq. ft. in a surface 24 ft. long and 18 ft. wide?
2. How many sq. yd. in a surface 48 ft. long and 32 ft. wide?
3. How much will it cost to sod a yard 96 ft. long by 48 ft. wide, at 25 cents per sq. yd.
4. A room is 36 ft. long, 24 ft. wide and 20 ft. high. How many sq. ft. in the ceiling? How many sq. ft. in the two sides? How many sq. ft. in the two ends? How many sq. yd. in all? How much will it cost to lath and plaster the room at 21 cts. per sq. yd., allowing 324 sq. ft. for doors and windows?
5. At 30 cts. a sq. yd. what will it cost to lath and plaster a room 24 ft. 6 in. long, 18 ft. 8 in. wide, and 12 ft. high, allowing for two doors and two windows each 4x8 ft.?
6. How many square feet in a box of glass containing 72 panes, each 12x16 in.?

7. How many sq. rd. in a field 160 rods long and 40 rods wide?

8. How many acres in a road 4 rods wide and 3 miles long?

9. What will it cost to pave a rectangular park which is 225 feet long and 120 feet wide at $62\frac{1}{2}$ cts. per sq. yd.?

10. A hall is 75 ft. long, 50 ft. wide, and 16 ft. high. What will it cost to plaster the same at 30 cts. per sq. yd., allowing for three doors 10x8 ft., 6 skylights 6x8 ft., and wainscoting 4 ft. high? (Doors 10 ft. high and 8 ft. wide.)

11. How many shingles, covering 4x4 in. each, will it take to cover a roof 40x20 ft.?

12. The roof of a barn is 50 ft. long and each side is 30 ft. wide, allowing the shingles to extend 5 inches to the weather and average 4 in. in width, what will the shingles cost at \$3.75 per M, the first course being doubled?

13. What will it cost at \$3.50 per M to shingle a roof 80 ft. long and 20 ft. wide, with shingles 4 in. wide and $4\frac{1}{2}$ in. to the weather, the first course being doubled, and the roof to project 1 ft. over all sides?

14. At 30 cts. a square yard, what will it cost to lath and plaster a hall of the following dimensions: Length 60 ft., width 40 ft., and height 24 ft., allowances to be made for 4 doors each 8x4 ft., 8 windows each 9x4 ft., and a wainscoting 4 ft. in height, reaching 1 ft. above the lower edge of the windows?

15. How many yards of carpeting are required for a room 24 ft. long and 18 ft. wide, the carpet being 3 ft. wide? 1 ft. 6 in. wide? 24 in. wide? 2 ft. 6 in. wide?

16. Find the number of yards of carpeting for the following rooms, the carpet being laid lengthwise.

1. A room 20 ft. long, 16 ft. wide, with carpet 18 in. wide.

2. A room 36 ft. long 24 ft. wide, with carpet 24 in. wide.

3. A room 12 ft. square, with carpet 2 ft. wide.

17. Find the cost of carpeting the following:

1. A room 30 ft. long, 24 ft. wide, with carpet 24 in. wide, at 75 cts. per yard.

2. A room 24 ft. long, 18 ft. wide, with carpet 1 ft. 6 in. wide, at \$1.25 per yd.
3. A room 16 ft. square with carpet 30 in. wide at \$2.50 per yd.

LAND MEASURE.

52. The **Acre** is the unit of land measure. Government Lands are generally surveyed into rectangular tracts, the boundary lines conforming to the cardinal points.

53. A **Base Line** is a starting line running east and west, from which parallel lines 6 miles apart are run.

54. A **Principal Meridian** is a starting line running north and south, from which other meridians 6 miles apart are run.

55. A **Township** is a tract of land 6 miles square, formed by the intersections of parallels and meridians. It is divided into 36 equal square parts. Townships are numbered 1, 2, 3, 4, etc., north or south of the base line.

56. A **Section** is one thirty-sixth of a township. Sections are subdivided into half-sections, quarter-sections, half-quarter-sections, quarter-quarter-sections, and lots.

57. A **Lot** is an irregular tract of land, also a descriptive term of city property.

58. A **Range** is a tier of townships extending north or south from a base line. Ranges are designated as Range 1, 2, 3, 4, etc., east or west of a principal meridian.

59. A **Correction Line** is a secondary base line used to correct the width of townships and sections that grow narrower as we go north, on account of the converging of all meridians toward a common point.

Since 1802, the United States surveyors have followed the above plan in laying out tracts of land, and all states having land surveyed by the government since that date use a very brief method of describing and locating land.

In the states whose land was surveyed previous to 1802, the description of tracts of land refer to natural objects near by; such as, a certain tree, or boulder, or pond of water, or an established road.

Forty acres of land in the regular survey would be described, as follows:

The North East quarter (N. E. $\frac{1}{4}$), of the North West quarter (N. W. $\frac{1}{4}$) of Section twenty-one (S. 21), Township thirty-five north (T. 35 N.), Range fifteen (R. 15), East of the Third Principal Meridian.

Diagram No. 1 shows a township and its division into sections, and the method of numbering same.

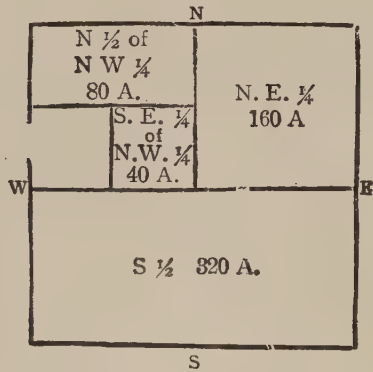
Diagram No. 2 shows the division of a section and the method of describing same.

Diagram No. 3 shows the base line and principal meridian, ranges of townships, and set off of meridian on correction line.

No. 1.
Township, 6 miles.

	N						
	6	5	4	3	2	1	
	7	8	9	10	11	12	
	18	17	16	15	14	13	
W	19	20	21	22	23	24	E
	30	29	28	27	26	25	
	31	32	33	34	35	36	
	S						

No. 2.
1 mile.



No. 3.

			6	5	4	3	2	1		
					Correc. tion	Line				
T ₅ N. R ₆ W.			7	8	9	10	11	12	5	
			13	14	15	16	17	18	4	
			19	20	21	22	23	24	3	
			25	26	27	28	29	30	2	
			31	32	33	34	35	36	1	
VIII	VII	VI	V	IV	III	II	I	T ₁ S. R ₁ W.	T ₁ S. R ₁ E.	II
					T ₂ S. R ₂ W.				2	T ₂ S. R ₂ E.

WRITTEN PRACTICE.

18. How many square rods in a field 80 rods long and 64 rods wide? 5120

19. How many acres in a field 120 rods long and 80 rods wide? 60

20. How many acres in a township of land?

21. How many acres in three half-sections of land?

22. How many acres in five half-quarter-sections of land?

23. I sold a quarter-section of land for \$6432. How much did I receive per acre?

24. A speculator sold a half-section of land at \$25 per A., a quarter-section at \$15 per A., a half-quarter-section at \$12.50 per A., a quarter-quarter-section at \$14.75 per A., and a lot containing $\frac{3}{4}$ of a quarter-quarter-section at \$8.50 per A. How much did he receive in all?

25. A man bought the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$, the W. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$, and the N. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$. How many acres of land did he buy? (Let the student make a diagram.)

26. How many acres of land in the following described property: The N. E. $\frac{1}{4}$; the S. E. $\frac{1}{4}$; the N. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$; the N. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$; and the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$: all of Section 14, Township 12 North, Range 3 East of the 3d Principal Meridian?

27. Bought 5 alternate sections of land of the Union Pacific Railway, at \$5 per acre. I sold as follows: The first section at \$6.25 per A., the second at \$4.75 per A., the third as follows: The E. $\frac{1}{2}$ at \$7.50 per A.; the N. W. $\frac{1}{4}$ and the W. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ at \$10.50 per A.; the balance of the section at \$4 per A.; the fourth section at cost. What did the land cost, and how much did I gain on the part sold?

28. The road way being 4 rods wide, what does the right of way for a railroad cost, which passes through a township at right angles, at \$125 per acre?

29. A township is separated from the adjoining townships by a road 1 chain wide, and all the sections of said township are separated by roads of the same width. Counting one-half of the boundary road, how many acres of land are in roads in the township?

VOLUME.

Applications of cubic measure. Review Article 41.

Cubic measure relates to the measuring of bodies having three dimensions—length, breadth, and height.

The solidity, or volume, is found by finding the number of cubic units in a given solid.

GIVEN.—The three dimensions to find solidity.

RULE.—*Find the product of the three dimensions expressed in units of the same denomination.*

GIVEN.—Two dimensions and the solidity.

RULE.—*Divide the solidity by the product of the two dimensions.*

WRITTEN PRACTICE.

30. How many cubic feet in a solid 8 ft. long, 6 ft. wide, and 4 ft. high?

31. What is the volume of a solid 6 ft. 3 in. long, 4 ft. 8 in. wide, and 6 ft. high?

32. How many cubic feet of earth are removed in excavating a cellar 24 ft. long, 20 ft. wide, and 8 ft. deep?

33. At 50 cents per load (cu. yd.) what will it cost to gravel a road 9 miles long, if the gravel is spread 9 ft. wide and 10 in. deep?

34. How many cubic yards of sifted gravel are required for a roof 60 ft. by 45 ft., if it is spread 1 inch thick?

35. There are 160 pupils in a room, each breathing 10 cu. ft. of air per minute. How long will it take them to breathe as much air as the room contains, if the dimensions are: Length 180 ft., width 75 ft., and height $16\frac{1}{2}$ ft.?

36. A contractor entered a bid on the following road work: The road to be 100 rods in length and 25 feet wide, the soil to be excavated to the depth of 1 foot, at a cost of $37\frac{1}{2}$ cents per cu. yd.; rubble to be laid 9 inches deep at 25 cents per cubic yard, and gravel to be laid on top 8 inches thick at 50 cents per cubic yard. Find the amount of his bid.

WOOD MEASURE.

A cord of wood is 8 feet long, 4 feet wide, and 4 feet high, and contains 128 cubic feet.

To find the number of cords, divide the number of cubic feet by 128.

WRITTEN PRACTICE.

37. How many cubic feet in a pile of wood 40 ft. long, 6 ft. high, and 4 ft. wide?
38. How many cord feet in a pile of wood 24 ft. long, 8 ft. high, and 8 ft. wide?
39. How many cords of wood in a shed 32 ft. long, 24 ft. wide, and $7\frac{1}{2}$ ft. high?
40. A pile of wood 20 ft. long, 4 ft. wide, and 8 ft. high contains how many cords of wood?
41. How many cords of wood in 6 piles, each 30 ft. long, 6 ft. 6 in. high, and 4 ft. wide?
42. At \$7.50 per cord, what is the value of a pile of wood, $16\frac{1}{2}$ ft. long, 8 ft. wide, and 6 ft. high?
43. There are 120 cords of wood in a pile 8 ft. wide and 4 ft. high. What is the length?
44. What is the value of a pile of tan-bark 80 ft. long, 48 ft. wide, and 8 ft. high, at \$5.50 per cord?
45. How long must be a pile of wood which is 8 ft. high and 8 ft. wide, to contain 480 cords?

LUMBER MEASURE.

Lumber as a term, applies to all kinds of sawed boards, planks, joists, etc.

The unit is a board foot, 12 inches long, 12 inches wide, and 1 inch thick.

Lumber less than one inch thick is counted the same as lumber an inch thick.

To find the number of feet of lumber when 1 inch, or less, in thickness.

RULE.—Multiply the length in feet by the width in inches, and divide by 12.

When more than 1 inch in thickness.

RULE.—Multiply the length in feet by the width and thickness in inches, and divide by 12.

Cancellation will shorten the work.

WRITTEN PRACTICE.

46. How many feet of lumber in a board 14 ft. long, 12 in. wide, and 1 in. thick?

SOLUTION.— $14 \times 12 \div 12 = 14$ feet of lumber.

47. How many feet in a plank 16 ft. long, 14 in. wide, and 2 in. thick?

48. How much lumber in 20 two-inch planks 18 ft. long, and 10 in. wide? (Described briefly, 20 planks $2 \times 10-18$.)

49. What will be the cost of 12 planks $2 \times 12-16$, 10 planks $3 \times 12-14$, 24 joists $6 \times 2-20$, 30 scantling $2 \times 4-18$, at \$22 per M?

50. What will it cost to enclose a quarter-section of land with a fence 5 boards high, of 1 inch material, the bottom board being 8 inches wide, the other boards each being 6 inches wide, posts to be placed 2 to the rod; the lumber costing \$15 per M, and the posts \$21 per C?

51. What will be one-half of the cost of a line fence between two adjoining sections of land, the fence to be 5 boards high, of boards 16 ft. long, 1 ft. wide, and 1 in. in thickness, the posts to be placed 8 feet apart, at a cost of \$16 per M for lumber and \$25 per C for posts?

52. At \$18 per M for lumber and \$22 per C for posts, what will be the cost of lumber and posts for enclosing a lot 40×160 ft. with a picket fence; the pickets being 4 ft. long, 3 in. wide, and 1 in. thick, allowing 3 in. space between pickets, the posts being placed 8 ft. apart, two 2×4 's being used as stringers, and a base board 10 in. wide extending below the pickets?

53. What is the difference in the cost of the following fences: The first around a regular section, the second around 640 acres of land 2 miles in length, each being fenced at a cost of 75 cents a rod?

54. The Geo. E. Spry Lumber Co. sold the following bill of lumber:

20 pc. $2 \times 6-12$, 12 pc. $4 \times 4-14$, 15 pc. $4 \times 4-16$, 30 pc. $2 \times 4-16$, 25 pc. $2 \times 6-14$, at \$15 per M; 125 pc. $2 \times 12-20$, 35 pc. $2 \times 6-22$, 50 pc. $2 \times 8-18$, 40 pc. $2 \times 12-16$, 30 pc. $2 \times 6-18$, at \$17.50 per M; 1850 posts, round, at \$23 per C; 30 pc. $6 \times 8-24$ at \$20 per M; 1650 posts, split, at \$18 per C; 120 windows 24×32 , 2 lights, at \$2.50.

per window; 80 bunches XXXX shingles at \$3.50 per M. Find the amount of the bill. (250 shingles per bunch.)

55. How many feet of plank two inches thick required for a walk 8 ft. wide, extending along the front and side of a lot 60 by 200 ft.?

56. What is the thickness of a piece of timber 50 ft. long and 12 in. wide, that contains 400 feet of lumber?

BRICK AND STONE-WORK, OR MASONRY.

Brick-work is usually estimated by the thousand brick, or in cubic feet.

Stone-work is usually estimated by the perch or cord. A perch of stone is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high. A cord of stone is usually estimated as 100 cubic feet.

In estimating material, allowance is made for openings. In estimating the work of laying a wall, outside measurement is generally taken and no allowance is made for openings.

The contract between parties usually settles the question of openings. The reason for using outside measurement and making no allowance for openings, is based on the fact of extra work in making corners and openings.

A completed stone wall has but 22 cu. ft. of stone per perch of masonry, $2\frac{3}{4}$ cubic ft. being allowed for mortar.

The following is a table in general use by contractors in estimating the number of bricks per square foot in a wall, one-eighth being allowed for mortar:

NAME.	SIZE.	Wall 1 brick in thickness.	Wall 2 bricks in thickness.	Wall 3 bricks in thickness.	Wall 4 bricks in thickness.
Common	8x4x2	8	16	24	32
Milwaukee	$8\frac{1}{2} \times 4\frac{1}{8} \times 2\frac{3}{8}$	$6\frac{1}{4}$	$12\frac{3}{4}$	19	$25\frac{1}{4}$
North River	$8 \times 3\frac{1}{2} \times 2\frac{1}{4}$	$7\frac{1}{2}$	$14\frac{1}{2}$	$21\frac{1}{2}$	$28\frac{1}{2}$
Maine	$7\frac{1}{2} \times 3\frac{3}{8} \times 2\frac{3}{8}$	7.2	14.4	21.6	28.8
Philadelphia	$8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$	6.5	13.	19.5	26.

WRITTEN PRACTICE.

How many bricks, common size, required for a wall 50 ft. long, 20 ft. high, and 3 bricks thick?

SOLUTION:

$$50 \times 20 = 1000 \text{ sq. ft. surface.}$$

$$1000 \times 24 = 24000 \text{ bricks.}$$

57. How many common bricks are required for a wall 20 ft. long, 10 ft. high, and 2 bricks in thickness?

58. How many North River bricks would be required for the same wall?

59. How many bricks required for the front of a building 25 ft. wide, 40 ft. high, and 3 bricks in thickness? The facing to be of Milwaukee brick, and common brick to be used for the remainder, allowance being made for 12 openings 8x4 feet.

60. How many cu. ft. of stone in a wall 40 ft. long, 8 ft. high, and 3 ft. thick? No allowance being made for mortar.

61. What will be the cost of a pile of stone 45 ft. long, 12 ft. wide, and 6 ft. high, at \$7.50 per cord?

62. How many perches of stone in a wall 75 ft. long, 8 ft. high, and 3 ft. thick?

63. How many cords of stone in a cellar wall, the outside dimensions of which are: Length 30 ft., width 20 ft., and height of wall 8 ft., the wall being 2 feet thick?

64. How much will it cost to lay the stone at \$1.25 per perch?

65. What will it cost to lay the wall for a cellar 60 ft. long, 30 ft. wide, 8 ft. high, and $1\frac{1}{2}$ ft. in thickness, at \$1.25 a perch?

66. What will it cost for the material for the above wall at \$5.25 per perch?

67. A contractor bid in the following work: To excavate a cellar 36x24x6 ft. at 30 cts. per cubic yard; to erect the wall to project 2 feet above ground, $1\frac{1}{2}$ ft. thick, at \$1.50 per perch; and to furnish the stone and lime at \$5.25 per perch. What was the amount of his bid?

CAPACITY.

Review Articles 42 and 43.

The Standard Bushel, also called stricken measure, contains 2150.4 cubic inches.

It is used for measuring wheat, oats, shelled corn, and other grains.

The Heaped Bushel contains 2747.7 cubic inches.

It is used for measuring vegetables, fruits, roots, etc.

To find the number of bushels in a bin, divide the number of cubic inches by 2150.4 for stricken measure, or by 2747.7 for heaped measure.

To find the number of gallons in a tank, divide the number of cubic inches by 231.

For approximate results:

Stricken measure, multiply the number of cubic feet by .8, to find the number of bushels.

Heaped measure, multiply the number of cubic feet by .63, to find the number of bushels.

Liquid measure, multiply the number of cubic feet by $7\frac{1}{4}$, to find the number of gallons.

WRITTEN PRACTICE.

68. Find the number of bushels in a bin having a capacity of 137625.6 cu. in.?

69. How many bushels of oats in a bin 20 ft. long, 8 ft wide, and 6 ft. deep?

70. A bin 20 ft. long, 6 ft. wide, and 4 ft. deep, will hold how many bushels of potatoes?

71. What is the weight of the wheat that will fill a wagon-bed, which is 12 ft. long, 3 ft. 8 in. wide, and 2 ft. 6 in. deep?

72. What is the depth of a bin that will hold 250 bushels of apples, if the bin is 20 ft. long and 4 ft. wide?

73. How many gallons will it take to fill a tank having a capacity of 30954 cu. in.?

74. Find the number of gallons in a tank of kerosene 12 ft. long, 4 ft. wide, and 3 ft. deep?

75. How many gallons of vinegar will a tank hold that is 8 ft. long, 4 ft. wide, and 2 ft. 6 in. deep?

76. What is the depth of a tank, 4 ft. square that will hold 8 barrels of water?

77. How many hogsheads of water will a cistern hold, that is 8 ft. square and 12 ft. deep?

78. A box that holds 25 bushels of wheat, will hold how many gallons?

79. A tank that will hold 75 bushels of barley, will hold how many gallons of water?

80. A bin that will hold 550 gallons of water, will hold how many bushels of rye?

81. A box that will hold 25 bushels of apples, will hold how many gallons of water?

82. A bin having a capacity of 420 bushels of potatoes, would be what part full, if it contained 420 bushels of oats?

83. A trader bought 64 quarts of berries by wine measure and sold them by dry measure. How much did he lose, his buying and selling price being 5 cts. per quart?

84. A dealer bought 10 bushels of cherries at \$2.50 per bushel. He sold one-half of them at 10 cts. per quart dry measure, and the remainder at 10 cts. per quart liquid measure. How much did he gain?

TIME.

Find the difference between the following dates by compound subtraction:

Apr. 4, 1892, to Jan. 24, 1896.

SOLUTION.

1896-1-24

1892-4-4

3-9-20

EXPLANATION.—Write Jan. 24, 1896, as 1896 yr. 1 mo. 24 da., and Apr. 4, 1892, as 1892 yr. 4 mo. 4 da., and subtract as in compound numbers.

WRITTEN PRACTICE.

Find the difference between the following dates by compound subtraction:

85. Mar. 1, 1891, and Aug. 4, 1896.

86. Apr. 1, 1892, and July 20, 1895.

87. May 4, 1892, and June 24, 1895.

88. June 19, 1893, and Aug. 31, 1896.

89. Aug. 21, 1894, and Nov. 24, 1896.

90. Mar. 1, 1890, and Jan. 4, 1894.
 91. Apr. 4, 1890, and Feb. 20, 1894.
 92. July 15, 1890, and Mar. 24, 1895.
 93. Aug. 20, 1891, and July 6, 1896.
 94. Sept. 14, 1891, and Mar. 7, 1896.
 95. What is the length of time from 20 min. past 9 o'clock, A. M. Aug. 4, 1895, to 36 min. past 11 o'clock, A. M. Nov. 24, 1897?
 96. Find the length of time from 26 min. before 10 o'clock, A. M., Oct. 15, 1893, to 26 min. past 9 o'clock, P. M., July 4, 1896.
 97. What is the length of time from 25 min. before 10 o'clock P. M. July 13, 1893, to 8 min. past 8 o'clock, A. M. Sept. 24, 1897?

Find the exact time between the following dates:

98. From Apr. 16, 1893 to Nov. 13, 1893.

SOLUTION.

14 da. left in Apr.

31 in May.

30 in June.

31 in July.

31 in Aug.

30 in Sept.

31 in Oct.

13 in Nov.

211 days.

99. From Mar. 16, 1892, to Nov. 12, 1892.
 100. From Apr. 3, 1892, to Jan. 16, 1893.
 101. From June 12, 1893, to Mar. 1, 1894.
 102. From Aug. 23, 1893, to Apr. 12, 1894

LONGITUDE AND TIME.

60. The **Longitude** of a place is the distance east or west of a given meridian.

61. A **Meridian** is an imaginary line passing from north to south through any place.

It is noon at any place when the sun is directly over the meridian of that place. The meridian is from the Latin word *meridies*, meaning midday. The abbreviations, **A. M.** are for the Latin words *ante meridiem*, meaning before midday; and **P. M.** for *post meridiem*, meaning after midday.

Since the earth revolves on its axis from west to east once in 24 hours, the sun appears to pass from east to west around the earth once in 24 hours. The circle of the earth's circumference consists of 360 degrees of longitude; hence, the sun appears to us to pass through 360 degrees of longitude in 24 hours, or through $\frac{1}{24}$ of 360 degrees in one hour. $\frac{1}{24}$ of 360 is 15.

LONGITUDE AND TIME COMPARED.

15° of longitude make 1 hr. difference in time.

1° of " makes 4 min. " "

15' of " make 1 min. " "

1' of " makes 4 sec. " "

15" of " make 1 sec. " "

The royal observatory at Greenwich, England, is the point from which English and Americans usually reckon longitude.

Since the sun appears to move from east to west, it will be over the meridian (noon time) of your place 1 hour sooner than at a point 15° west of you, and it will be 1 hour past noon, or 1 o'clock **P. M.**, at a point 15° east of you.

MENTAL PRACTICE.

1. If it is noon now, what is the time 15°E. of here? 30°? 45°? 60°? 75°? 7½°? 22½°? 30'? 45'?

2. If it is noon now, what is the time 15°W. of here? 30°? 45°? 60°? 90°? 120°? 180°?

3. Is your watch too slow or too fast when you go east? When you go west?

4. When it is 1 o'clock A. M. in Chicago, what is the time 15° E. of Chicago? 30° E.? 45° E.? 15° W.? 30° W.? 45° W.? 45' W.?

5. I travel E. 30°, and on looking at my watch find it to be 3 o'clock P. M. What correction should I make?

6. I travel W. 45°, and on looking at my watch find it to be 6 o'clock A. M. What is the correct time?

TABLE OF LONGITUDES.

Boston.....	71° 3' 30" W.
Bombay.....	72° 54' .." E.
Berlin.....	13° 23' 45" E.
Calcutta.....	88° 19' 2" E.
Cincinnati.....	84° 29' 31" W.
Chicago.....	87° 37' 45" W.
New York.....	74° 3' .." W.
San Francisco....	122° 26' 45" W.
St. Louis.....	90° 15' 15" W.
Washington.....	77° ..' 15" W.

EXAMPLES.

1. What is the difference of longitude between Boston and Chicago?

87° 37' 45" W. longitude of Chicago.

71° 3' 30" W. longitude of Boston.

16° 34' 15" difference of longitude.

EXPLANATION.—Since both places are west of Greenwich, their difference of longitude will be found by subtraction, as in compound numbers.

2. What is the difference of longitude between Calcutta and San Francisco?

88° 19' 2" E. longitude of Calcutta.

122° 26' 45" W. longitude of San Francisco.

210° 45' 47" difference of longitude.

360° 00' 00"

210° 45' 47"

149° 14' 13" actual difference.

EXPLANATION.—Since one place is E. and the other is W., their difference will be found by adding as in compound numbers. Since this sum is greater than 180°, the number of degrees in a half circle, it is evidently shorter in the other direction, consequently subtract the sum from 360°.

WRITTEN PRACTICE.

Find the difference of longitude between

103. Washington and Bombay.
104. Boston and Berlin.
105. Chicago and Calcutta.
106. Cincinnati and St. Louis.
107. New York and San Francisco.
108. San Francisco and Bombay.

What is the difference in time between Chicago and Washington?

SOLUTION.

$$\begin{array}{r}
 87^{\circ} 37' 45'' \text{ W.} \\
 77 \quad 0 \quad 15 \text{ W.} \\
 15 \overline{) 10^{\circ} 37' 30''} \\
 \underline{42 \text{ min. } 30 \text{ sec.}}
 \end{array}$$

EXPLANATION.—Since both are in W. longitude, the distance between points is found by subtracting. Since 15° of longitude make 1 hour difference in time, divide the difference by 15 to find the difference in time.

Find the difference in time between

109. Washington and Greenwich.
110. Chicago and San Francisco.
111. New York and St. Louis.
112. Boston and Bombay.
113. Chicago and Calcutta.
114. Berlin and Cincinnati.

When it is midnight at the first named, what is the time at the second named of the following places:

115. Boston and Calcutta?
116. Bombay and San Francisco?
117. Berlin and Cincinnati?
118. Calcutta and Washington?

When it is noon at the first place, what is the time at the second of the following:

119. Chicago and San Francisco?
120. New York and St. Louis?
121. Berlin and Washington?
122. New York and San Francisco?

INVOLUTION.

62. Involution is the process of finding the required power of a number.

63. A Power of a number is the product obtained by using it a number of times as a factor.

64. The Square of a number is the second power.

65. The Cube of a number is the third power.

66. The Exponent is a figure written to the right of the factor and a little above, indicating the number of times the factor is to be used.

WRITTEN PRACTICE.

1. What is the square or second power of 25?

SOLUTION.— $25 \times 25 = 625$.

2. What is the third power of 75? 85? 95? 65? 55?

3. What is the fourth power of 4? 8? 16? 15?

4. What is the third power of $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$?

5. Expand the following: 5^3 , 6^2 , 8^4 , 10^5 , and 25^3 .

6. Find the required power of the following: $(\frac{7}{8})^3$, $(\frac{4}{5})^4$, $(2\frac{1}{2})^3$, $(14\frac{2}{7})^2$, $(5.06\frac{1}{4})^3$.

7. Find the difference between 25^3 and 40^2 .

EVOLUTION.

67. Evolution, the reverse of involution, is the process of finding the repeated factor.

The Root of a number is one of the equal factors.

68. The Square Root of a number is one of the two equal factors.

69. The Cube Root of a number is one of the three equal factors.

EXAMPLE.

What is the largest square that can be formed of 15625 sq in.?

SOLUTION BY ANALYSIS.

$$\begin{array}{r|l}
 & 5 \\
 1'56'25 & 20 \\
 \hline
 1\ 00\ 00 & 100 \\
 \hline
 200 & 56\ 25\ 125 \\
 220 & 44\ 00 \\
 \hline
 240 & 12\ 25 \\
 245 & 12\ 25
 \end{array}$$

SOLUTION BY RULE.

$$\begin{array}{r}
 1'56'25\ 125 \\
 \underline{1} \\
 20\ 56 \\
 \underline{22\ 44} \\
 240\ 1225 \\
 \underline{245\ 1225}
 \end{array}$$

EXPLANATIONS.

The square of any number having but units place will not occupy more than two places.

The square of any number having units and tens will not occupy more than four places, etc.

Therefore, point off the number into periods of two figures each, beginning at the right.

The number of periods shows the number of places in the root.

15625 is the square; 125 is the root; 20 or 200 is the first trial divisor; 22 or 220 is the first complete divisor, etc.

Fig. 1. Shows the first assumed square.

Fig. 2. Shows the two additions, the additions that produce the trial divisor.

Fig. 3. Shows the first complete divisor.

Fig. 4. Shows the second complete square, or a square containing 15625 sq. in.

RULE—I. Separate the number into periods of two figures each, beginning at the right.

II. Find the greatest square in the left hand period, and place its root on the right. Subtract this square from the period, and to the remainder annex the next period for a new dividend.

III. Double the root figure already found, add a cipher, and write it at the left for a trial divisor; find how many times it is contained in the dividend, writing the trial figure at the right and adding it to the trial divisor for a complete divisor.

IV. Multiply the complete divisor by the trial figure in the root, subtract and bring down the next period, with which proceed as before.

NOTE—1. If the number is not a perfect square, periods of ciphers may be annexed and the root continued as a decimal.

2. If the trial divisor is not contained in the dividend, the next period must be brought down, and ciphers added to both the divisor and the root.

3. To find the square root of decimals, there must be an even number of places, if the decimal has an odd number a cipher must be added.

4. To find the square root of fractions, find the root of the numerator for a new numerator, and of the denominator for a new denominator, or find the root decimally.

MENTAL PRACTICE.

Find the square root of the following:

1. 9	5. 36	9. 225	13. $\frac{1}{4}$
2. 16	6. 49	10. 625	14. $\frac{9}{16}$
3. 25	7. 64	11. 169	15. $\frac{25}{49}$
4. 81	8. 144	12. 256	16. $\frac{36}{81}$

WRITTEN PRACTICE.

Find the square root of the following:

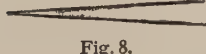
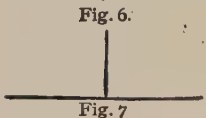
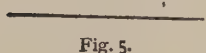
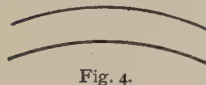
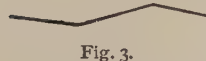
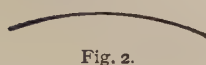
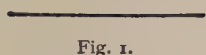
1. 1225	5. 5625	9. 53361	13. $\frac{225}{441}$
2. 2025	6. 7225	10. 17424	14. $\frac{169}{289}$
3. 3025	7. 15129	11. 97344	15. .1225
4. 4225	8. 103041	12. 45369	16. .007225

MENSURATION.

70. Mensuration is the branch of mathematics that has to do with finding the length of lines, the area of surfaces, and the volume of solids.

71. A Point has position but no dimension. A point in motion produces a line, dimension—length; a line in motion laterally produces a surface, dimensions—length and breadth; a surface in motion laterally produces volume, dimensions—length, breadth, and height.

LINES.



72. A Straight Line is a line that does not change its direction. (Fig. 1.)

73. A Curved Line is a line that continuously changes its direction. (Fig. 2.)

74. A Broken Line is a line formed of two or more straight lines. (Fig. 3.)

75. Parallel Lines are lines that continue the same distance apart. (Fig. 4.)

76. A Horizontal Line is a line that runs parallel with the water level. (Fig. 5.)

77. A Perpendicular Line is a line that joins or crosses another line so that the angle on one side equals the angle on the other. (Fig. 6.)

78. A Vertical Line is a line that is perpendicular to a horizontal line. (Fig. 7.)

ANGLES.

79. An Angle is the divergence of two lines that meet. (Fig. 8.)

80. A Right Angle is an angle formed by the meeting of perpendicular lines. It equals 90° . (Fig. 9.)

81. An Acute (sharp) Angle is less than a right angle, or less than 90° . (Fig. 10 B.)

82. An Obtuse (blunt) Angle is more than a right angle, or more than 90° . (Fig. 10 A.)

SURFACE.

83. Surface is that which has two dimensions—length and breadth.

84. A Plane Figure is a plane surface bounded by straight or curved lines.

85. The Area of a plane figure is the surface included within the boundary lines.

86. A Polygon is a plane figure bounded by straight lines. Three straight lines, at least, are necessary.

87. A Regular Polygon is one in which all the sides and angles are equal.

88. The Perimeter is the broken line which forms the boundary of a polygon.

TRIANGLES.

89. A Triangle is a polygon bounded by three sides.

With reference to sides they are:

90. Equilateral, when the three sides are equal. (Fig. 11.)

91. Isosceles, when two sides only are equal. (Fig. 12.)

92. Scalene, when the three sides are unequal. (Fig. 13.)

With reference to angles they are:

93. Right-Angled, when the triangle has one right angle. (Fig. 14.)

94. Obtuse-Angled, when the triangle has one obtuse angle. (Figs. 13 and 15.)

95. Acute-Angled, when the triangle has three acute angles. (Fig. 11.)

96. The Altitude of a triangle is the perpendicular distance from the base, or the extended base, to the vertex of the angle opposite.

The altitude is represented by the dotted lines in Figs. 11, 12, 13, and 15.

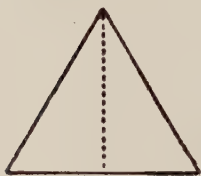


Fig. 11.

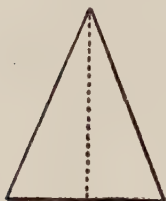


Fig. 12.

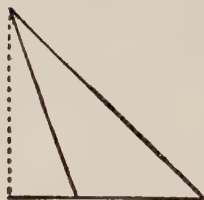


Fig. 13.

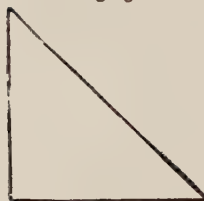


Fig. 14.



Fig. 15.

QUADRILATERALS.

97. A **Quadrilateral** is a polygon having four sides.



Fig. 16.

The Quadrilateral is called:

98. A **Trapezium**, when the sides are unequal. (Fig. 16.)

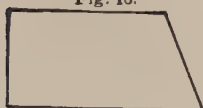


Fig. 17.

99. A **Trapezoid**, when two sides are parallel. (Fig. 17.)

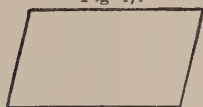


Fig. 18.

100. A **Parallelogram**, when the opposite sides are parallel. (Fig. 18.)

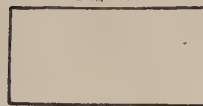


Fig. 19.

101. A **Rectangle**, when the angles are equal. (Fig. 19.)

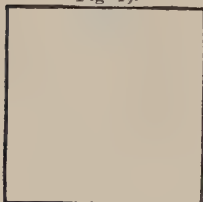


Fig. 20.

102. A **Square**, when the sides and angles are equal. (Fig. 20.)

Names of Polygons having more than four sides: Pentagon, Hexagon, Heptagon, Octagon, Nonagon, Decagon, Undecagon, Duodecagon, etc.

CIRCLES.

103. A **Circle** is a plane figure bounded by a curved line, called the circumference, everywhere equally distant from a point within called the center. (Fig. 21.)



Fig. 21.

104. A **Diameter** of a circle is a straight line passing through the center and terminating at the circumference.

105. A **Radius** of a circle is a straight line passing from the center to the circumference; one-half a diameter.

VOLUME.

106. **Volume**, or **Solidity**, is that which has three dimensions—length, breadth, and height.

PRISMS.

Fig. 22.



Fig. 23.

107. A **Prism** is a solid whose bases or ends are any similar, equal, and parallel plane figures, and whose lateral faces are parallelograms. Prisms are named, from the form of the base, triangular, quadrangular, pentagonal, hexagonal, etc. (Figs. 22 and 23.)

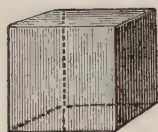


Fig. 24.

108. A **Parallelopipedon** is a prism bounded by six parallelograms. (Fig. 22.)

109. A **Cube** is a parallelopipedon having equal faces. (Fig. 24.)



Fig. 25.

110. A **Cylinder** is a solid whose sides are bounded by a uniformly curved surface, and whose ends are equal parallel circles. (Fig. 25.)

A Cylinder may be generated by the revolution of a rectangle about an axis that bisects parallel sides.

PYRAMIDS AND CONES.

Fig. 26.

111. A **Pyramid** is a solid having a polygon for a base, and whose faces are triangles, meeting at a common point called the vertex. (Fig. 26.)

112. A **Cone** is a solid that tapers uniformly from a circular base to a point. (Fig. 27.)

A Cone may be generated by the revolution of a right-angled triangle upon its altitude or base.



Fig. 27.

113. The **Altitude** of a pyramid or cone is the perpendicular distance from the vertex to the plane of its base.

114. The **Slant height** of a pyramid is a straight line from the vertex meeting the base of one of the faces at right angles.

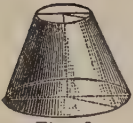


Fig. 28.

115. The **Slant height** of a cone is a straight line from the vertex to the circumference of the base.



Fig. 29.

116. The **Frustum** of a pyramid or cone is that part which remains, after cutting off the top by a plane parallel to the base. (Figs. 28 and 29.)

THE GLOBE OR SPHERE.

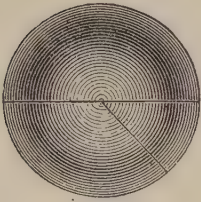


Fig. 30.

117. A **Sphere** is a solid, every part of whose surface is equidistant from a point within called the center. (Fig. 30.)

It is generated by the revolution of a circle upon a diameter as an axis.

118. A **Diameter** of a sphere is a straight line passing through the center and terminating at the surface.

119. A **Radius** of a sphere is a straight line passing from the center to the surface. One-half of a diameter.

120. To find the area of a quadrilateral, square, or rectangle.

RULE.—*Multiply the length by the width.* (Figs. 19 and 20.)

121. To find the area of a parallelogram.

RULE.—*Multiply the length by the altitude.* (Fig. 18.)

122. To find the area of a trapezoid.

RULE.—*Multiply one-half of the sum of the parallel sides by the altitude.* (Fig. 17.)

WRITTEN PRACTICE.

1. How many acres of land in a rectangular field 40 chains long, and 24 chains wide?

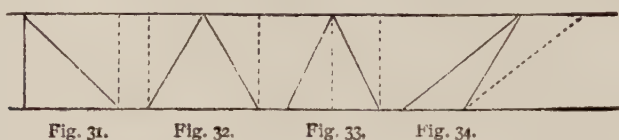
2. The base of a parallelogram is 20 ft. 8 in., and its altitude 12 ft. 6 in. What is the area?

3. What is the area of a trapezoid, the parallel sides being 40 rods and 30 rods respectively, the perpendicular distance between the sides being 24 rods?

4. A rectangular field containing 40 acres is 200 rods long. What is its width?

5. A township map is drawn on a scale of 3 inches to the mile. The map is $4\frac{1}{2}$ feet long, and $2\frac{1}{4}$ feet wide. How many acres are there in the township?

Figures showing the relation of the triangle to the parallelogram.



123. PRINCIPLE.—The area of a triangle is one-half of the area of a parallelogram having the same base and altitude.

124. GIVEN.—Base and altitude, to find area.

RULE.—*One-half of the product of the base and altitude equals the area.*

Conversely.—Twice the area divided by one of the dimensions equals the other.

WRITTEN PRACTICE.

Find the area of the following triangles:

6. Base 20 feet, altitude 24 feet.
7. Base 36 rods, altitude 48 feet.
8. Isosceles; base 27 feet, altitude 25 feet. (Fig. 33)
9. Right angled; base 36 rods, perpendicular 80 rods (Fig. 31.)

Find the factor of the following triangles:

10. Area 300 sq. ft., base 40 ft.
11. Area 125 sq. rd., altitude 40 rods.
12. Area 120 A., base 60 rods.

13. What are the dimensions of an isosceles triangle containing 8 acres, whose base is the side of a square containing 10 acres?

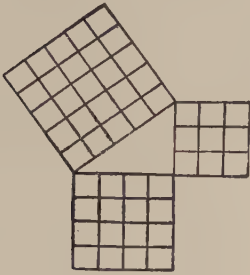


Fig. 35.

PRINCIPLES.—1. The square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. The square of either of the smaller sides is equal to the square of the hypotenuse less the square of the other side.

NOTE.—The longest side of a right-angled triangle is called the hypotenuse, the side upon which it rests the base, and the upright side the perpendicular.

125. GIVEN.—Base and perpendicular, to find hypotenuse.

RULE.—*Extract the square root of the sum of the squares of the base and perpendicular.*

CONVERSELY.—*Extract the square root of the difference between the squares of the hypotenuse and given side, to find the other side.*

WRITTEN PRACTICE.

Find the hypotenuse of the following:

14. Base 30 feet, perpendicular 40 feet.

15. Base 25 feet, perpendicular 75 feet.

Find the base of the following:

16. Perpendicular 40 feet, hypotenuse 50 feet.

17. Perpendicular 25 rods, hypotenuse 60 rods.

Find the perpendicular of the following:

18. Base 20 rods, hypotenuse 45 rods.

19. Base 75 rods, hypotenuse 120 rods.

20. A farm is in the form of a right-angled triangle, whose base is 120 rods and the hypotenuse is 200 rods. How many acres of land in the farm?

21. A footpath goes along two sides of a rectangle, whose length is 200 rods, and its width 150 rods. What is the distance saved in going diagonally across the field?

22. The distance between two posts is 16 feet, the height of

one post is 8 feet, and the height of the other is 20 feet. What is the length of a beam that will reach from the top of one post to the top of the other?

23. A rides east at the rate of 15 miles per hour; B rides south at the rate of 18 miles an hour. How far apart are they at the end of 45 minutes?

126. GIVEN—The three sides of a triangle, to find the area.

RULE.—From half the sum of the three sides subtract each side separately, and extract the square root of the product of the half-sum and the three remainders.

WRITTEN PRACTICE.

24. Find the area of a scalene triangle whose sides are 40, 50, and 60 ft.

SOLUTION.

$$(40+50+60) \div 2 = 75; \quad 75-40=35; \\ 75-50=25; \quad 75-60=15. \quad \sqrt{75 \times 35 \times 25 \times 15} = 992 + \text{sq. ft.}$$

25. What is the area of an isosceles triangle whose sides are 50, 50, and 20 rods?

26. The dimensions of a piece of land are: 30, 60, and 75 rods. What is the value of the land at \$75 per acre?

27. What is the area of an equilateral triangle, the sides of which are 60 feet respectively?

28. A quadrilateral field has the following dimensions: 40, 30, 35, and 25 rods respectively. The angle formed by the first two sides is a right angle. What is the area of the figure?

127. GIVEN—Perimeter of irregular surfaces, to find area

RULE.—Divide the figures into triangles and find the area of each triangle.

WRITTEN PRACTICE.

29. What is the area of the trapezium, as per Fig. 36?

30. What is the area of the pentagon, as per Fig. 37?

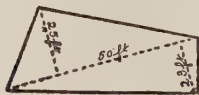


Fig. 36.



Fig. 37.

Figures showing relation of diameter to circumference.

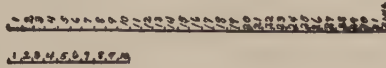


Fig. 38.



Fig. 39.

PRINCIPLE.—The ratio of the diameter of a circle to the circumference is 1 to 3.1416.

128. GIVEN.— { Circumference, to find diameter.
Diameter, to find circumference.

RULE.—*Divide circumference by 3.1416. Conversely.—Multiply the diameter by 3.1416 to find the circumference.*

WRITTEN PRACTICE.

31. What is the diameter of a circle whose circumference is 100 feet?

32. What is the diameter of a circle whose circumference is 180 rods?

33. What is the diameter of a race track whose circumference is 1 mile?

34. What is the circumference of a circle whose diameter is 100 feet?

35. What is the circumference of a tree whose diameter is 6 feet?

36. What is the circumference of a circular pond whose radius is 25 rods?

37. What is the greatest circumference that can be struck with a string 20 feet long?

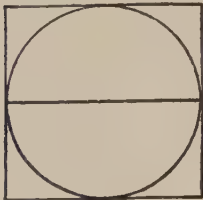


Fig. 40.

PRINCIPLE.—The circle occupies .7854 of the area of the circumscribing square. The diameter of the circle equals one of the sides of the square.

129. GIVEN.—Diameter, or diameter and circumference, to find the area.

RULE.—*I. Multiply the square of the diameter by .7854*

II. Take one-fourth of the product of the diameter and circumference.

WRITTEN PRACTICE.

38. What is the area of a circle whose diameter is 10 feet?

SOLUTION.— $10^2 \times .7854 = 78.54$, the area.

39. What is the area of a circular grass plat whose diameter is 36 feet?

40. What is the area of a circle whose radius is 3 yards?

41. What is the area of a circle whose circumference is 78.54 yards and its diameter 25 yards?

42. What is the area of a circular park whose diameter is 75 ft., and its circumference 235.62 ft?

43. What is the area of a circle whose circumference is 250 feet?

SUGGESTION.—Find the diameter first.

44. A goat is tethered to a post by a rope 40 feet long. Over what part of an acre can he graze?

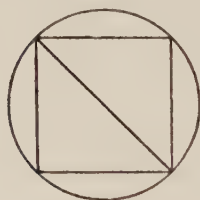


Fig. 41.

PRINCIPLE.—The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

130. GIVEN.—Diameter or circumference, to find inscribed square.

RULE.—*Extract the square root of one-half of the square of the diameter, to find one side of the square.*

The diameter is the hypotenuse of a right-angled triangle.

WRITTEN PRACTICE.

45. If the diameter of a circle is 25 feet, what is the side of the inscribed square?

SOLUTION.— $25^2 \div 2 = 312.5$; $\sqrt{312.5} = ?$

46. Find the side of the inscribed square, the radius being 60 feet.

47. Find the side of the inscribed square, the circumference being 600 feet.

48. The area of an inscribed square is 5 acres. What is the area of a square circumscribing the same circle?

Figures showing that the lateral surface of a prism or cylinder is rectangular when extended:



Fig. 42.

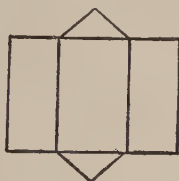


Fig. 43.



Fig. 44.

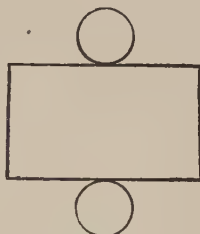


Fig. 45.

PRINCIPLE.—The perimeter of a prism or cylinder compares with the length, and the height or altitude with the width of a rectangle.

131. GIVEN.—Dimensions, to find lateral surface of prism or cylinder.

RULE.—*Multiply the perimeter of the base by the altitude. Add the surface of the ends to find the total surface.*

WRITTEN PRACTICE.

49. What is the lateral surface of a rectangular prism whose altitude is 30 feet and its base 4 feet by 3 feet?

SOLUTION.— $(4+3) \times 2 \times 30 = 420$ sq. ft.

50. What is the lateral surface of a cylinder 40 feet long and 15 feet in circumference? What is the total surface?

51. What is the lateral surface of a triangular prism whose altitude is 24 feet and the edges of its base are 6, 8, and 10 feet respectively? What is the total surface?

52. What is the total surface of a cylindrical pile 40 feet long and 1 foot in diameter?

53. What is the lateral surface of a duodecagonal prism 20 feet long and whose edges are each 6 inches?

132. GIVEN.—Perimeter and altitude, to find solidity of prism or cylinder.

RULE.—*Multiply the area of the base by the altitude.*

WRITTEN PRACTICE.

54. What is the solidity of a 6 foot cube?

55. What is the solidity of a rectangular prism 16 feet long, and whose edges are each 3 feet?

56. What is the solidity of a triangular prism 12 feet long, and whose edges are each 4 feet?

57. What is the solidity of a cylinder 20 feet long and 8 feet in diameter?

58. What is the capacity in gallons of a cistern 10 feet deep, and 314.16 inches in circumference?

Figures showing the relation of surface of cone and pyramid to triangle:



Fig. 46.



Fig. 47.



Fig. 48.



Fig. 49.



Fig. 50.

PRINCIPLE.—The area of a triangle is one-half the product of the base by the altitude.

133. GIVEN.—Dimensions to find lateral surface of cone or pyramid.

RULE.—*Multiply the perimeter by one-half of the slant height.*

WRITTEN PRACTICE.

59. What is the lateral surface of a cone whose slant height is 25 feet, and its circumference 30 feet? What is its total surface?

60. What is the lateral surface of a rectangular pyramid whose slant height is 18 feet, and its base a foot square? What is its total surface?

61. What is the cost at 25 cents per square yard to paint a church steeple that is in the form of a heptagonal pyramid, whose slant height is 100 feet, and each face at the base is 12 feet wide?

62. What is the total surface of a cone that is 25 feet high, and 10 feet in diameter?

SUGGESTION.—Find slant height first.

Figures showing the relation of solidity of the cone to the cylinder, each having the same base and altitude.

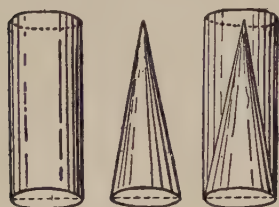


Fig. 51. Fig. 52. Fig. 53.

PRINCIPLE.—The solidity of a pyramid or cone is one-third of the solidity of a prism or cylinder having the same base and altitude.

134. GIVEN—Dimensions, to find the solidity of cone or pyramid.

RULE.—*Multiply area of the base by one-third of the altitude.*

WRITTEN PRACTICE.

63. What is the volume or solidity of a cone 16 feet high, and 4 feet in diameter?

64. What is the solidity of a rectangular pyramid 14 feet high, and its base being 6 feet on each edge?

65. What is the capacity in bushels of a bin in the form of an inverted cone 8 feet in depth, and 4 feet in diameter?

Figure showing the relation of a sphere to a pyramid.



Fig. 54.

PRINCIPLE.—The sphere is made up of pyramids, the base of the pyramids being the surface, and the vertex the center.

135. GIVEN.—Dimensions, to find the surface.

RULE.—*Multiply the circumference by the diameter.*

WRITTEN PRACTICE.

66. What is the surface of a sphere 5 inches in diameter?
 67. What will it cost to bronze a globe 6 feet in circumference, at 2 cents a square inch?

136. GIVEN.—Dimensions, to find solidity.

RULE.—*Multiply the surface by one-sixth of the diameter, or
 Multiply the cube of the diameter by .5236.*

WRITTEN PRACTICE.

68. How many cubic feet in a globe 12 feet in diameter?
 69. How many cubic feet in a globe 12 feet in circumference?
 70. What is the capacity in gallons of a hollow globe 10 feet in diameter?
 71. A tank in the form of a hemisphere 20 feet in diameter will hold how many bushels of oats?

PROMISCUOUS WRITTEN PRACTICE.

72. What is the distance from one lower corner to the opposite lower corner of a room, that is 24 feet long, and 16 feet wide?
 73. The base of a right-angled triangle is 75 feet, the perpendicular is 150 feet. What is the hypotenuse?
 74. Find the cost of a triangular piece of land at \$125 per acre, the base being 20 rods, and the altitude being $33\frac{1}{2}$ rods
 75. What will it cost to paint the gable ends of a barn at 20 cents per sq. yd., the width of the barn being 40 ft., and the altitude of the gable being 25 ft.?
 76. A 50 ft. ladder is placed in an alley so that, without moving the foot of it, the top will reach a window on one side that is 30 ft. from the ground, and will reach one on the other side 25 ft. from the ground. What is the width of the alley?
 77. An army contains 105,625 men; the general wishes it massed into a square. How many men in each file?
 78. What is the length of the thread of a screw that passes around a cylinder twice to each foot, the cylinder being 10 feet long, and 10 inches in diameter?

79. A tree which is 75 feet high and 2 feet in diameter, has a hole in it 50 feet from the ground; a squirrel in reaching the hole goes by the most direct line from the base, but goes around the tree once. How many feet does he go?

80. What is the distance from a lower corner to the opposite upper corner of a room of the following dimensions: Length 24 feet, width 15 feet, height 12 feet?

81. What is the length of a path that goes diagonally across a section of land?

82. What is the shortest distance a fly will travel in crawling from one lower to the opposite upper corner of a room that is 30 feet long, 25 feet wide, and 16 feet high?

83. What will it cost to bronze a sphere that is 50 inches in diameter, at 3 cents per sq. in.?

84. A sphere 3 feet in diameter is converted into a 6 inch cylinder. What is the length?

85. How many board feet in the largest square sill that can be sawed from a log 16 feet long, and 1 foot in diameter?

86. A triangle, a circle, and a square each contain the same area. The dimensions of the triangle are: Altitude 75 feet, base 50 feet. What are the dimensions of each of the other figures?

87. Three men buy a grindstone, the radius of which is 30 inches. How much must each grind off to obtain $\frac{1}{3}$ of the stone, if no allowance is made for the aperture?

88. A trapezoid contains 4 acres; its parallel sides are 15 rods, and 25 rods. What is the perpendicular distance between the sides?

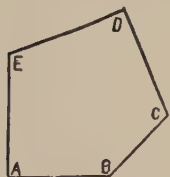


Fig. 55.

89. Figure 55 represents an irregular piece of land. The measurements are: $AB=30$ rods; $BC=28$ rods; $CD=40$ rods; $DE=50$ rods; and $EA=40$ rods. The angles at A and at D are right angles. How many acres in the field?

90. If it requires 12 turns of the handle to raise a bucket of water from the bottom of a well, what is the depth, if the axle on which the chain is wound is 15 inches in diameter?

91. What is the solidity of a figure generated by revolving a 6 ft. square upon one of its diagonals?

92. What is the solidity of a figure generated by revolving a door 8 ft. by 4 ft., upon its hinges? By revolving it upon a line bisecting the ends?

93. What is the solidity of a figure generated by revolving an 8 foot circle upon one of its diameters?

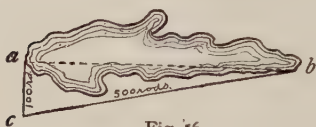


Fig. 56.

94. Wishing to find the length (Fig. 56) of a certain lake, I measured from a line at right angles to A B to C, a distance of 100 rods, and found the distance from C to B to be 500 rods. What is the length of the lake?

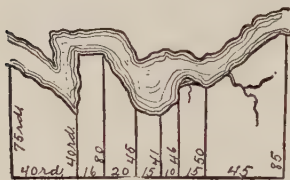


Fig. 57.

95. Figure No. 57 represents an irregular tract of land with measurements. How many acres in the field?

96. Each of the four sides of a lot measures 20 rods, but on account of a ravine the surface on one side following the depression measures $22\frac{1}{2}$ rods. How many feet of lumber will be required for the pickets and base board to build a picket fence around the same, the pickets being 4 feet long, 3 inches wide, of 1-inch lumber, placed vertically, the horizontal distance between the pickets being 3 inches, the base board being 1x10 inches.

PROPORTION.

137. Proportion is an equality of ratios.

Thus, $5 : 3 :: 10 : 6$ expresses a proportion of equality. The sign (=) may be used instead of the double colon (::).

Four terms are necessary to each proportion.

The Extremes are the first and fourth terms.

The Means are the second and third terms.

The product of the Extremes equals the product of the Means.

138. GIVEN.—The mean terms and one extreme term, to find the other extreme term.

RULE.—*Divide the product of the mean terms by the given extreme term, the result will be the required term.*

139. GIVEN.—The extreme terms and one mean term, to find the other mean term.

RULE.—*Divide the product of the extreme terms by the given mean term, the result will be the required term.*

WRITTEN PRACTICE.

Find the required term in the following proportions:

1. $20 : 36 :: 40 : ()$ 7. $() : 1.25 :: 6.25 : 2.5$

2. $18 : 72 :: 24 : ()$ 8. $\frac{2}{3} : \frac{1}{2} :: \frac{3}{4} : ()$

3. $36 : 16 :: () : 48$ 9. $\frac{3}{8} : () :: \frac{2}{7} : 2$

4. $25 : 75 :: () : 120$ 10. $() : 2 :: 12 : \frac{4}{9}$

5. $18\frac{3}{4} : 25\frac{1}{4} :: 12\frac{1}{2} : ()$ 11. $8\frac{1}{2} : 76\frac{1}{2} :: () : 153$

6. $() : \$24 :: 12 \text{ da.} : 48 \text{ da.}$ 12. $21 \text{ yd.} : 45 \text{ yd.} :: () : \50

13. If 20 bushels of wheat cost \$12, what will 35 bushels cost?

SOLUTION.

$$20 \text{ bu.} : 35 \text{ bu.} :: \$12 : (\$).$$

$$\frac{35 \times 12}{20} = \$21, \text{ cost of 35 bu.}$$

14. If 12 horses cost \$600, how many horses can be bought for \$750?

SOLUTION.

$$12 \text{ horses} : (\text{horses}) :: \$600 : \$750.$$

$$\frac{12 \times 750}{600} = 15, \text{ number of horses.}$$

15. If 12 yds. of silk cost \$21, what will 25 yds. cost?
16. If 50 yds. of carpeting 1 yd. wide will cover a floor, how many yards $\frac{3}{4}$ yd. wide will be required to cover the same floor?
17. If 250 bu. of oats are worth 225 bu. of corn, what amount of corn can be procured for 800 bu. of oats?
18. A tank holds 1000 gal., Liquid measure; how many bushels of grain will it hold?
19. If the freight on wheat is 20 cts. per cwt., how much will it be for 10 bushels?
20. A man failed in business with property valued at \$2250, and debts amounting to \$3750. How much can he pay to Mr. B., to whom he owes \$1000?
21. If the shares of two partners in the profits of their business for one year be \$975 and \$1150 respectively, what would be each one's share of a gain of \$3500 divided in the same ratio?
22. What must be the height of a load of wood containing 3 tiers to hold 1 cord, if each tier is 4 ft. long, and 3 ft. wide?
23. A dog is chasing a rabbit, which has 20 rods the start of him. The dog runs 20 rods while the rabbit runs 18 rods. How far must the dog run before overtaking the rabbit?
24. If it requires 20 mechanics to complete a house in 25 days, how many would be required to complete one having $\frac{1}{2}$ more work in $\frac{1}{2}$ less time?

SUGGESTION.—A term in proportion may consist of two or more factors; as, 20 men working 25 days, equivalent to 1 man working 500 days.

25. If a merchant has flour worth \$3.75 per cwt., and a farmer has butter worth 18 cts. per lb., how much should the merchant ask for his flour in trade, if the farmer asks 21 cts. per lb. for his butter?
26. If it takes 1000 bricks, 8 by 4 in., to lay a pavement of a certain length and width, how many bricks, $8\frac{1}{2}$ by $4\frac{1}{2}$ in., will be required to lay a pavement twice as long and twice as wide?
27. If a stone slab 10 ft. long, 6 ft. wide, and 4 in. thick weighs 550 lbs., what will be the weight of a slab 12 ft. long, 8 ft. wide, and 6 in. thick?
28. If 10 men mow a field of grass, working 5 days of 10 hours each, how many men will be required to mow a field $2\frac{1}{2}$ times the size, in 8 days of $8\frac{1}{2}$ hours each?

PERCENTAGE.

140. Percentage (an application of decimals) is the process of calculating by hundredths.

141. The term **Per Cent** (from the Latin *per centum*) means by the hundred, or simply hundredths; i. e., the result of dividing a number into 100 equal parts.

This term has usually been written as an abbreviation of the Latin *per centum*, indicating the contraction by a period after the word cent. We have followed later usage in treating these words as commonly accepted English, dropping the period.

TERMS EMPLOYED.

142. The **Base** is the number on which the percentage is computed.

143. The **Rate Per Cent** is the number of hundredths of the base to be found.

144. The **Percentage** is the number bearing the same ratio to the base that the per cent does to 1.

The terms *amount* and *difference* being merely so many hundredths of the base, come within the definition of percentage, and should be treated as such.

RELATIONSHIP.

1. Since percentage is a number of hundredths of the base, it is found by taking the number of hundredths indicated; that is, by multiplying the base by the rate.

2. Percentage is related to multiplication, the rate being the multiplier, and the percentage being the product.

3. If several factors are given, their product may be found by multiplication.

4. If the product of two factors and one of the factors is given, the other factor is found by dividing the product by the known factor.

5. Two questions for students of percentage to answer:

(a) Are the numbers given, factors?

(b) Are the numbers given, a factor and a product?

If factors, then the first operation will be a multiplication. If factor and product, then the first operation will be a division.

INTRODUCTION TO PERCENTAGE.

100 % = the whole of anything, or the number taken as a standard.

25 % equals $\frac{25}{100}$ or $\frac{1}{4}$; conversely, $\frac{1}{4}$ of a thing or quantity equals $\frac{1}{4}$ of 100 %, or 25 %.

1. What per cent of a thing or quantity is $\frac{1}{2}$ of it? $\frac{1}{2}$? $\frac{1}{2}$? $\frac{1}{2}$? $\frac{1}{2}$? $\frac{1}{2}$? $\frac{1}{2}$?

2. What per cent is $\frac{1}{4}$? $\frac{1}{4}$? $\frac{1}{4}$? $\frac{1}{4}$? $\frac{1}{4}$?

3. What per cent is $\frac{1}{8}$? $\frac{1}{8}$? $\frac{1}{8}$? $\frac{1}{8}$? $\frac{1}{8}$? $\frac{1}{8}$? $\frac{1}{8}$?

4. What per cent is $\frac{1}{16}$? $\frac{1}{16}$? $\frac{1}{16}$? $\frac{1}{16}$? $\frac{1}{16}$? $\frac{1}{16}$? $\frac{1}{16}$?

5. Count by $6\frac{1}{4}$'s to 100; $8\frac{1}{3}$'s to 100; $11\frac{1}{3}$'s to 100; $12\frac{1}{2}$'s to 100; $14\frac{1}{2}$'s to 100.

6. What part of a thing or quantity is 10 % of it in 10ths? 20 %? 30 %? 40 %? What part in 4ths is 25 %? 50 %? 75 %? 100 %? What part in 8ths is $12\frac{1}{2}$ %? 25 %? $37\frac{1}{2}$ %? 50 %? $62\frac{1}{2}$ %? 75 %? $87\frac{1}{2}$ %? What part in 40ths is $2\frac{1}{2}$ %? 5 %? $7\frac{1}{2}$ %? 10 %? $12\frac{1}{2}$ %? 15 %? $17\frac{1}{2}$ %? 20 %? $22\frac{1}{2}$ %? 25 %? $27\frac{1}{2}$ %? 30 %? $32\frac{1}{2}$ %?

7. Count by $3\frac{1}{3}$'s to 100; $2\frac{1}{2}$'s to 100; $16\frac{2}{3}$'s to 100.

8. Count by $12\frac{1}{2}$'s to 100, and return by deducting $16\frac{2}{3}$'s.

9. Count by $14\frac{2}{7}$'s to 100, and return by deducting $11\frac{1}{3}$'s.

10. Count by $3\frac{1}{3}$'s to 100, and return by deducting $2\frac{1}{3}$'s.

11. What is the ratio of 25 % to 100 %? to 50 %? to 75 %?

12. What is the ratio of 10 % to 40 %? to 90 %?

13. What is the ratio of $12\frac{1}{2}$ % to 75 %? to $87\frac{1}{2}$ %? to $62\frac{1}{2}$ %?

14. What is the ratio of $6\frac{1}{4}$ % to $18\frac{3}{4}$ %? to $56\frac{1}{4}$ %?

15. What is the ratio of $37\frac{1}{2}$ % to $62\frac{1}{2}$ %? to $87\frac{1}{2}$ %?

16. What is the ratio of $18\frac{3}{4}$ % to 25 %? to $37\frac{1}{2}$ %?

17. What is the ratio of $16\frac{2}{3}$ % to $33\frac{1}{3}$ %? to 50 %?

RELATIONSHIP TO FRACTIONS AND DECIMALS.

Words.	Commercial Terms.	Decimal Fractions.	Common Fractions.	Lowest Terms.
1 per cent	1%	.01	$\frac{1}{100}$	$\frac{1}{100}$
2 per cent	2%	.02	$\frac{2}{100}$	$\frac{1}{50}$
3 per cent	3%	.03	$\frac{3}{100}$	$\frac{3}{100}$
4 per cent	4%	.04	$\frac{4}{100}$	$\frac{1}{25}$
5 per cent	5%	.05	$\frac{5}{100}$	$\frac{1}{20}$
6 per cent	6%	.06	$\frac{6}{100}$	$\frac{3}{50}$
8 per cent	8%	.08	$\frac{8}{100}$	$\frac{2}{25}$
10 per cent	10%	.10	$\frac{10}{100}$	$\frac{1}{10}$
15 per cent	15%	.15	$\frac{15}{100}$	$\frac{3}{20}$
20 per cent	20%	.20	$\frac{20}{100}$	$\frac{1}{5}$
25 per cent	25%	.25	$\frac{25}{100}$	$\frac{1}{4}$
30 per cent	30%	.30	$\frac{30}{100}$	$\frac{3}{10}$
50 per cent	50%	.50	$\frac{50}{100}$	$\frac{1}{2}$
75 per cent	75%	.75	$\frac{75}{100}$	$\frac{3}{4}$
100 per cent	100%	1.00	$\frac{100}{100}$	1
125 per cent	125%	1.25	$\frac{125}{100}$	$1\frac{1}{4}$
250 per cent	250%	2.50	$\frac{250}{100}$	$2\frac{1}{2}$
1000 per cent	1000%	10.00	$\frac{1000}{100}$	10
$\frac{1}{2}$ per cent	$\frac{1}{2}$ %	.005	$\frac{5}{1000}$	$\frac{1}{200}$
$\frac{1}{4}$ per cent	$\frac{1}{4}$ %	.0025	$\frac{25}{10000}$	$\frac{1}{400}$
$\frac{3}{4}$ per cent	$\frac{3}{4}$ %	.0075	$\frac{75}{10000}$	$\frac{3}{400}$
$\frac{1}{3}$ per cent	$\frac{1}{3}$ %	.00 $\frac{1}{3}$	$\frac{1}{300}$	$\frac{1}{300}$
$\frac{2}{3}$ per cent	$\frac{2}{3}$ %	.00 $\frac{2}{3}$	$\frac{2}{300}$	$\frac{1}{150}$
$\frac{1}{8}$ per cent	$\frac{1}{8}$ %	.00125	$\frac{125}{100000}$	$\frac{1}{800}$
$\frac{3}{8}$ per cent	$\frac{3}{8}$ %	.00375	$\frac{375}{100000}$	$\frac{3}{800}$
$6\frac{1}{4}$ per cent	$6\frac{1}{4}$ %	.06 $\frac{1}{4}$	$\frac{625}{10000}$	$\frac{1}{16}$
$8\frac{1}{3}$ per cent	$8\frac{1}{3}$ %	.08 $\frac{1}{3}$	$\frac{25}{300}$	$\frac{1}{12}$
$12\frac{1}{2}$ per cent	$12\frac{1}{2}$ %	.12 $\frac{1}{2}$	$\frac{125}{1000}$	$\frac{1}{8}$
$33\frac{1}{3}$ per cent	$33\frac{1}{3}$ %	.33 $\frac{1}{3}$	$\frac{100}{300}$	$\frac{1}{3}$
$37\frac{1}{2}$ per cent	$37\frac{1}{2}$ %	.37 $\frac{1}{2}$	$\frac{375}{1000}$	$\frac{3}{8}$
$62\frac{1}{2}$ per cent	$62\frac{1}{2}$ %	.62 $\frac{1}{2}$	$\frac{625}{1000}$	$\frac{5}{8}$
$66\frac{2}{3}$ per cent	$66\frac{2}{3}$ %	.66 $\frac{2}{3}$	$\frac{200}{300}$	$\frac{2}{3}$

Change the following per cents, expressed in words, to the following forms: Decimals, common fractions, and common fractions reduced to their lowest terms:

1 per cent, $2\frac{1}{2}$ per cent, $3\frac{1}{3}$ per cent, $4\frac{1}{2}$ per cent, $6\frac{1}{4}$ per cent, $8\frac{1}{8}$ per cent, $12\frac{1}{2}$ per cent, $14\frac{2}{7}$ per cent, $16\frac{2}{3}$ per cent, $18\frac{3}{4}$ per cent, $31\frac{1}{4}$ per cent, $37\frac{1}{2}$ per cent, $43\frac{3}{4}$ per cent, $56\frac{1}{4}$ per cent, $57\frac{1}{2}$ per cent, $62\frac{1}{2}$ per cent, $66\frac{2}{3}$ per cent, $67\frac{1}{2}$ per cent, $72\frac{1}{2}$ per cent, $77\frac{1}{2}$ per cent, $87\frac{1}{2}$ per cent, $93\frac{1}{3}$ per cent, $96\frac{2}{3}$ per cent, $\frac{1}{8}$ per cent, $\frac{2}{3}$ per cent, $\frac{5}{8}$ per cent, $\frac{1}{10}$ per cent, $\frac{3}{10}$ per cent, $\frac{7}{10}$ per cent, $112\frac{1}{2}$ per cent, $133\frac{1}{3}$ per cent, $166\frac{2}{3}$ per cent, 225 per cent, 275 per cent, $333\frac{1}{3}$ per cent, and 1500 per cent.

CASE I.

145. GIVEN.—Base and Rate, to find Percentage.

HINT.—Factors, to find product.

RULE.—*The percentage being the product of the base and rate, is found by multiplying the base by the rate.*

FORMULA.— $B \times R = P$.

MENTAL PRACTICE.

1. What part of a thing or quantity is $12\frac{1}{2}\%$ of it?

SOLUTION.— $12\frac{1}{2}\%$ is $\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$.

2. What is $12\frac{1}{2}\%$ of 200?

SOLUTION.—Since $12\frac{1}{2}\%$ is $\frac{1}{8}$ of a thing or quantity,
 $12\frac{1}{2}\%$ of 200 is $\frac{1}{8}$ of 200, or 25. Therefore, 25 is
 $12\frac{1}{2}\%$ of 200.

3. What is $12\frac{1}{2}\%$ of 40? 80? 120? 240?
4. What is 10% of \$60? \$45? \$75? \$95?
5. What is 20% of 20? 40? 2000? 85?
6. What is $16\frac{2}{3}\%$ of \$120? \$24? \$72? \$96? \$144?
7. What part of a thing or quantity is $14\frac{2}{7}\%$ of it?
8. What is $14\frac{2}{7}\%$ of 14 bu.? 56 bu.? 490 bu.? 84 bu.?
9. What is $66\frac{2}{3}\%$ of 120 men? 240 men? 36 men?
10. What is 75% of \$200? \$240? \$280? \$300?
11. What is 250% of \$60? \$150? \$900? \$1200?
12. What is 125% of 120 oxen? 560 oxen? 640 oxen?
13. What is $\frac{1}{4}\%$ of 2000 bu.? 3600 bu.? 144 bu.?

WRITTEN PRACTICE.

1. What is 24% of \$560?

SOLUTION.

100% of it equals \$560.00

1% " " \$5.60

24% " " 24 times 1% $.24 \times \$560 = \134.40 .\$5.60 \times 24 equals \$134.40

2. What is 35% of 177 pounds?
3. What is 42% of 864? 962?
4. What is $12\frac{1}{2}\%$ of \$736? \$1152?
5. What is $37\frac{1}{2}\%$ of 240 bu.? 2376 bu.?
6. What is 75% of 1464 bu.? 2376 bu.?
7. What is 125% of 1296 pounds? 2480?
8. What is 225% of \$4162? \$3624?
9. What is $\frac{5}{8}\%$ of \$24? \$36? \$42?
10. What is $\frac{3}{8}\%$ of $\frac{3}{4}$?
11. A man invests \$17280 as follows: 25% in real estate, $37\frac{1}{2}\%$ in bank stock, and the remainder in city lots. How much did he invest in each?
12. What will $37\frac{1}{2}\%$ of 480 bushels of wheat cost at \$1.25 per bushel?
13. A man buying a house and lot, paid \$1500 for the lot, and $37\frac{1}{2}\%$ more than that for the house. What did both cost?
14. A farm contained a half section; 25% of it was sold at \$45 per acre, 40% at \$37.50 per acre, and the balance at \$30 per acre. How much was received for the farm?
15. A man has a yearly salary of \$2400, and spends 33% of it the first year, 45% the second, and 64% the third. How much does he save in the three years?
16. A man deposited in a bank \$1875; he withdrew 40% of the deposit, and with $9\frac{1}{8}\%$ of the amount withdrawn purchased a gun. What was the cost of the gun?
17. A man having a salary of \$3600, spends 20% of it for board, $12\frac{1}{2}\%$ for clothing, 10% for books and lecture fees, 5% for incidentals, and deposits the remainder in the bank. How much does he deposit?

18. A man had a bank account of \$6000; he withdrew $33\frac{1}{3}\%$ of it, and expended 75% of the money thus withdrawn in the purchase of a piano, depositing the unexpended balance. What is his present bank account?

19. A farmer having 896 bushels of wheat, sold at one time 25% of it, and at another time $37\frac{1}{2}\%$ of what remained. How much did he sell?

20. A speculator bought a section of land at \$25 per acre; he sold 25% of the land at \$40 per acre, 40% at \$22.50 per acre, and the remainder at cost. How much did he gain?

21. A man started in business with a cash capital of \$7200, and remained in business 3 years. The first year he gained 25% of his capital, the second year he gained $33\frac{1}{3}\%$ of his increased capital, and the third year he lost $12\frac{1}{2}\%$ of his increased capital. How much had he at the end of 3 years?

CASE II.

146. GIVEN.—Base and Percentage, to find Rate.

HINT.—Product and factor, to find factor.

RULE.—*The percentage being the product of the base and rate, the rate is found by dividing the percentage by the base, expressing the result in hundredths.*

FORMULA.— $P \div B = R$.

MENTAL PRACTICE.

1. What per cent of \$75 is \$15?

SOLUTION.—Since \$15 is $\frac{1}{5}$ of \$75, it is $\frac{1}{5}$ of 100%, or 20%.

2. What part of 200 bu. is 100 bu.? 50 bu.? 150 bu.? 25 bu.? 75 bu.? What per cent?

3. The whole of a number equals what per cent of it?

4. What per cent of \$50 is \$10? \$20? \$30? \$40? \$50?

5. What per cent of 20 pecks is 5 pecks? 10 pks.? 15 pks.? 20 pks.? 30 pks.? 40 pks.?

6. What per cent of 60 feet is 10 feet? 20 ft.? 30 ft.? 40 ft.? 50 ft.? 60 ft.?

7. What per cent of \$500 is \$250? \$200? \$300? \$400? \$100?

8. What per cent of 20 quarts is 20 quarts? 30 qts.? 40 qts.? 50 qts.? 60 qts.? 25 qts.?

9. What per cent of $\frac{3}{4}$ is $\frac{1}{4}$? $\frac{1}{2}$? $1\frac{1}{2}$?
10. What per cent of 480 is 240? is 120? is 360? is 960?
11. What per cent is 360 min. of 360 min.? of 720 min.? of 180 min.?
12. What per cent is $\$33\frac{1}{3}$ of $\$100$? of $\$16\frac{2}{3}$? of $\$8\frac{1}{3}$?
13. What per cent is $66\frac{2}{3}$ of 200? is $33\frac{1}{3}$? is $16\frac{2}{3}$? is $8\frac{1}{3}$?
14. What per cent of 120 is $16\frac{2}{3}\%$ of 180?
15. What per cent of 200 is 25% of 400?
16. What per cent of .48 is .24? is .12? is .36? is .96?
17. $33\frac{1}{3}\%$ of 180 is what per cent of 25% of 180?
18. $16\frac{2}{3}\%$ of $\$90$ is what per cent of $12\frac{1}{2}\%$ of $\$240$?
19. What per cent of an hour is 30 min.? is 20 min.? is 15 min.? is 12 min.?
20. How much more is 300 than 200? than 150?
21. What part more is 300 than 200? than 150?
22. What per cent more is 300 than 200? than 150?
23. What per cent more than 200 is 210? is 230? is 250?
24. What per cent more than $\$450$ is $\$500$? is $\$540$? is $\$630$?
25. What per cent more than 500 bu. is 700 bu.? is 800 bu.? is 900 bu.?
26. What per cent less than 90 is 80? is 70? is 60? is 40?
27. What per cent less than 200 is 190? is 180? is 150? is 120? is 100? is 75? is 50?
28. What per cent less than $\$12\frac{1}{2}$ is $\$10$? is $\$7\frac{1}{2}$? is $\$5$? is $\$2\frac{1}{2}$?
29. What per cent of 25 is $\frac{3}{4}$ of 100? $\frac{7}{8}$ of 200?
30. What per cent of 50 is 75 per cent of 200?
31. Two-thirds of 75 is what per cent of three-fourths of 80?
32. 20% of 250 is what per cent more than $16\frac{2}{3}\%$ of 240?
33. $14\frac{2}{7}\%$ of 840 bu. is what per cent less than $33\frac{1}{3}\%$ of 540 bu?

WRITTEN PRACTICE.

22. What per cent of $\$540$ is $\$180$?

SOLUTION.

$\$540.00$ equals 100%	OR
5.40 " 1%	$\$180 \div 540 = .33\frac{1}{3}$.
$\$180 \div 5.40$ " $33\frac{1}{3}$, or $33\frac{1}{3}\%$.	

23. What per cent of \$450 is \$360? is \$6.75?
24. What per cent of 360 is 240? is 135?
25. What per cent is \$2.25 of \$3.75? of \$6.75?
26. What per cent is 84 of 336? of 756?
27. What per cent more is \$720 than \$480?
28. What per cent more is 560 than 490?
29. What per cent less than 750 hr. is 615 hr.?
30. What per cent less is 784 than 896?
31. What per cent of $\frac{3}{8}$ is $\frac{3}{4}$? is $\frac{5}{8}$? is $\frac{1}{8}$?
32. In a brigade of 7500 men, there were 1500 Germans, 2400 Russians, and the remainder Austrians. What is the per cent of each?
33. A man traveled 24 miles the first day, 18 miles the second, and 30 miles the third. What per cent of the whole journey did he travel each day?
34. A merchant bought 500 barrels of flour, selling at one time 25 per cent of it, and at another $33\frac{1}{3}$ per cent of the remainder. How many barrels has he left, and what per cent did he sell?
35. An army during an engagement lost 2400 men, and now numbers 9600 men. What was the per cent loss?
36. A merchant having 544 barrels of flour, sold $\frac{1}{4}$ of it at one time, and $\frac{1}{2}$ of the remainder at another. What per cent remained unsold? How many barrels were left?
37. Forming a partnership, A invests \$6400 and B invests \$9600. What per cent of the firm's capital does each invest? What per cent is each one's money of the other's?
38. An army being reinforced by a regiment of 1200 men, now numbers 6000. What was the per cent of increase?
39. A firm's real estate is valued at \$6000, personal property at \$4800; the former increases 25% in value, and the latter decreases $16\frac{2}{3}\%$. What per cent is each property of the other at present valuation?
40. J. V. Farwell & Co. imported goods invoiced at £1264 12s., and paid £316 3s. tariff. What per cent was the tariff of the invoice price?

41. A liberty pole 96 feet in height fell, breaking into three pieces, the second piece being twice as long as the first, and the third as long as the other two. What per cent is each piece of the total length?

CASE III.

147. GIVEN.—Percentage and Rate, to find Base.

HINT.—Product and factor, to find factor.

RULE.—*The percentage being the product of the base and rate, the base is found by dividing the percentage by the rate expressed in hundredths.*

FORMULA.— $P \div R = B$.

MENTAL PRACTICE.

1. 75 is 10% of what number?

SOLUTION.—10% equals $\frac{1}{10}$. The number is 10 times 75 or 750; or since 75 is 10%, 1% which is $\frac{1}{10}$ of 10%, is $\frac{1}{10}$ of 75, or 7.50, and 100% is 100 times 7.50, or 750.

2. 200 is 20% of what number? 25%? $33\frac{1}{3}\%$?

3. \$1.20 is 6% of what number? 40%? 50%? 1%?

4. 400 is 20% less than what number? 25%?

5. 250 pounds is 25% more than what number? 50%?

6. What number increased by $16\frac{2}{3}\%$ of itself equals 350?
875?

7. What number diminished by $33\frac{1}{3}\%$ of itself equals 72?
420?

8. What number diminished by 25% of itself equals .48?
.72? .96? 9.6? 24? 2.4? 240?

9. What number increased by 25% of itself equals $\frac{5}{8}$?

10. $33\frac{1}{3}\%$ of \$240 is 25% of what number?

11. $66\frac{2}{3}\%$ of 360 is 20% more than what number?

12. $91\frac{2}{3}\%$ of 144 oxen is $14\frac{2}{3}\%$ less than what number?

13. If $16\frac{2}{3}\%$ of a number is 84, what is 50% of the same number?

14. If $133\frac{1}{3}\%$ of a number is 248, what is $16\frac{2}{3}\%$ of the same number?

15. If $\frac{1}{4}\%$ of a number is 24, what is 2% of the same number?

16. If $87\frac{1}{2}\%$ of a number is 490, what is $62\frac{1}{2}\%$ of the same number?

17. If a number increased by $16\frac{2}{3}\%$ of itself equals 840, what will it equal if increased by 25% of itself?

18. If a number increased by 25% of itself equals 450, what will it equal if decreased by $37\frac{1}{2}\%$ of itself?

19. If a number decreased by $33\frac{1}{3}\%$ of itself equals 720, what will it equal if decreased by 25% of itself?

20. If a number decreased by $14\frac{2}{3}\%$ of itself equals 54, what will it equal if increased by $11\frac{1}{3}\%$?

WRITTEN PRACTICE.

42. 375 is 25% of what number?

SOLUTION:

375 equals 25% of the number

1% equals $\frac{1}{25}$ of 375, or 15

100% equals 100 times 15, or 1500.

OR
 $375 \div .25 = 1500.$

43. 111 is 37% of what number?

44. $5\frac{1}{2}\%$ is 25% of what number? 40% ?

45. $\frac{3}{8}$ is 60% of what number? 20% ?

46. What number minus $33\frac{1}{3}\%$ of itself equals 80?

47. What number plus 25% of itself equals 80?

48. $12\frac{1}{2}\%$ of 640 is 20% of what number?

49. 36% of 900 is 8% more than what number?

50. What number increased by 50% of itself equals 75% of 600?

51. 10% of 90 is 10% less than 10% of what number?

52. 10% of 220 is $8\frac{1}{8}\%$ less than $\frac{1}{2}$ of what number?

53. A man spends yearly \$1750, which is $83\frac{1}{3}\%$ of his salary. What is his salary?

54. A paid me \$324.25, which is $12\frac{1}{2}\%$ of the sum he still owes me. How much did he owe me?

55. A man having a two-thirds interest in a piece of property, sells 40% of his share for \$1600. What is the value of the property?

56. A man having bought an invoice of goods amounting to \$1050, finds that this purchase is 24% of his whole stock. What was his original stock?

57. I bought two farms for \$9408. I paid 24% more for one than I did for the other. How much did I pay for each?

58. The expenses of a manufacturing plant for two years were \$70125, the expenses this year were $12\frac{1}{2}\%$ greater than last year. Find the expenses for last year.

59. A merchant's sales for two years were \$8800, the sales for last year being $16\frac{2}{3}\%$ less than the sales for this year. Find the sales for each year.

60. The building, machinery, and stock of a manufacturing plant were valued at \$72000. If the building is worth 40% less than the machinery, and the stock 25% more than the other two, what is the value of each?

61. During the first year in business I gained 25%, the second year I gained 20% of my increased capital, and the third year I lost 8% of my increased capital, quitting the business with \$65136. How much was my capital stock?

62. A man withdraws 60% of the money he has deposited in a bank, and spends $16\frac{2}{3}\%$ of the money thus withdrawn in the purchase of a house, paying \$2700. How much money has he left in the bank?

63. A merchant increases his bank account 50%; he then withdraws 80% of the whole, and invests the same in real estate; his bank account is now \$3600. How much did he have in bank at first?

PROMISCUOUS MENTAL PROBLEMS.

1. $37\frac{1}{2}\%$ of a flock of sheep were killed. What per cent remained?

2. A farmer sold $12\frac{1}{2}\%$ of a flock of 200 sheep. How many sheep had he remaining, and what per cent?

3. If $16\frac{2}{3}\%$ of a man's bank account is \$500, what is $66\frac{2}{3}\%$ of his bank account?

4. A man sold 160 acres from a section of land. What per cent of a section had he remaining?

5. What per cent is 5 sq. mi. of 5 miles square? of 10 miles square?

6. What per cent of 25 square rods is 25 rods square?

7. What is the sum of 10%, 12%, and 15% of \$200?

8. What is the sum of 20% of \$80, and $33\frac{1}{3}\%$ of \$75?

9. $33\frac{1}{3}\%$ of a hogshead of molasses was sold at one time, and 50% of the remainder at another. How many gallons remained?

10. What is the cost of 25% of 160 bushels of oats at $37\frac{1}{2}\phi$ per bushel?

11. A farmer sold a horse for \$90, which was $12\frac{1}{4}\%$ more than it cost. What did it cost him?

12. Wheat was sold at 77 cts. per bushel, which is $12\frac{1}{2}\%$ less than was paid for it. How much was paid for it?

13. I deposited \$1200 in gold and silver, and $16\frac{2}{3}\%$ of that amount in bank bills. How much did I deposit in all?

14. A man paid \$6 for a pair of shoes and \$4.50 for a hat, which was 25% of his money left. How much money had he at first?

15. From a cask containing 45 gallons of kerosene, 15 gallons were sold. What per cent was sold?

16. A gentleman having \$1200, lost 25% of it. How much had he left?

17. If 10% of a journey is 25 miles, what is 20% of the same journey? 30%? 40%? 50%? 75%? 100%?

18. 25% of a man's salary is \$80. What is $6\frac{1}{4}\%$ of it? $12\frac{1}{2}\%$? $18\frac{3}{4}\%$? $31\frac{1}{4}\%$? etc.

19. In a box of berries $\frac{1}{4}$ were damaged. What per cent were good?

20. If a pint of water is added to every gallon of milk, what per cent of the whole is pure milk?

21. A man, owning $\frac{3}{8}$ of a factory, sells $\frac{3}{8}$ of his share. What per cent of the factory has he left?

22. A grocer, after selling $\frac{7}{8}$ of a barrel of sugar, has 35 pounds left. What per cent did he sell?

23. A man's property invested at 5% yields him an income of \$120 per month. How much has he invested?

24. Three sections of land are what per cent of a government township?

25. The population of a city is 17280, showing an average gain of 20% per year for three years. What was its population three years ago?

26. A city of 12500 has gained 25% on its population of five years ago, which was 25% gain on its population of ten years ago. What was its population ten years ago?

27. A father is 60% older than his son, and the difference between their ages is 30 years. How old is the father?

28. A horse and carriage cost \$300, the horse costing 50% more than the carriage. How much did each cost?

29. $\frac{2}{3}$ of A's money is 75% of B's, and $\frac{2}{3}$ of B's is $33\frac{1}{3}\%$ of C's. C has \$24. How much has A?

30. A man owning 60% of a store sells $33\frac{1}{3}\%$ of his share for \$2000. What is the remainder of his interest worth? What is the entire store worth?

31. A man's income is \$1600 per year, from which he saves \$400. What per cent does he spend?

32. A man had a half interest in a foundry and sold one-half of his share for \$2500. What did the foundry cost, the sale being made at the rate of 25% more than the cost?

33. A store rents for \$600 per year, which is 10% of its value. What was its cost, provided its value is 20% more than its cost?

34. If a pupil attends school 15 days in March, 18 days in April, and 17 days in May, what is his percentage of attendance, the term of school lasting 12 weeks of 5 days per week?

35. If a man's debts amount to \$1800 and his property to \$1200, what per cent can he pay on his debts?

36. If an agent sells merchandise for his principal to the amount of \$4000 and remits to him cash \$3950, what per cent of the sales does he retain?

37. If a farm yields 300 bushels of wheat one year and 400 bushels in the next, what is the per cent of the increase?

38. A man withdraws 40% of his bank deposit, spending 25% of the amount thus withdrawn in the purchase of a piano, paying \$240. How much money had he in the bank?

39. If I own 75% of a mill and sell $33\frac{1}{3}\%$ of my share for \$1800, what would be the value of the mill at the same rate?

PROMISCUOUS WRITTEN EXERCISE.

GIVEN.—Two terms, to find the third.

	Base.	Rate.	Percentage.
64.	16	11%	?
65.	2.25	112%	?
66.	$37\frac{1}{2}$	$\frac{1}{2}\%$?
67.	$\frac{3}{4}$	$\frac{2}{3}\%$?
68.	900	?	30
69.	4.5	?	5
70.	$\frac{7}{8}$?	$\frac{1}{4}$
71.	$\frac{3}{8}$?	.625
72.	$\frac{3}{8}$?	4.25
73.	?	5%	75
74.	?	14%	560
75.	?	$\frac{4}{5}\%$	$\frac{4}{5}$
76.	?	$\frac{3}{4}\%$	$\frac{3}{8}$
77.	?	$33\frac{1}{3}\%$	1080
78.	5.4	2.5%	?
79.	?	$201\frac{1}{2}\%$	161.20
80.	.006	.06	?
81.	$\frac{3}{8}$?	.05
82.	$\frac{3}{4}$?	$\frac{7}{8}$

SUGGESTIONS.—The above table will make an excellent exercise for the class-room, if the student is required, (1) To state the terms given and the process necessary to find the wanting term. (2) To construct a problem, using the given numbers in each example. (3) To give the answer required in each example.

PROMISCUOUS WRITTEN PROBLEMS.

83. A drover lost 15% of a flock of sheep and had 340 head left. How many did he lose?

84. In a city of 25000, the deaths for a certain year were 1050. What was the per cent of deaths?

85. If there are 30 scholars enrolled in a winter school, whose term is 16 weeks, and there are altogether 500 days lost by different scholars, what is the average per cent of attendance?

86. A man whose income is \$2000, spends \$1850 of it. What per cent does he save?

87. A man having bought a section of land gave a half-quarter to each of his five children. What per cent of the section had he left, and how many acres?

88. A farmer sold 25% of 72% of 3000 head of sheep, at \$2.75 per head. How much did he receive?

89. If you lose \$1824 by selling at a loss of $12\frac{1}{2}\%$, what would you receive if you sold the same at a profit of $12\frac{1}{2}\%$?

90. If I pay \$70 for the use of \$350, what is the rate per cent?

91. A house cost \$2500 and sold for \$3000. What was the per cent of selling price?

92. A horse that cost \$75 was sold for \$60. What was the per cent of selling price?

93. A man owning 50% of a foundry, exchanged 50% of his share for a business lot 40 by 140 feet, at \$1.25 per sq. ft. At that rate, what was the foundry worth?

94. A merchant pays \$600 for a year's rent of his store room, which amount is 20% of his income, and his income is 10% of his sales. How much are his yearly sales?

95. A man owes \$6500, which is 25% more than his property is worth. What per cent can he pay?

96. A man drew 20% of his money from the bank, and paid 20% of the money thus withdrawn for three months' rent at the rate of \$960 per year. How much has he left in the bank?

97. The net profits of a business for two years were \$6400, the second year's profit being $66\frac{2}{3}\%$ greater than the profits of the first year. What were the profits each year?

98. I paid for two farms \$22500. If 75% of the cost of one farm is equal to 150% of the other, what did I pay for each?

99. D has 10% more money than C, and C has 20% more and B 8% less than A, and they all have \$33300. Required the amount of A's money and also of C's money?

100. I paid \$58 for a coat; the trimmings cost $66\frac{2}{3}\%$ less, and the making 40% less than the material. What was the cost of each?

101. In the purchase of oats, wheat, and barley, a merchant expended equal sums. In the sales he gained 7% on the oats, 9% on the wheat, but on the barley he lost 21%; the total sales were \$2212.50. What sum did he invest in each kind of grain?

102. If 50% of A's money equals 40% of B's, and 90% of B's equals 75% of C's, and 120% of C's equals 50% of D's, how much has each, if 25% of D's is equal to \$360?

103. A man planted an orchard of 2880 apple and pear trees; there were 120% more apple trees than pear trees. How many of each?

104. The profits of a grocery business for three years were \$5632. The second year's profits were 20% greater than the first, and the third year's profits were 10% greater than the second. What were the profits of each year?

105. A has \$360, B has \$540. What per cent is A's money of B's? What per cent is B's money of A's?

106. The profits of a business were 25% less the second year than the first year, and during the two years the profits were \$13125. What were the profits for the first year? What per cent greater were the first year's profits than the second?

107. B and C together have \$2100; 10% of B's money equals 25% of C's money. How much has each?

108. A man left \$28750 to his two sons, giving to the older 30% more than to the younger; at the end of five years the younger has gained 30% on what he received, and the older lost 30%. How much has each one now?

109. A and B each desire to sell a house and lot to C. A asks 50% more than B; A then lowers his price 20%; B lowers his in the same ratio. C now takes both properties, paying \$14780. What was each one's asking price?

110. A paid \$55000 for a bill of merchandise. He sold part for \$60000, of which $\frac{1}{3}$ was at a profit of 25%, and $\frac{2}{3}$ at a profit of $33\frac{1}{3}\%$. Find the cost price of the remaining merchandise.

111. In building a house, the cost of the material was to the cost of the labor as 3 is to 2. Had 5% more for material and 10% less for labor been paid, the house would have cost \$44550. What was the cost?

112. I bought a horse and saddle; I paid three times as much for the horse as for the saddle. Had I paid $12\frac{1}{2}\%$ more for the horse and 5% less for the saddle, the cost would have been \$648.75. What did I pay for each?

113. A's money is 20% of B's, and 25% of C's. B has \$300 more than C. How much money has A?

114. Two brothers receive by bequest \$3600 each. One's money increases at the rate of 12% per annum for 5 years, while the other's increases at the rate of 10% per annum for the same time. How much more has the one than the other?

115. After I have spent 50% of my money and 25% of the remainder, I find that I have left \$3375. How much money had I at first?

116. A merchant invested \$42500 in dry goods. The first year he gained 20% ; the second year he gained $12\frac{1}{2}\%$; the third year he lost $16\frac{2}{3}\%$; and the fourth year he gained 5% . Find the amount invested in business at the end of the fourth year, the gain being added or the loss subtracted each year?

117. A's farm is 1 mile long and 120 rods wide. He sold 40% of it at \$36 per acre at one time, and $37\frac{1}{2}\%$ of the remainder at \$40.50 per acre at another time. What was the total amount received, if the balance was sold at \$25 per acre?

118. On commencing business a merchant had \$5000. The first year he gained 25% , the second year he gained $37\frac{1}{2}\%$, and the third year he gained 20% . What is his present capital, if he left 80% of each year's gain in the business?

PERCENTAGE AS AN APPLICATION.

The following subjects are applications of percentage of the first class, and have terms corresponding to all the percentage terms: Profit and Loss, Commission and Brokerage, Trade Discount, Insurance, Stocks, Taxes, etc.

The rules relating to the applications of percentage of the first class are the same as in percentage, therefore they will not be repeated. The student may make his own application.

COMMON SENSE IN ARITHMETIC.

The student should endeavor to use his judgment, as well as his memory in solving problems. Anyone who understands the simplest processes of arithmetic can obtain answers to the most intricate problems, if they come to him labeled with topic, case, section, and rule.

The man of business does not meet his problems thus carefully labeled. He must use judgment as to what principles of arithmetic to apply. He cannot keep a thousand rules stored up in his crowded brain, or if he succeeds in memorizing them, he cannot keep them all polished ready for immediate use. He must make himself master of principles and use common sense in their application.

Realizing this fact, we have endeavored to offer such examples and problems for solution, as the student will be likely to meet in practical life. They are not assorted and grouped under each case, nor are the rules given, where the principles are so plain that the student can formulate his own rules.

Before working an example, the student is advised to study its statements in all their relations to each other and, reasoning from the nature of the case, obtain an approximate answer. A moment's thought will frequently save the student a half hour's extra calculation by starting him right and, what is still more important, will often aid him in detecting an absurd answer obtained from a proper application of the rule, the result of inaccuracies in calculation.

Let us use the high endowments of reason and judgment whenever it is possible, and not allow ourselves to degenerate into mere mathematical machines.

PROFIT AND LOSS. .

148. Profit and Loss are terms relative to gains and losses in buying and selling. Their processes are based on the principles of percentage and have corresponding elements.

149. The Cost of any article is the basis or Base, on which the gains and losses are estimated.

150. The Per Cent of profit or loss is the Rate

151. The Gain or Loss is the Percentage.

152. The Selling Price is also a percentage, being a certain per cent of the cost price. When the selling price is at a profit, it is sometimes called the amount; when at a loss, the difference.

The different problems or cases may be found in the following table in which the student is to find all the terms in columns marked with a question mark:

As compared with Percentage terms

Cost = Base.

Rate of Gain or Loss = Rate.

Gain or Loss = Percentage.

Selling Price $\left\{ \begin{array}{l} \text{At Gain} = \text{Amount.} \\ \text{At Loss} = \text{Difference.} \end{array} \right\} = \text{Percentage.}$

MENTAL PRACTICE.

1. Sold a horse for \$100 that cost \$75. How much did I gain? What part of the cost did I gain? What per cent?

2. Sold a cow for \$50 that cost \$60. How much did I lose? What part of the cost did I lose? What per cent?

3. By selling at \$60 I gain \$20. What would be the selling price if sold at a gain of \$34?

4. By selling at \$75 I lose \$25. What would be the selling price if I sold at a loss of \$40?

5. By selling at \$65 I lose \$20. What would be the selling price if sold at a gain of \$22.50?

6. By selling at \$.45 I gain \$.15. What would be the selling price if sold at a loss of \$.09?

Find the Gain or Loss:

7. Buying at \$20, and selling at a gain of 10%. 20%.
25%. 30%.

SUGGESTION.—Gain at 10 per cent = $\frac{1}{10}$ of \$20.

8. Buying at \$64, and selling at a gain of $6\frac{1}{4}\%$. $12\frac{1}{2}\%$.
9. Buying at \$14, and selling at a gain of $14\frac{2}{7}\%$. $28\frac{4}{7}\%$.
10. Buying at \$18, and selling at a gain of $11\frac{1}{3}\%$. $22\frac{2}{3}\%$.
11. Buying at \$.25, and selling at a loss of 10%. 20%.
30%. 40%.

12. Buying at \$.75, and selling at a gain of $33\frac{1}{3}\%$. $66\frac{2}{3}\%$.
 $133\frac{1}{3}\%$. $233\frac{1}{3}\%$.

13. Buying at \$.80, and selling at a loss of $12\frac{1}{2}\%$. $37\frac{1}{2}\%$.

Find the Selling Price:

14. Buying at \$20, and selling at a profit of 10%. 20%.
25%. 30%.

SUGGESTION.—Selling price = $\frac{11}{10}$ of \$20. Why?

15. Buying at \$24, and selling at a profit of $12\frac{1}{2}\%$. 25%.
16. Buying at \$80, and selling at a profit of $2\frac{1}{2}\%$. 5%.
 $7\frac{1}{2}\%$. 10%. $12\frac{1}{2}\%$. 15%.
17. Buying at \$.25, and selling at a profit of 10%. 20%.
30%. 40%. 50%.

18. Buying at \$.54, and selling at a loss of $11\frac{1}{3}\%$. $22\frac{2}{3}\%$.

19. Buying at \$.60, and selling at a loss of $3\frac{1}{3}\%$. $6\frac{2}{3}\%$.

Find the Gain or Loss per cent:

20. Buying at \$5, and selling at \$6. \$7. \$8. \$9. \$10.

SUGGESTION.—Difference between Cost and Selling Price = Loss or Gain. Loss or Gain is what per cent of Cost?

21. Buying at \$10, and selling at \$11. \$12. \$13. \$14. \$15.

22. Buying at \$1, and selling at \$1. \$1. \$1. \$1. \$1.

23. Buying at \$5, and selling at \$4. \$3. \$2. \$1. \$0.

24. Buying at \$10, and selling at \$9. \$8. \$7. \$6. \$5.

25. Buying at \$12 $\frac{1}{2}$, and selling at \$15. \$17 $\frac{1}{2}$. \$20. \$22 $\frac{1}{2}$.

26. Buying at \$8, and selling at \$9. \$7. \$10. \$6. \$11. \$5.

Find the Cost Price:

27. Selling at \$80, when the gain per cent was 25%. $33\frac{1}{3}\%$

SUGGESTION.—\$80 is $\frac{4}{3}$ of the Cost. Why?

28. Selling at \$9, when the gain per cent was $12\frac{1}{2}\%$. 20%.
 29. Selling at \$126, when the gain per cent was 20%. $16\frac{2}{3}\%$.
 30. Selling at \$560, when the loss per cent was $11\frac{1}{3}\%$. $22\frac{2}{3}\%$.
 31. Selling at \$12, when the loss per cent was 25%. 50%.

Find the Cost Price, and Selling Price, when:

32. By selling at a gain of \$25, I gain 20%. 25%.

SUGGESTION.—\$25 = $\frac{1}{4}$ of Cost Price.

33. By selling at a gain of \$.24, I gain $16\frac{2}{3}\%$. $33\frac{1}{3}\%$.
 34. By selling at a loss of \$.36, I lose 25%. $11\frac{1}{3}\%$.
 35. By selling at a loss of \$.15, I lose $33\frac{1}{3}\%$. 20%.
 36. By selling at a gain of \$12 $\frac{1}{2}$, I gain 25%. $12\frac{1}{2}\%$.

Find the second Selling Price, when:

37. By selling at \$125, I gain 25%. What would be the selling price if sold at a gain of $37\frac{1}{2}\%$?

SUGGESTION.—\$125 = $\frac{5}{8}$ of Cost. Second selling price is $\frac{9}{8}$ of Cost.

Or ratio of $\frac{1}{8}$ to $\frac{5}{8}$ is $\frac{1}{5}$: $\frac{1}{10}$ of \$125 is \$13 $\frac{1}{2}$.

38. By selling at \$60, I gain $33\frac{1}{3}\%$. What would be the selling price if sold at a gain of $11\frac{1}{3}\%$?
 39. By selling at \$.77, I lose $12\frac{1}{2}\%$. What would be the selling price if sold at a loss of $37\frac{1}{2}\%$?
 40. By selling at \$.87, I lose $3\frac{1}{3}\%$. What would be the selling price if sold at a loss of $2\frac{1}{2}\%$?
 41. By selling at \$.24, I gain $14\frac{2}{7}\%$. What would be the selling price if sold at a loss of $33\frac{1}{3}\%$?

PROMISCUOUS MENTAL PROBLEMS.

42. I bought wheat at \$.60 per bushel, and sold it at \$.75 per bushel. What per cent did I gain?
 43. I bought wheat at \$.75 a bushel, and sold it at \$.60 per bushel. What per cent did I lose?
 44. How much must I ask for sugar that cost me 10 cts. per lb. in order to gain 10%?
 45. I sold 20 lbs. coffee for \$4, gaining 25%. How much did it cost me per lb.?
 46. What was the per cent of loss on oranges bought for \$5 per hundred, and sold at 4 cts. apiece?
 47. Mr. A. bought a horse for \$90, and sold him for \$105. What per cent did he gain?

48. Seven gallons of oil leaked from a barrel holding 42 gallons. What per cent was there left?

49. A boy bought apples at the rate of 3 for 2 cts., and sold them at the rate of 2 for 3 cts. What per cent did he gain?

50. A merchant sold a carpet for \$35, gaining 25%. What did it cost?

51. A jeweler sold a watch for \$27, losing 10%. How much did he lose?

52. When any article is sold for $\frac{3}{4}$ of its cost, what is the per cent gain?

53. When any article is sold for $\frac{5}{6}$ of its cost, what is the per cent loss?

54. What is the greatest per cent loss that any article may have? The greatest per cent gain?

55. Paid \$10 for a dog, and sold him so as to gain 100%. What did I receive for him?

56. By selling peaches at 3 for 5 cts., I gain 25%. What must I sell them for to gain 50%?

57. Paid \$1 for 8 qts. of berries, 2 qts. of which soured and became unsalable. What was my per cent loss?

58. If I buy apples at the rate of $1\frac{1}{2}$ for 1 ct., and sell them at the rate of $1\frac{1}{2}$ cts. apiece, what is my per cent gain?

59. One half of my property is what per cent of $\frac{3}{4}$ of it?

60. Sold a farm for \$500 less than it cost, thereby losing 8%. What did it cost?

61. A newsboy gains $\frac{1}{3}$ of the cost of his papers by selling them at 4 cts. What per cent would he gain by selling them at 5 cts.?

62. Coal costs \$4.50 per ton, and sells for \$5.25. What is the per cent gain?

63. If the materials in a glass of soda water cost $1\frac{1}{4}$ cts. and it sells for 5 cts., what is the per cent gain?

64. I paid \$200 for a horse. How much shall I ask for him that I may gain 20%?

65. I paid \$800 for a house, and after spending \$200 for repairs, I sold it for \$1250. What per cent did I gain?

66. Sold a quantity of fruit at a profit of $16\frac{2}{3}\%$, and gained \$150. What did the fruit cost me?

67. Sold a farm for \$1200, losing 20%. For how much should I have sold it to have gained 20%?

68. I sold 2 horses at \$100 each. On the first I gained 20%, on the second I lost 20%. How much did I gain or lose on the whole transaction?

WRITTEN PRACTICE.

Find the Gain on the following:

1. On merchandise that cost \$720, and sold at a gain of 24%.
SUGGESTION.—Cost \times Rate = Gain.
2. On goods that cost \$840, and sold at a gain of $16\frac{2}{3}\%$.
3. On goods that cost \$1.60, and sold at a gain of $62\frac{1}{2}\%$.
4. On goods that cost \$120, and sold at a gain of $33\frac{1}{3}\%$.

Find the Loss on the following:

5. On goods that cost \$240, and sold at a loss of $37\frac{1}{2}\%$.
SUGGESTION.—Cost \times Rate = Loss.
6. On goods costing \$36, to lose $33\frac{1}{3}\%$.
7. On goods costing \$.75, to lose $16\frac{2}{3}\%$.
8. On goods costing \$.84, to lose $12\frac{1}{2}\%$.

Find Selling Price on the following:

9. On goods costing \$120, and selling at a gain of $37\frac{1}{2}\%$.
SUGGESTION.—Cost \times Rate = Gain, Gain + Cost = Selling Price;
or Cost \times (1 + Rate) = Selling Price.
10. On goods that cost \$.30, and sold at a gain of 25%.
11. On a horse that cost \$55, and sold at a profit of 60%.
12. On sheep costing \$4 per head, and sold at a profit of 40%.
13. On goods that cost \$240, and sold at a loss of $8\frac{1}{3}\%$.
14. On goods that cost \$75, and sold at a loss of 24%.
15. On goods that cost \$.78, and sold at a loss of $2\frac{1}{2}\%$.
16. On goods costing \$120, and sold at a loss of $11\frac{1}{2}\%$.

Find Gain or Loss per cent on the following:

17. On goods costing \$160, that sold for \$180.
SUGGESTION.—Gain or Loss \div Cost = Rate per cent.
18. On goods that cost \$180, and sold for \$160.
19. On goods costing \$160, that sold for \$172.
20. On a house that cost \$2400, and sold for \$2238.

Find Cost Price and Selling Price of the following:

21. On goods that sold at a gain of \$76, or $12\frac{1}{2}\%$ gain.
SUGGESTION.—Gain or Loss \div Rate = Cost. Cost + Gain (or — Loss) = Selling Price.

22. On goods that sold at a loss of \$114, or $28\frac{1}{2}\%$ loss.

23. On goods selling at a profit of $33\frac{1}{3}\%$, or \$84.

24. On goods selling at a loss of \$145, or 5%.

Find the Cost Price of the following:

25. On goods selling for \$1260, at a gain of 5%.

SUGGESTION.—Selling Price $\div 1 + \text{Rate}$, or $1 - \text{Rate} = \text{Cost Price}$.

26. On a house and lot that sold for \$3900, at a gain of $8\frac{1}{3}\%$.

27. On a wagon selling for \$100, at a gain of $11\frac{1}{3}\%$.

28. On a sale of dry goods for \$1174.92, at a loss of 7%.

29. On a team of horses that sold for \$200.25, at a loss of 11%.

30. On the sale of a yoke of oxen for \$74.88, at a loss of 22%.

PROMISCUOUS WRITTEN EXERCISE.

	(Base.) Cost.	(Percentage.) Selling Price.	(Percentage.) Loss.	(Percentage.) Gain.	(Rate per cent.) Rate of Loss or Gain
31.	\$125	\$150		?	?
32.	275	250	?		?
33.	280	300		?	?
34.	480	360	?		?
35.	325	?	25		?
36.	240	?	60		?
37.	327.50	?		32.75	?
38.	329.60	?		16.48	?
39.	?	?	75		5%
40.	?	?	125		$6\frac{1}{4}\%$
41.	?	?		328	8%
42.	?	?		121	$16\frac{1}{3}\%$
43.	?	?	$3.33\frac{1}{3}$		$83\frac{1}{3}\%$
44.	?	?	$86\frac{1}{2}$		40%
45.	?	180		?	20%
46.	?	1140		?	25%
47.	?	1080		?	44%
48.	?	$8.33\frac{1}{3}$?		$8\frac{1}{3}\%$
49.	?	9595		?	$16\frac{1}{3}\%$
50.	?	650	50		?
51.	?	2944		384	?
52.	?	3200	800		?
53.	?	1440	560		?
54.	?	880		280	?

The above exercise should be conducted on the plan suggested on page 152.

PROMISCUOUS WRITTEN PROBLEMS.

55. Bought a horse for \$100, paid \$25 for his keeping and sold him for \$150. What per cent did I gain on the entire investment?

56. A grocer gained \$50 by selling a car load of potatoes at a profit of $12\frac{1}{2}\%$. How much did he receive for them?

57. Tea sold at 75 cts. per pound produces a loss of $16\frac{2}{3}\%$. What would be the gain per cent if sold at \$1 per lb.?

58. If I pay \$800 per acre for a piece of land 40 rods square and, after opening a street 4 rods wide through the center of it, I divide the land on each side into lots 4 rods wide, what per cent will I gain on the investment by selling the lots at \$600 each, my expenses being \$1000? How much land is in each lot?

SUGGESTION.—\$1000 is not a part of the investment.

59. Mr. B. bought a section of land of the government, paying \$2.50 per acre. He sold $\frac{3}{4}$ of it at \$7.50 per acre, and the remainder at \$8 per acre. What was his gain per cent?

60. Paid \$4 per ream for paper, and sold it at the rate of 4 sheets for 5 cts. What per cent did I gain?

61. A sold a farm to B and gained 20%. B sold it to C for \$11000, losing 20%. What did the farm cost A?

62. A merchant's expenses are as follows: Rent \$600, clerk hire \$1500, sundry items \$400, advertising \$250. What must be the amount of his annual sales, in order that they may average $16\frac{2}{3}\%$ above the cost, and allow him a gain of \$2000 above expenses?

63. If I buy berries by the quart dry measure, and sell them at the same price per quart wine measure, what per cent profit do I make?

64. If I buy berries by the quart wine measure, and sell them at the same price per quart dry measure, what per cent do I lose?

65. I sold two houses at \$6000 each. On one I gained 25%, and on the other I lost 25%. Did I gain or lose on the whole and how much?

66. I bought two houses at \$6000 each. In the sales I cleared 25% on one, and lost 25% on the other. Did I gain or lose on the whole, and how much?

67. A house and lot were offered for sale at an advance of 50% above their cost; but the agent sold the house at 20% below the asking price, receiving \$500 more than the cost. What is the owner's per cent gain, after allowing the agent 2% for his services?

68. If sugar costs $4\frac{1}{2}$ cts. per lb., and loses 10% in drying out, down weights, and bad accounts; how much must a grocer ask for the sugar in order to make a net profit of $12\frac{1}{2}\%$?

69. If a grocer sells $\frac{1}{2}$ of a bushel of apples for the same that he paid for $\frac{3}{4}$ of a bushel, what is his per cent gain?

70. Which merchant makes the greater average gain and how much, A, who sells at an average profit of $12\frac{1}{2}\%$, changing his stock every four months; or B, who sells at an average profit of $16\frac{2}{3}\%$, changing his stock every six months?

71. A grocer bought 200 barrels of apples at \$1.50 per barrel, and sold 75 barrels at a profit of $16\frac{2}{3}\%$, and 100 barrels at a profit of 25%. At what price per barrel must he sell the remainder, to net 20% on the whole deal?

72. A speculator bought a farm for \$9000. It cost him \$350 a year to pay the taxes and cost of repairs. If he receives a rental of \$3375 for a three years' lease, what rate per cent per annum does he make on the investment?

73. A merchant bought carpeting at \$1.20 per yard, and marked it to sell at an advance of 50%, but sold at a reduction of 10% from his asking price, thereby making \$39.90. How many yards did he sell?

74. A man sold goods at an advance of 40%; he lost 10% of his sales in bad debts, and paid 10% for collecting. What was his net gain per cent?

75. I invest and sell at a loss of 25%; I invest the proceeds of sale and sell at a gain of 25%. Do I gain or lose on the two transactions, and what per cent?

76. I sell at 25% gain, and invest the proceeds and sell at an advance of 10%. I again invest the proceeds and sell at

a loss of 5%, and find that I have \$7440.40. What amount did I start with?

77. A merchant bought 320 yards of broadcloth at \$4 per yard, and sold 25% of it at a profit of 25 cts. per yard, and 50% of the remainder at \$4.50 per yard. At what price per yard must he sell the remainder to net $37\frac{1}{2}\%$ on the whole?

78. My agent sold an invoice of flour at an advance of $16\frac{2}{3}\%$. I instructed him to invest the sum in corn, which he sold at a profit of 30%. I realized on the two transactions \$537, the agent retaining \$75 of the last sale for his work. What did he receive for each sale?

79. A man bought $\frac{2}{3}$ of a farm and sold $\frac{1}{4}$ of his share for \$8100, which was at a profit of $33\frac{1}{3}\%$. What was the cost of the farm?

80. What is my per cent gain, if 20% of what I receive for an article is gain?

81. I bought land at \$60 per acre. How much must I ask an acre for it, that I may allow a discount of $33\frac{1}{3}\%$ and still gain 25%?

82. I bought land at \$36 per acre, intending to ask \$72 per acre for it. What abatement may I allow and still make 25% profit?

83. I bought land at \$120 per acre, intending to ask \$250 per acre, and allow a discount of 30%. What would be my per cent profit?

84. A grocer bought 400 barrels of apples at \$2.40 per barrel. He sold 90 barrels at a gain of $16\frac{2}{3}\%$, 164 barrels at a gain of $33\frac{1}{3}\%$, 100 at a loss of 5%, and the remainder at \$1.50. What was his per cent of loss or gain?

85. A merchant's loss at wholesale was 4%, his retail price being 25% in advance of the wholesale price. What was his per cent gain in retail?

86. A jeweler bought a watch for \$24. How much must he ask for it, that he may offer an abatement of 25% and still net 25% profit?

87. A merchant's sales for one year were \$50000, $\frac{1}{4}$ of the sales being at a profit of 25%, $\frac{1}{3}$ at a loss of $33\frac{1}{3}\%$, and the balance at a gain of 50%. What did the goods cost?

88. A merchant buys a bill of goods for \$7500 and sells $\frac{1}{2}$ at a profit of 25%, he sells the rest at an advance of 10 cts. per yard, receiving 40 cts. per yard. What is his total profit?

89. A jeweler sold by mistake gold by avoirdupois weight. What was his per cent loss and what was the purchaser's per cent gain?

90. A grocer and tailor sell to each other at cost. The tailor sells the grocer a suit of clothes, the profit on which at \$75 is $33\frac{1}{3}\%$. The grocer pays in tea and coffee in equal sums, the profit on the tea at 60 cts. being 20%, and the profit on the coffee at 28 cts. being $16\frac{2}{3}\%$. How many pounds of each does the tailor receive; which one throws off the larger amount and how much?

91. I invest and sell at a loss of 15%; I invest the proceeds again and sell at a gain of 15%. Do I gain or lose on the two speculations, and how many per cent?

92. I sold at 8% gain, invested the proceeds and sold at an advance of $12\frac{1}{2}\%$. I invested the proceeds again and sold at 4% loss, and quit with \$2332.80. What was the sum I started with?

93. When beef is selling at \$12.50 per barrel, I lose 20%. What would be my loss or gain if sold at \$20 per bbl.?

94. A real estate dealer sold two building lots for \$6000, thereby gaining 20% on one, and losing 20% on the other. The selling price of the first lot was 50% more than the other. How much was the cost of each?

95. A merchant's sales for two years were \$39600; his sales this year were 25% more than his sales last year. What were his sales last year?

COMMISSION AND BROKERAGE.

153. **Commission and Brokerage** relate to the percentages allowed an agent for the transaction of business. They are estimated at a certain per cent on the sales or purchases.

154. A **Commission Merchant** is an agent who buys and sells for another, called a principal.

155. The **Consignment** is the property sent for sale.

156. The **Consignor** is the party who ships the property.

157. The **Consignee** is the party to whom the shipment is made.

158. The **Net Proceeds** is the amount left after all deductions have been made.

159. The **Prime Cost** is the first cost or sum paid for the goods by an agent.

160. The **Gross Cost** is the prime cost plus all charges for buying.

161. An **Account Sales** is a statement rendered to the principal by the agent, showing sales and expenses.

162. **Guaranty** is an additional commission allowed for guarantying a time sale.

163. An **Account Purchase** is an agent's statement of goods bought and charges added.

164. **Commission** is the compensation allowed the agent for his work.

165. **Brokerage** is a compensation allowed for negotiating a sale.

The commission or brokerage is estimated upon the entire amount of the sale or purchase, usually being a per cent of the selling or purchase price, but in some kinds of merchandise, especially grain and produce, the commission is frequently a certain amount per pound, bushel, barrel, etc., according to the standard of estimating; as, grain by the bushel, cotton by the bale, flour by the barrel.

The same problems are involved in commission that have already been explained in percentage, the only change being a change of terms, as follows:

Amount of Sale or Purchase = Base.

Commission or Brokerage = Percentage.

Rate of Com. or Bro. = Rate.

Net Proceeds or Gross Cost = Percentage.

MENTAL PRACTICE.

1. What is an agent's commission on sales amounting to \$600 at 2%?

SUGGESTION.—2% of \$600 is the Commission.

2. What is an agent's commission on a purchase of goods amounting to \$500 at $2\frac{1}{2}\%$?

3. An agent's commission is 3%. How much does he receive on a sale of 120 bushels of oats at 40 cts. per bushel?

4. What commission must be paid an agent for collecting a bill of \$750 at 2%?

5. What commission does an agent earn by collecting 75% of a claim of \$800 at $\frac{3}{4}\%$ commission?

6. Find the commission at $1\frac{1}{2}\%$ on a sale of grain for \$640.

7. An agent sells 12 shares of stock at \$100 per share, and 15 shares of railroad stock at \$120 per share. What is his commission at $\frac{3}{4}\%$?

8. A commission merchant sold 325 barrels of apples at \$4 per barrel. What is his commission at 3%?

9. My agent sold 120 bushels of corn at 50 cents per bushel, and 240 bushels of wheat at $62\frac{1}{2}$ cents per bushel. What is his total commission at $2\frac{1}{2}\%$?

10. What is the commission on the sale of a farm for \$6400 at 3%?

11. How much do I receive from sales amounting to \$300, less a commission of 2%?

SUGGESTION.—Subtract the Commission from the Sales.

12. My agent sells flour for \$725, and deducts a commission of 2%. How much do I receive?

13. Find the net proceeds of a sale of flour amounting to \$720, less a commission of 3%.

14. An agent sells 150 bushels of wheat at 80 cents a bushel, and deducts a commission of 2% and a drayage charge of \$5. What are the net proceeds?

15. How much must I send my broker who buys 12 shares of stock at \$100 per share, and who charges a brokerage of $\frac{3}{4}\%$?

SUGGESTION.—Add the Brokerage to the cost of stock.

16. How much must I send an agent to pay for the purchase of 400 bushels of wheat at 75 cents a bushel and his commission of $2\frac{1}{2}\%$?

17. What is the commission, if an agent receive \$408 to invest, after deducting a commission of 2%?

SUGGESTION.—The sum received, \$408, is $\frac{188}{100}$ of the sum invested.

18. What is the investment, if an agent receives \$618 to invest, after deducting a commission of 3%?

19. An agent receives \$515 to invest in apples after deducting his commission of 3%. How many barrels does he buy at \$2 per barrel?

20. An agent deducted \$72 as commission at 2% for selling corn. How much did he receive for the corn?

SUGGESTION.—\$72 is 2 per cent of the Sales.

21. An agent's commission is \$50 for selling land. What were the sales, if his rate of commission is $2\frac{1}{2}\%$?

22. An agent is paid \$8 for buying corn at a commission of 2%. How many bushels does he buy at 25 cents per bushel?

23. An agent receives \$40 for buying wheat at $3\frac{1}{3}\%$ commission. How many bushels did he buy at 50 cents per bushel?

WRITTEN PRACTICE.

Find Commission on the following:

1. The sales amounting to \$960, the rate of commission being 3%.

SUGGESTION.—Sale or Purchase \times Rate = Commission.

2. The sales amounting to \$3125, the rate of commission being $2\frac{1}{2}\%$.

3. The sale of 420 bu. of wheat at $87\frac{1}{2}$ cents per bushel, the rate of commission being $2\frac{1}{3}\%$.

4. The purchase of 560 bu. of barley at $56\frac{1}{4}$ cents per bu., 824 bu. of oats at 32 cents per bu., 428 bu. of corn at $48\frac{3}{4}$ cents

per bu., and 287 bu. of wheat at $75\frac{1}{4}$ cents per bu., the rate of commission being 2%.

Find Net Proceeds of the following:

5. The sales amounting to \$750, and the rate of commission being 4%.

SUGGESTION.—Sales—Charges = Net Proceeds.

6. The sales amounting to \$864, and the rate of commission being $3\frac{1}{4}$ %.

7. The sale of 2465 pounds of pork at 9 cents per lb., the rate of commission being $2\frac{1}{4}$ %.

8. The sales of 460 bushels of rye at $\$.62\frac{1}{2}$ per bushel, 387 bushels of flax seed at \$1.04 per bushel, and 529 $\frac{1}{2}$ bushels of wheat at $\$.83\frac{1}{8}$ per bushel. Charges: Rate of commission 3%, freight \$176.75, insurance $\frac{1}{8}$ % of sales, drayage \$18.50, inspection \$4.

Find the Amount of Sales of the following:

9. The net proceeds being \$980, and the rate of commission 2%.

SUGGESTION.—1. Net Proceeds \div (1 — Rate) = Sales.

2. (Net Proceeds \div charges) \div (1 — Rate) = Sales.

10. The net proceeds of a shipment being \$291, and the rate of commission 3%.

11. The account sales of a shipment of corn show the net proceeds to be \$1095, the rate of commission $2\frac{1}{2}$ %, and other charges amounting to \$75.

12. I received a check for \$1504 as the net proceeds of a consignment of butter and cheese, the rate of commission 2%, freight \$24, drayage and cooperage \$16, and a guaranty commission of $1\frac{1}{2}$ %.

Find the Rate of Commission in the following:

13. Sales amounting to \$1000, the net proceeds being \$970.

SUGGESTION.—1. Sales — Net Proceeds = Commission. Commission \div Sales = Rate of Com.

2. Sales — (Net Proceeds \div Charges) = Commission. Commission \div Sales = Rate of Com.

14. The sale of 500 bushels of barley at $\$.63\frac{1}{4}$ per bushel, \$303.60 being remitted as the net proceeds.

15. I received \$939.24 as the net proceeds of a sale of 8960 lbs. beef at 11 cents per lb., after deducting \$36.50 freight charges.

16. \$27134.16 was remitted me as the net proceeds of a sale of a section of land, 50% being sold at \$48 per acre, 50% of the remainder at \$37.50 per acre, and the balance at \$42.50 per acre. The charges being for surveying and abstracting \$350.

Find the Rate of Commission in the following:

17. The net proceeds of a sale being \$291, and the commission \$9.

SUGGESTION.—1. Net Proceeds + Commission = Sales. Commission ÷ Sales = Rate of Com.

2. Net Proceeds + Com. + Charges = Sales. Commission ÷ Sales = Rate of Com.

18. The net proceeds being \$877.50, and the commission \$22.50.

19. The net proceeds of the sale of a farm being \$10971.50, the commission \$55.50, and other charges \$73.

20. The net proceeds of a consignment of cotton being \$6920.83, the commission \$145.27, freight \$125, and insurance 1% of sales.

21. The net proceeds of a shipment of molasses being \$1044.60, and the charges being as follows: Commission \$25.20, freight \$90, drayage \$75, guaranty commission 2%.

Find the Gross Cost of the following:

22. Goods costing \$360, commission at 5%.

SUGGESTION.—1. Cost + Commission = Gross Cost.

2. Commission ÷ Rate = Cost.

23. Of the purchase of 80 baskets of peaches at \$1.25 per basket, commission 3%, and other charges \$6.50.

24. Of the purchase of 96 cases of eggs at \$4.50 per case, 24 dozen dressed chickens, 658 lbs. at $14\frac{1}{2}$ cts. per lb., and 9 firkins of butter averaging 54 lbs. each, at 18 cts. per lb. Charges: Commission 3%, freight \$5.25, drayage \$1.75.

25. The commission being \$12.50, the rate of commission 4%.

26. Of the purchase of a shipment of cheese, the commission being \$37.50, the rate of commission 2%, and other charges amounting to \$42.50.

27. Of the purchase of a consignment of cotton, the commission being \$332.50, the rate of commission $2\frac{1}{2}\%$, the rate of guaranty $1\frac{1}{2}\%$, and other charges amounting to \$145.50.

Find the Cost or Investment in the following:

28. I send my agent \$408 to invest, after deducting a commission of 2%.

SUGGESTION.—1. Gross Cost $\div (1 + \text{Rate}) = \text{Cost, or Prime Cost.}$

2. Gross Cost $- \text{Charges} \div (1 + \text{Rate}) = \text{Cost.}$

29. Smith & Co. send their agent \$2568.50 to invest in wheat, after deducting a commission of 3%, and paying incidental expenses of \$45.

30. The gross cost is \$4119.20, the rate of commission $2\frac{1}{2}\%$, and other charges \$19.20.

PROMISCUOUS WRITTEN EXERCISE.

	Sales.	Rate of Com.	Commission.	Net Proceeds.	Other Charges.	Gross Cost.	Prime Cost.
31.	\$2500.00	2%	?	?			
32.	\$ 936.50	4%	?	?			
33.	\$ 584.20	$2\frac{1}{2}\%$?	?	\$24.30		
34.	\$ 927.38	3%	?	?	\$35.20		
35.	?	3%	\$.93.30	?			
36.	?	$2\frac{1}{2}\%$	\$150.00	?			
37.	?	$2\frac{1}{4}\%$	\$ 27.00	?	\$23.60		
38.	?	4%	\$ 84.40	?	\$16.20		
39.	?	?	\$ 12.50	\$612.50			
40.	?	?	\$ 18.00	\$582.00			
41.	?	?	\$ 15.00	\$470.28	\$14.72		
42.	?	?	\$ 16.40	\$800.15	\$ 3.45		
43.	\$2400.00	?	\$ 12.00	?			
44.	\$1300.00	?	\$ 6.50	?			
45.	\$ 464.00	?	\$ 23.20	?	\$12.15		
46.	\$ 372.00	?	\$ 9.30	?	\$ 9.64		
47.	\$ 840.00	?	?	\$821.10			
48.	\$ 436.00	?	?	\$430.55			
49.	\$ 284.00	?	?	\$272.54	\$ 4.36		
50.	\$ 825.00	?	?	\$786.13	\$14.12		
51.	?	?	?			\$489.60	\$480.00
52.	?	?	?			\$580.75	\$575.00

Sales.	Rate of Com.	Com- mission.	Net Proceeds.	Other Charges.	Gross Cost.	Prime Cost.
53.	?	?		\$12.00	\$640.68	\$624.00
54.	?	?		\$13.62	\$348.37	\$325.00
55.	2%	?			?	\$324.50
56.	4%	?			?	\$187.75
57.	5%	?		\$11.90	?	\$295.60
58.	2½%	?		\$ 4.37	?	\$324.40
59.	?	\$10.95			?	\$438.00
60.	?	\$ 7.25			?	\$362.50
61.	?	\$26.38		\$35.25	?	\$527.60
62.	?	\$ 6.16		\$ 9.85	?	\$246.40
63.	2%	?			\$510.00	?
64.	1½%	?			\$609.00	?
65.	5%	?		\$13.00	\$353.83	?
66.	2½%	?		\$13.50	\$298.86	?
67.	?	\$11.25			\$386.25	?
68.	?	\$ 9.55			\$391.55	?
69.	?	\$10.52		\$ 8.24	\$281.76	?
70.	?	\$ 8.40		\$ 3.75	\$348.15	?

Follow the suggestions given on page 152.

PROMISCUOUS WRITTEN PROBLEMS.

71. What is an agent's commission for selling goods to the amount of \$1250 at 3%?

72. An agent sells merchandise to the amount of \$750, and retains \$15 for his commission. What rate does he charge?

73. An agent charges 2½% on the sales of a consignment of cigars, receiving for his commission \$100. The other expenses being \$75, what are the consignor's net proceeds?

74. How much can a commission merchant invest from a remittance of \$1230, after deducting his purchasing commission of 2½%?

75. A consignor's net proceeds are \$1500.50, after allowing \$100 expenses and 3% commission. What was the amount of the sale?

76. Paid a cotton broker \$50 for selling 50 bales of cotton, averaging 500 pounds each, at 8 cents per pound. What is the rate of commission?

77. A broker charges $\frac{1}{4}\%$ brokerage for purchasing stock for me. How much does he purchase, his brokerage being \$24.25?

78. My commission merchant sent me \$1100 as the net proceeds of a consignment, after deducting \$125 for charges and 2% commission. What was the amount of the sale?

79. The net proceeds of a consignment are \$675.50, and the rate of commission $3\frac{1}{2}\%$. What is the amount of the sale?

80. How many pounds of sugar at $4\frac{1}{4}$ cts. per lb. can be bought by an agent for \$897.75, after deducting \$5 paid for drayage, and a purchasing commission of 5%?

81. An agent collected part of an account for me, and, after deducting his commission of 5%, he sent me \$427.50. What per cent of the original debt of \$900 remains unpaid?

82. My agent in Birmingham bought 100 barrels of sweet potatoes for me at \$1.75 per barrel. His charges were: Commission $2\frac{1}{2}\%$, drayage \$14.50, sundry charges \$2.75. For what sum must he draw, allowing 75 cents for collection?

83. I sent my agent at Big Rapids \$1000 to invest in apples at \$1.75 per barrel, after deducting charges as follows: Commission 4%, drayage 5 cents per barrel. How many barrels did he buy, and what was the unexpended balance on hand?

84. A speculator bought 2500 bu. of wheat, at 75 cents per bushel. He immediately shipped the same to Chicago, to be sold on a commission of 3%, other charges being: Freight \$175, drayage \$23, insurance $\frac{1}{8}\%$ of sale. How much did he gain, the wheat being sold at 95 cents per bushel?

85. A commission merchant sells 600 bu. of oats at 32 cents per bu. for cash, 750 bu. of rye at 64 cents per bu., on account 30 da. What is his total commission, if, besides the regular commission of 3%, he charges $2\frac{1}{2}\%$ for guaranty?

86. An agent charged me 5% for selling corn, and 2% for investing the proceeds in cotton. His commission amounted to \$280; what was the selling price of the corn?

87. My agent in New Orleans charges 3% for buying and 2% for selling, and an additional commission of 2% for guarantying quality of goods purchased. I make him a shipment of wheat with instructions to sell, and invest in a shipment of cotton to Liverpool. What was the selling price of the wheat, and what was the cost of the cotton, his total commission being \$1031.25?

88. A commission merchant bought for his principal 12000 bu. wheat at \$1.20 per bu., and immediately sold it at \$1.30, investing the proceeds in the purchase of oats in car load lots of 1000 bu. each, at \$.35 per bu. His commission being 2% for buying, and $2\frac{1}{2}\%$ for selling. Find the following: Number of car loads, unexpended balance, and total commission.

89. A commission merchant received 4000 bushels of wheat and 24000 pounds of beef, with instructions to sell, and invest \$2500 of the proceeds in cotton, and remit the balance after deducting the charges. He sold the wheat at $62\frac{1}{2}$ cents per bushel, and the beef at $9\frac{1}{2}$ cents per pound. He paid \$375 for freight, \$24 for drayage, and charged a commission of $\frac{7}{8}$ cents per bushel on the wheat, $2\frac{1}{2}\%$ on the beef, and $2\frac{1}{4}\%$ on the cotton. How much did he remit?

90. Find the net proceeds of the following account sales:

CHICAGO, Jan. 1, 1896.

Sold for account of SAM'L B. WILLEY,

By W. J. DURAND & Co., Commission Merchants.

1895.							
Nov.	1	250 bu. Beans	1.06	***			
"	16	400 lbs. Cheese	.10	**			
Dec.	4	40 bbls. Flour	3.80	***			
"	28	1200 bu. Wheat	.64	***		****	
Charges.							
Freight \$84.60.		Drayage \$12.85.		Storage \$24.30.			
Inspection \$4.		Insurance $\frac{1}{8}\%$.		Commission 3%.			
				Total charges		***	**
				Net proceeds		****	**

MARKING GOODS.

166. In **Marking Goods**, merchants adopt various methods to prevent the gain from being known to the purchasers.

167. The **device** or key may consist of letters, words, or symbols containing 10 characters, one to represent each letter and an extra one to be used as a repeater.

b l a c k h o r s e x
1 2 3 4 5 6 7 8 9 0

By this scale, goods costing \$2.35 and selling at \$4.55 would be marked lak.

c.kx

MENTAL PRACTICE.

Mark the following, using "Cash profit" as a key, with x as a repeater:

	Cost.	Selling		Cost.	Gain.
1.	\$3.20.....	\$5.25	7.	\$3.60.....	25%
2.	\$1.20.....	\$3.45	8.	\$1.25.....	20%
3.	\$.75.....	\$1.26	9.	\$.80.....	50%
4.	\$.60.....	\$1.50	10.	\$.12.....	33 $\frac{1}{3}$ %
5.	\$.16.....	\$.32	11.	\$.06.....	16 $\frac{2}{3}$ %
6.	\$.08.....	\$.12	12.	\$.18.....	33 $\frac{1}{3}$ %

WRITTEN PRACTICE.

Mark the following, using "Don't be lazy" for the cost price, and "Now be sharp" for the selling price:

- | | Cost. | Selling. | | Cost. | Gain. |
|----|-------------|----------|----|-------------|--------------------|
| 1. | \$1.20..... | \$2.35 | 5. | \$1.50..... | 60% |
| 2. | \$3.40..... | \$5.25 | 6. | \$1.44..... | 16 $\frac{2}{3}$ % |
| 3. | \$.56..... | \$.84 | 7. | \$.36..... | 33 $\frac{1}{3}$ % |
| 4. | \$.36..... | \$.72 | 8. | \$3.50..... | 10% |
9. Hats were bought at \$18 per dozen, and sold at \$2.25 each. Mark them by the key "Hard moneyes."
10. Cassimere costing \$2.40 per yard, was sold at a profit of 35%. Mark the goods by the key "Gambolines."
11. Mark shoes, bought at \$27 per dozen, and sold at a profit of 33 $\frac{1}{3}$ %, using the key "Cumberland."
12. By selling broadcloth at \$9.60 per yard, I gain 25%. Mark goods by the key "Now be sharp."

TRADE DISCOUNT.

168. Trade Discount is an allowance made to purchasers from catalogue prices.

169. Many classes of merchandise have printed lists of prices, which are higher than the actual selling prices. These are so arranged, that a change of price lists may not be required every time that the value of these goods change. The selling price is found by taking a certain discount from the printed price. If the goods rise in value, the rate of discount is decreased; if they fall in value, the rate of discount is increased.

170. Price Lists are usually estimated on the basis of the credit sales, so that the cash buyer may have a second discount for cash, thus making it an inducement to him to pay cash.

171. One buyer may purchase large quantities of certain articles, and thereby receive a third discount for quantity. So that certain articles may have a list price, and be discounted to find the selling price, and this price may be again discounted to large purchasers, and the proceeds of this second discount be again discounted to cash purchasers of large quantities.

172. The wholesale and jobbing business is from 6 months to 1 year in advance of the retail trade. The retail merchant buys his goods several months before his sales begin, consequently his bills from the wholesaler or jobber bear two dates, one the date of sale, and the other the date from which the sales begin to the general trade. His cash discount term begins with the latter date. Since he holds these goods several months before he commences the sale of them, he has the privilege of paying and receiving an additional discount, generally from $\frac{1}{2}\%$ to 1% per month, for prepayment; this is called anticipating bills.

List price = Base.

Rate of Discount = Rate.

Discount=Percentage.

Net Proceeds, or net=Percentage.

EXAMPLE.

What single discount is equivalent to 25%, 20% and 5% off?

SOLUTION.

1.00=Base.	1.00	1.00	1.00
.25=25% of 1.00	.25	.20	.05
<u>.75=Net after first discount.</u>	<u>.75</u>	<u>.80</u>	<u>.95</u>
.15=20% of .75.	1.00	— .57	= .43
<u>.60=Net after 2d discount.</u>			
.03= 5% of .60.			
<u>.57=Net after 3d discount.</u>			
1.00— .57= .43, or 43%=the single discount.			

OR

WRITTEN PRACTICE.

What single discount is equivalent to the following discount series?

- | | |
|-----------------------------|-----------------------------------|
| 1. 10% and 10% | 11. 20%, 25% and 10% |
| 2. 20% " 10% | 12. 25%, 33 $\frac{1}{3}$ % " 10% |
| 3. 10% " 5% | 13. 20%, 20% " 10% |
| 4. 25% " 20% | 14. 50%, 20% " 5% |
| 5. 33 $\frac{1}{3}$ % " 10% | 15. 30%, 20% " 10% |
| 6. 50% " 50% | 16. 20%, 10% " 2% |
| 7. 16 $\frac{2}{3}$ % " 10% | 17. 10%, 10% " 10% |
| 8. 50% " 25% | 18. 10%, 5% " 2% |
| 9. 12 $\frac{1}{2}$ % " 75% | 19. 20%, 5% " 2 $\frac{1}{2}$ % |
| 10. 45% " 10% | 20. 25%, 12 $\frac{1}{2}$ % " 5% |

EXAMPLE.

What is the net price of a bill of dry goods listing \$720, less discounts of 25% and 20%?

SOLUTION.

1.00=Base.	100%—25%=75%
.25=25% of 1.00.	100%—20%=80%
<u>.75=Net, or 2d Base.</u>	<u>.80</u> × <u>.75</u> = <u>.60</u>
.15=20% of .75.	.60×\$720=\$432
<u>.60=Net of 1.00, or 3d Base.</u>	
.60×\$720=\$432, the net price.	

WRITTEN PRACTICE.

Find the Net price in the following:

LIST PRICE.	DISCOUNT.
21. \$625	10 % and 10 %
22. \$324	25 % and 20 %
23. \$600	$33\frac{1}{3}$ % and 25 %
24. \$720	$16\frac{2}{3}$ % " $33\frac{1}{3}$ %
25. \$327.85	50 % and $12\frac{1}{2}$ %
26. \$348.20	20 %, 25 % and 10 %
27. \$462	10 %, 10 % " 10 %
28. \$850	30 %, 20 % " 10 %
29. \$325.50	25 %, $12\frac{1}{2}$ % " 10 %
30. \$144	20 %, 5 % " $2\frac{1}{2}$ %
31. \$360	50 %, 20 % " 5 %
32. \$ 45.50	60 %, 20 % " $2\frac{1}{2}$ %
33. \$120	65 %, 10 % " 10 %
34. \$324	$33\frac{1}{3}$ %, 10 % " 20 %
35. \$127	$66\frac{2}{3}$ %, 10 % " 5 %
36. \$316.80	40 %, 25 % " 10 %
37. \$415	10 %, 10 % " 5 %
38. \$243.50	5 %, $2\frac{1}{2}$ % " $2\frac{1}{2}$ %
39. \$142	$12\frac{1}{2}$ %, $2\frac{1}{2}$ % " $2\frac{1}{2}$ %
40. \$421	$37\frac{1}{2}$ %, 5 % " $2\frac{1}{2}$ %

EXAMPLE.

What is the list price of a bill of groceries that net \$432, after allowing discount of 25% and 20%?

SOLUTION.

OR

$$1.00 = \text{Base.}$$

$$.75 \times .80 = .60$$

$$.25 = 25\% \text{ of } 1.00$$

$$\$432 \div .60 = \$720$$

$$.75 = \text{Net, or 2d Base.}$$

$$.15 = 20\% \text{ of } .75.$$

$$.60 = \text{Net, or 3d Base.}$$

$$\$432 = \text{Net of Bill.}$$

$$\$432 \div .60 = \$720, \text{ List price.}$$

WRITTEN PRACTICE.

Find the List price in the following:

NET PRICE.	RATE OF DISCOUNT.
41. \$ 450	10 %
42. \$ 525	25 %
43. \$ 420.75	$66\frac{2}{3}\%$
44. \$1053	10 % and 10%
45. \$ 144.60	$33\frac{1}{3}\%$ " 10%
46. \$ 148.20	25 % " 20%
47. \$ 65.52	40 % " 25%
48. \$ 89.46	20 % " 10%
49. \$ 97.65	$12\frac{1}{2}\%$ " 10%
50. \$ 89.78	30 % " 5%
51. \$ 73.71	25 %, 20 % and 10%
52. \$ 140.51	20 %, 25 % " 10%
53. \$ 106	$33\frac{1}{3}\%$, 10 % " 10%
54. \$ 155.70	50 %, $12\frac{1}{2}\%$ " 10%
55. \$ 256.50	40 %, 25 % " 5%
56. \$ 121.50	40 %, 10 % " 10%
57. \$ 142.50	$16\frac{2}{3}\%$ and 10%
58. \$ 331.50	10 %, 5 % and $2\frac{1}{2}\%$
59. \$ 141.75	20 %, $12\frac{1}{2}\%$ " 10%
60. \$ 188.10	60 %, 10 % " 5%

PROMISCUOUS WRITTEN EXERCISE.

LIST PRICE.	RATE OF DISCOUNT.	NET PRICE.	DISCOUNT.
61. \$640	20%	?	?
62. \$320	25%, 20%	?	?
63. \$240	$33\frac{1}{3}\%$?	?
64. \$120	$33\frac{1}{3}\%$, 10%	?	?
65. \$248	20%, 25%, 10%	?	?
66. \$360	25%, $33\frac{1}{3}\%$, 10%	?	?
67. \$280.60	10%, 5%, $2\frac{1}{2}\%$?	?
68. ?	20%	\$ 240	?
69. ?	20%, 10%	\$ 288	?
70. ?	$33\frac{1}{3}\%$, 10%	\$ 480	?
71. ?	10%, 10%	\$2025	?
72. ?	?	\$ 450	\$ 90

PROMISCUOUS WRITTEN EXERCISE.

LIST PRICE.	RATE OF DISCOUNT.	NET PRICE.	DISCOUNT.
73. ?	?	\$ 360	\$ 40
74. ?	?	\$ 324	\$162
75. ?	?	\$227.50	\$ 97.50
76. ?	15%, 10%	?	\$188
77. ?	10%, 5%	?	\$ 40.31
78. ?	50%, 20%, 10%	?	\$207.36
79. ?	10%, 10%, 10%	?	\$162.60
80. \$484	?	?	\$ 96.80
81. \$375	?	?	\$112.50
82. \$472	?	\$401.20	?
83. \$522	?	\$348	?
84. \$460	?	?	\$ 23
85. \$640	?	?	\$256

Follow the suggestions given on page 152.

PROMISCUOUS WRITTEN PROBLEMS.

86. What is the net value of a bill of goods amounting to \$120 per price list, the rate of discount being $33\frac{1}{3}\%$, and 5%?

87. Find the net of a bill of wire screens amounting to \$1265.70, discount off 25%, 20%, and 10%.

88. Bought a bill of hardware amounting to \$1560: of this \$125 less 10% and 10%; \$268 less 40% and 10%; \$600 less 25% and 20%; the balance of the bill $33\frac{1}{3}\%$ and 10%. What is the net price?

89. The list price of a bill of hose pipes, couplings, etc., was \$760.50. What was the net price, discounts of 25% and 20% being allowed?

90. What is the cost of 15 governors at \$15.50 each, the rates of discount being $33\frac{1}{3}\%$ and 10%?

91. The gross amount of a bill of boots and shoes was \$1260: of which \$200 was subject to discounts of 25% and 10%; \$350 to 20%, 10%, and 10%; and the balance to $33\frac{1}{3}\%$. What was the net of the bill?

92. Bought the following bill of hardware of H. B. & S Aug. 24, 1896, 6% for cash, 5% 10 da., 4% 30 da:

\$ 72	less a discount of 3-10's (10, 10, and 10%)	
\$124	" " " " 25%, 20%, and 10%	
\$324.60	less a discount of 60% and 10%	
\$125.53	" " " " 40% and 25%	
\$163.82	" " " " 60%, 10%, and 5%	
\$224	" " " " 20%, $12\frac{1}{2}\%$, and $2\frac{1}{2}\%$	
\$216.30	" " " " 10%, 5%, $2\frac{1}{2}\%$, and $2\frac{1}{2}\%$	
\$125	" " " " 50% and 50%	
\$178.60	" " " " 25%	
\$194.50	" " " " 75% and 10%	
\$175	" " " " 50%	
\$142	" " " " 60% and 25%	
\$ 36	" " " " $62\frac{1}{2}\%$ and 10%	
\$120	billed	Net
\$142	"	Net

What was the net on the date of sale? How much would pay the bill Sept. 2? How much would pay the bill any time between Sept. 14 and Sept. 23 inclusive?

93. I sold a house and lot at 25% gain. Had this price been discounted 25% I would have lost \$250. What was the cost?

94. Wishing to sell at cost damaged goods that cost \$4 per dozen, I mark them up 25%. What per cent discount must I give?

95. How much better is a single discount of 60%, than a discount series of 25%, 20%, and 15%?

96. I sell bicycles at \$150 each; I am allowed a discount of $33\frac{1}{3}\%$ from \$150. What per cent profit do I make?

97. A piano dealer instructed his clerk to mark a piano, so that by allowing a discount of 25% he would realize a profit of $33\frac{1}{3}\%$. By mistake the clerk marked the piano at \$300, thereby producing a loss of $16\frac{2}{3}\%$ on the sale. What was the intended marking price and what was the loss?

98. A merchant bought goods at 25% and 20% off, and sold at 20% and 10% off. What per cent profit did he make?

99. I bought a bicycle for \$96. I wish to make 25% profit and yet offer 33 $\frac{1}{3}$ % and 10% off. How much must I ask for it?

100. A marked goods to sell at 50 per cent profit; he lost 10 per cent of sales in bad debts, and paid 10 per cent for collecting. What was his net gain per cent?

101. My retail profit is 50% and I wholesale at 20% less than retail. What is my per cent profit at wholesale?

102. Bought hams at 9¢ a pound; the wastage is 10%. What must I ask to gain 30%, and offer a discount of 25%?

103. I buy goods at 25%, 20%, and 10% off. What is the net price of the goods, the total discount being \$211.14?

104. A bill for 25 stoves amounted to \$567 net, after a cash discount of 10% and a list discount of 30% had been allowed. What was the list price per stove?

105. —

CHICAGO, April 28, 1896.

MESSRS. GOODYEAR & Co.,
Indianapolis, Ind.

Bought of H. C. JEVNE & Co.

Terms: 30 days, discount 1% 10 days.

5	bbl. Empire A Sugar 388-21 381-20 382-18 375-19 376-18 1897-96 1801 lb. \$5.04	90 77
10	bbl. Fine Granulated Sugar 374-20 372-19 386-18 343-18 323-19 333-19 342-18 338-19 336-19 324-20 XXXX-XXX XXXX lb. 5.60	XXX XX
25	Boxes Morgan Evaporated Raspberries 25x50 XXXX lb. .20 $\frac{1}{2}$	XXX XX
30	Boxes Harlem Evaporated Blackberries 30x25 XXX lb. .07 $\frac{3}{4}$	XX XX
85	Cases Pacific 1 lb. Salmon 85x4 XXX doz. 1.45	XXX
25	Bags Lot 307 Apricots 1950 lb. .06 $\frac{1}{2}$	XXX XX XXXX XX

What is the net, and how much would settle the bill May 1, 1896?

106.

ST. LOUIS, MO., March 1, 1896.

MESSRS. JONES & BROWN.

Bought of JNO. M. SMYTHE FURNITURE CO.

Terms: Cash 5%, 10 days 2%.

2	doz. Kitchen Tables (each)	\$4.20	100	80		
	Less 10%		10	08	90	72
4	doz. Common Lounges (each)	9.00	XXX			
	Less 15%		XX	XX	XXX	XX
15	10 ft. Walnut Extension Tables	16.00	XXX			
	Less 12½%		XX		XXX	
6	doz. Dining Room Chairs					
	(per doz.)	11.25	XX	XX		
¾	doz. Cottage Bedsteads (each)	4.75	XX	XX		
7	Walnut Marble Top Center Tables	12.50	XX	XX		
			XXX	XX		
	Less 20%		XX	XX	XXX	XX
3	doz. Antique Oak Bedroom Sets					
	(each)	25.00	XXX			
	Less 15%		XXX		XXX	
1	doz. Cherry Bedroom Sets (each)	32.00	XXX			
	Less 25%		XX		XXX	
½	doz. Birds-eye Maple Bedroom Sets					
	(each)	75.00	XXX			
	Less 33⅓%		XXX		XXX	
7	Office Desks, Oak (each)	24.00	XXX			
	Less 2-10's		XX	XX	XXX	XX
¾	doz. Rockers, Upholstered					
	(per doz.)	36.00	XX			
	Less 20%		X	XX	XX	XX
3	doz. Fancy Baby Cabs (each)	11.50	XXX			
	Less 25%		XXX	XX	XXX	XX
					XXXX	XX

What is the net of the bill? How much would settle the bill on date of sale? How much would settle the bill March 11, 1896?

107. Bought of J. V. Farwell & Co., wholesale merchants: 15 bbl. A sugar, each 327-36, at 6¢; 10 sk. Rio Coffee, each 155-3, at 12¢; 9 ch. Young Hyson Tea, each 95-8, at 37½¢; 12 ch. Japan Tea, each 76-12, at 40¢; 14 bx. Laundry Soap, each 74-14, at 4½¢; 12 bbl. Kerosene, each 45 gal., at 14¢. I am allowed a list discount of 20%, and a second discount of 5% on the first three items. What is the net amount of the bill?

108.

OR. NO.	QUAN.	SIZE.	DESCRIPTION.	PRICE.	GROSS.		NET.	
11146	348-9*	1 inch	Pipe	11¾	40	98	13	40
			47½, 4-10's, 5%					
	109-4	5 inch	Pipe	1.42				
			57½, 3-10's, 7½%					
	16	1 inch	Globe Valves	2.90				
			67½, 10, 5%					
	60	1 x 8	Nipples	47				
			85, 7½%					
	75	2 inch	Couplings	60				
			65, 5%					
	25	1 x 6	Flanges	95				
			80, 10%					
	125	1¼	Ells	35				
			75, 10%					
	55	2½x2x1	Tees	1.75				
			65, 10, 5%					
	6		Fin. Water Gauges	12.00				
			55, 10, 2½%					
	10	2¼	Oil Cups	3.25				
			40, 5, 2½%					
	6	3 inch	Lubricators	6.50				
			70, 15%					
	6	2¾	Flue Cleaners	4.25				
			57, 15, 5%					

What is the net of the bill?

*348-9=348 feet 9 inches; the discounts on this item are 47½%, 10%, 10% 10%, 10%, and 5%.

109.

CHICAGO, March 1, 1896.

MESSRS. L. E. BARRETT & Co.,

Bought of FRANKLIN MACVEAGH & Co.

Terms: 4 months, discount 4% 10 days.

5	½ Chests A. R. & Co. No 89 Oolong Tea					
	63, 61, 60, 62, 62,					
	Tare 15 lb. each, 308-75	233 lb.	.59		137	47
10	¼ Chests R. M. Y. No. 72 Gun Powder Tea					
	47½, 46½, 44, 48, 49½,					
	49¾, 49½, 47½, 48½, 48¾,					
	Tare 9 lb. each, XXX ^x -XX	XXX ^x lb.	.27½	XXX	XX	
5	½ Chests R. J. & Co.					
	No. 176 Young Hyson Tea					
	86, 88, 88, 86, 89,					
	Tare 18 lbs. each, XXX-XX	XXX lb.	.32	XXX	XX	
				XXX	XX	

What is the net, and how much would settle the bill March 10, 1896?

110.

CHICAGO, April 25, 1896

MESSRS. GOODYEAR & Co., Minneapolis, Minn.

Bought of HIBBARD, SPENCER, BARTLETT & Co.,
Jobbers of Hardware.

Terms: 30 days: { If paid within 10 days from date,
1 per cent. may be deducted.

Terms: 60 days: { If paid within 10 days from date,
2 per cent. may be deducted.

Salesman, W. J. Durand.

Order No. 53721.

Via C. & N. W.

30 DA. 60 DA.

5	doz. 3½# to 4½# Axes	10	00			50	
2	" #1½ Nail Hammers	7	00			XX	
2	C ¾ x 6½ Carriage Bolts \$2.76	X	XX				
2	C ½ x 5 Carriage Bolts 3.80	X	XX				
		XX	XX				
	40-10-10%	X	XX	X	XX		
1	C ½ x 4 Log Screws	5	25				
2	C ½ x 5 " " 5.85	XX	XX				
3	C ¼ x 2½ " " 2.25	X	XX				
		XX	XX				
	60-5%	XX	XX	X	XX		
2	doz. #10 Hand Saws	18	00			XX	
3	" #1111 Mortise Locks	4	15			XX	XX
1	" #00 Plain Wash Basins					80	
3	" 30 Qt. Heavy Dish Pans	12	00			XX	
2	" #10 6 inch O & B Shears	3	75			X	XX
2	" " 7 " " " "	4	30			X	XX
2	" " 8 " " " " "	4	85			X	XX
2	" 6 inch Wilson's Butchers	2	25			X	XX
2	" 7 " " " " "	3	15			X	XX
2	" 8 " " " " "	4	10			X	XX
2	" #50 Razor Strops	2	25			X	XX
2	gro. #900 Knives and Forks	4	00			X	
2	doz. Putz Pomade		65			X	XX
				XX	XX	XXX	XX
	30 Days XX.XX						
	60 Days XXX.XX						
	\$XXX.XX						

What is the net? How much would settle on May 1, 1896?

III. I bought a case of prints containing 4500 yards at 4¢ per yard, less 5%, and by paying cash in 10 days an additional discount of 2% was allowed. I sold at 5¢ per yard, ½ of the amount being in jobbing lots was discounted at 4%. What per cent profit did I average on the net cost, after allowing for freight and drayage \$12 35, and taking advantage of the additional discount?

112.

January 1, 1897.

MR. A. N. PALMER.

Bought of THE I. W. PIERSON CO., LTD.

Terms: Cash 5%, 10 days 2½%.

30 Casks Hams.

387-75	395-69	402-68	413-76	412-88	392-81
384-72	396-90	364-83	403-83	401-71	400-69
384-80	375-70	382-81	384-92	365-75	390-95
386-72	416-83	420-80	416-83	412-86	412-90
413-83	410-70	403-71	394-82	410-85	412-92

@ 11½c. per lb.

Charges as follows: Packing, per cask, 90 cts.; drayage \$7.50; freight, prepaid, 33½ cts. per cwt. How much would settle the bill on date of sale?

113. Messrs. Palmer, Wright & Co., Burlington, Iowa, bought of Marshall Field & Co., Chicago, July 30, 1896. Terms: Apr. 1, 1897, cash 5%; 30 days 2½%; 60 days net; 1% per mo. for advance payment.

Case # 241, — pieces Silk and Wool Crepons.

40 ¹	43	44 ³	42 ¹	45 ¹	43 ³	42	41 ¹	45 ²	43 ³	41 ³	41 ²
42	43 ²	40 ¹	46	45 ²	40	41 ³	42	43			

@ \$1.25 per yard.

Case # 242, — pieces French Cheviots.

41 ¹	43 ³	41	41 ³	43	43 ²	41 ¹	43 ²	43	44	41 ²	43	43 ²
41	43 ²	43	44	43 ³	41	43 ³	44 ¹	45	40 ¹	41 ²	40	42 ²
43 ¹	44 ²											

@ \$1.37½ per yard.

Case # 245, — pieces Cravenette.

34 ¹	36 ²	33	34 ²	35	36	32 ²	33	34	35 ¹	36	32 ²	34
36 ³	35 ²											

@ \$2.12½ per yard.

Case # 250, — pieces Satin Brocade.

24 ²	26	27 ³	25 ²	24	27 ²	26	24 ²	26	27 ²	25 ²	24 ¹	25
24												

@ \$2.62½ per yard.

What is the net of the bill, and what would pay the bill on Sept. 1, 1896?

INSURANCE.

173. Insurance is an indemnity in case of loss.

174. Fire Insurance is an indemnity in case of loss by fire.

175. Marine Insurance is an indemnity in case of loss at sea.

176. The Parties to the contract of insurance are the insurer and the insured.

177. The Insurer is the party agreeing to pay the indemnity; he is sometimes called an underwriter.

178. The Insured is the party protected by the contract of insurance.

The insurance business is carried on by corporations regulated by state laws; they are mutual, stock, or both.

179. A Mutual Company is an association of individuals for their own benefit.

180. A Stock Company is one organized by stockholders and conducted for pecuniary profit.

181. The Policy is the contract of insurance.

182. The Premium is the sum paid by the insured to the insurer.

183. The Term of Insurance is the time for which the agreement of insurance is made.

184. Losses are adjusted according to the kind of policy

1. In an ordinary policy the sum paid covers the loss, provided the loss does not exceed the face of the policy.

2. The sum paid, when the policy contains the "average clause," is to the total loss as the amount of the policy is to the total value of the property.

ILLUSTRATION.—A man owns a building valued at \$10000, which he insures for \$5000. What sum would he receive in case of loss by fire to the extent of \$8000?

In No. 1 he would receive \$5000. Why?

In No. 2 he would receive \$4000. Why?

The average clause is generally used in marine insurance.

MENTAL PRACTICE.

1. How much must be paid for insuring property for \$3000 at $\frac{3}{8}\%$ premium?

SUGGESTION.—The premium is $\frac{3}{8}\%$ of \$3000.

2. How much will it cost to insure a vessel for \$10000 at $1\frac{1}{2}\%$ premium?

3. What is the insurance on household furniture for \$5000 at $\frac{3}{4}\%$ premium?

4. At $\frac{1}{2}\%$, what is the annual premium at $\frac{3}{4}$ valuation of a house costing \$8000?

5. At $\frac{3}{8}$ valuation, what is the annual premium at $1\frac{1}{4}\%$ on household furniture, valued at \$6000?

6. What is the premium on a house for five years, valued at \$5000, at $1\frac{1}{2}\%$ per annum?

7. What premium must I pay for insuring a building for \$3600 at $\frac{3}{4}\%$?

8. Insured grain for \$5600 at $\frac{7}{8}\%$. What is the premium?

9. An agent receives \$80 for insuring a vessel at 2% . What is the value of the vessel?

SUGGESTION—\$80 is .02 of the policy.

10. An agent's premium is \$25 for insuring a house at $\frac{3}{8}\%$. What is the face of the policy?

11. If a man insures his store for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$, paying a premium of \$150, what is the value of the store?

12. A merchant pays \$75 annually for insurance on his stock of goods at $\frac{3}{8}\%$. For what amount does the policy call?

13. An agent received \$20 for insuring a stock of grain at $\frac{3}{8}\%$. What was the value of the grain?

14. At $\frac{3}{4}\%$, an agent received \$37.50 for insuring a house at $\frac{3}{4}$ valuation. What was the value of the house?

15. An agent received \$75 as premium for insuring a house for three years at $\frac{3}{8}\%$. What was the amount of the policy?

16. At $\frac{3}{4}\%$ premium, an agent received \$72 for insuring a barn for three years at $\frac{3}{4}$ its value. What was the value of the barn?

17. A merchant has a store valued at \$8000, and a stock of goods valued at \$6000. He insures the store for $\frac{3}{4}$ of its valuation, and the stock of goods for $\frac{3}{8}$ of their valuation; what is his premium, estimated at $\frac{7}{8}\%$?

WRITTEN PRACTICE.

1. A merchant insures his stock of dry goods to the amount of \$8400 at $1\frac{1}{2}\%$. What premium does he pay?

SUGGESTION.—Amount of Insurance=Base.

2. What premium must be paid for insuring a house for \$7200 at \$1.05 per hundred?

3. A cargo valued at \$3640 was insured for the full amount at $2\frac{1}{4}\%$. If the cargo was totally lost, how much did the insurer lose? How much did the insured lose?

4. A merchant has a stock of goods valued at \$1200, and a store building valued at \$9000. At $\frac{3}{4}\%$, what amount of premium does he pay on a policy, if the insurance is on a $\frac{3}{4}$ valuation?

5. An insurance agent takes a risk of \$20000 on a vessel at $2\frac{1}{4}\%$, and immediately re-insures $\frac{1}{2}$ of the risk at $2\frac{3}{4}\%$. What net premium does he receive?

6. A manufacturing plant carries insurance as follows: On building \$12000, on machinery \$20000, and on stock \$30000, paying $1\frac{3}{4}\%$ premium. What is the net loss of the insurance company, if they pay the following losses: On machinery $\frac{1}{2}$ of valuation, on building $\frac{2}{3}$ of valuation, and on stock a total loss?

7. My agent insures a cargo of wheat costing \$6400, at the rate of $2\frac{1}{4}\%$ for an amount that will cover the cost of wheat and premium. What is the face of the policy?

SUGGESTION.—\$6400÷.97 $\frac{3}{4}$ =Face of Policy.

8. My agent buys 2000 bushels of wheat at \$.54 $\frac{1}{2}$ per bushel, 3200 bushels of oats at \$.23 per bushel, and 4100 bushels of corn at \$.48 $\frac{1}{4}$ per bushel, charging 2% commission for buying. I instruct him to ship to Buffalo, per steamer City of

Chicago, taking policy at $1\frac{1}{4}\%$, to cover cost of grain and all charges. What is the amount of insurance?

9. A commission merchant bought 40000 bu. of wheat at 75 cents per bu., which he insured at \$20000, taking a policy containing the "average clause." How much does he receive, the wheat being damaged to the extent of \$7500?

10. I took \$50000 insurance in 5 companies at 85 cents per \$100. Upon report of the board of survey I was rebated 18%. What sum was returned to me by the insurance companies?

11. A consignment of cotton which cost \$30000 was insured at $1\frac{1}{4}\%$ for that amount, and also to cover premium and a commission of 2%. While in transit the goods increased in value 15%, which sum was covered at $\frac{7}{8}\%$ premium. What was the total premium paid, and what was the net gain to the shipper, the goods being sold at an increase of 21% on the first cost?

12. My house cost \$6000, which was insured at $\frac{3}{8}$ valuation in the Wheeling Mutual, at $1\frac{1}{2}\%$ per annum, for 3 years. The house was totally destroyed by fire. What was my loss, and what was the loss of the company?

13. A farmer took the following insurance in the Half Day Mutual: House valued at \$6000 for $\frac{3}{4}$ valuation at $1\frac{1}{4}\%$; barn valued at \$4500 for $\frac{3}{8}$ valuation at $1\frac{1}{2}\%$; live stock for \$5500 for $\frac{3}{4}\%$; grain for \$3000 at $\frac{1}{2}\%$. What was his total premium?

14. The Prairie State National Bank carries \$6000 insurance in the Home Insurance Company, \$5000 in the Galena Insurance Company, \$7500 in the Royal Insurance Company, \$10000 in the Manchester Assurance Company, and \$4000 in the Charter Oak Insurance Company. How much does each insurance company pay, the bank being injured by fire causing a loss of \$18687.50?

15. A cargo valued at \$45000 was insured for \$10000 in the Continental Insurance Company, \$6000 in the Liverpool Insurance Company, and \$14000 in the Hamburg Insurance Company. The cargo was injured during a storm to the extent of \$9600. What sum did each company pay according to "average clause" insurance?

16.

CHICAGO, Mar. 1, 1896.

STRAUSS & SMITH,

285 Madison St.

To CASE & Co., Dr.

For Insurance as follows:

Date.	Policy No.	Company.	P'p'ty.	Am't.	Rate.	Premium.	
211 96	6114790	Royal	F. & F.	3000	80	XX	
212 "	74711	Home	F. & F.	5500	92	XX	XX
215 "	78084	Rockford	House	7400	105		
216 "	524603	Orient	Store	12500	120		
220 "	432340	Spring Garden	F. & F.	4800	90		
226 "	92365	North America	Barn	2400	125		

Find the net premium.

Less 15%.

LIFE INSURANCE.

185. **Life Insurance** is a contract whereby a company agrees to pay a certain sum of money at the death of a person, or upon the expiration of a term of years.

186. A **Whole Life Policy** continues during the life of the insured.

187. An **Endowment Policy** is payable to the insured at the expiration of a term of years, or to his estate if he dies sooner.

188. The **Beneficiary** is the person in whose favor the policy is written.

189. The **Expectation of Life** is the probability of life for a term of years; it is based upon the records of mortality.

190. The **Rate of Life Insurance** is expressed at a given sum on each \$1000.

191. A **Dividend** is a share of the earnings of a company payable to certain policyholders.

TABLE OF RATES.
SHOWING PREMIUMS FOR AN INSURANCE OF \$1000.

WHOLE LIFE POLICIES.

WITH PROFITS.

Age.	Pay- ments for life.	20 Pay- ments.	15 Pay- ments.	10 Pay- ments.	5 Pay- ments.	1 Pay- ment.	Age.
18	23 41	27 76	36 72	64 16	285 61	18
19	23 86	28 29	37 40	65 33	290 74	19
20	17 27	24 33	28 83	38 11	66 54	296 05	20
21	17 70	24 82	29 39	38 84	67 79	301 54	21
22	18 15	25 32	29 97	39 60	69 09	307 21	22
23	18 62	25 84	30 58	40 39	70 44	313 08	23
24	19 11	26 38	31 21	41 21	71 83	319 14	24
25	19 63	26 95	31 87	42 05	73 27	325 41	25
26	20 17	27 54	32 55	42 93	74 76	331 89	26
27	20 74	28 15	33 25	43 84	76 30	338 58	27
28	21 34	28 78	33 98	44 78	77 90	345 50	28
29	21 97	29 44	34 74	45 75	79 55	352 64	29
30	22 63	30 12	35 53	46 76	81 26	360 02	30
31	23 32	30 83	36 34	47 81	83 03	367 64	31
32	24 05	31 58	37 19	48 89	84 85	375 51	32
33	24 82	32 36	38 07	50 01	86 74	383 63	33
34	25 63	33 17	38 99	51 17	88 69	392 02	34
35	26 49	34 01	39 94	52 38	90 71	400 68	35
36	27 39	34 90	40 93	53 64	92 80	409 63	36
37	28 35	35 83	41 97	54 94	94 97	418 87	37
38	29 36	36 81	43 06	56 29	97 21	428 42	38
39	30 43	37 84	44 20	57 70	99 53	438 29	39

To find the semi-annual premium, multiply the annual by .5185.

ENDOWMENTS.
PAYMENTS FOR FULL TERM.
 WITH PROFITS.

Age.	Payable in 10 years.	Payable in 15 years.	Payable in 20 years.	Payable in 25 years.	Payable in 30 years.	Payable in 35 years.	Payable in 40 years.	Age.
18	99 79	62 91	45 03	34 76	28 31	24 03	21 14	18
19	99 89	63 02	45 16	34 91	28 46	24 21	21 35	19
20	100 00	63 14	45 29	35 05	28 62	24 40	21 57	20
21	100 11	63 27	45 43	35 21	28 79	24 60	21 81	21
22	100 23	63 40	45 58	35 37	28 97	24 82	22 07	22
23	100 35	63 54	45 74	35 54	29 17	25 06	22 35	23
24	100 49	63 69	45 90	35 72	29 39	25 31	22 64	24
25	100 63	63 84	46 07	35 91	29 63	25 58	22 96	25
26	100 78	64 01	46 25	36 12	29 88	25 87	23 31	26
27	100 93	64 18	46 44	36 35	30 14	26 18	23 68	27
28	101 10	64 37	46 65	36 59	30 42	26 52	24 08	28
29	101 27	64 56	46 87	36 85	30 73	26 89	24 51	29
30	101 45	64 76	47 10	37 13	31 07	27 29	24 98	30
31	101 64	64 98	47 35	37 43	31 44	27 73	25 49	31
32	101 84	65 20	47 62	37 76	31 83	28 20	26 03	32
33	102 06	65 44	47 92	38 12	32 25	28 70	26 62	33
34	102 28	65 71	48 24	38 50	32 71	29 24	27 26	34
35	102 51	65 99	48 58	38 92	33 21	29 83	27 95	35
36	102 76	66 29	48 95	39 37	33 76	30 49	36
37	103 03	66 62	49 36	39 87	34 36	31 19	37
38	103 33	66 99	49 82	40 42	35 01	31 96	38
39	103 65	67 40	50 32	41 02	35 73	32 79	39

To find the quarterly premium, multiply the annual by .2639.

WRITTEN PRACTICE.

1. What is the annual premium on \$1000 at the age of 35 years life plan? Endowment plan payable in 10 years.

SUGGESTION.—Find 35 in age column at left of table, and in "payment for life" column will be found the annual premium on \$1000. For endowment plan, apply same rule to endowment table.

2. A young man, 25 years of age, takes out life insurance amounting to \$3000 payable annually, life plan. How much has he paid at the end of 20 years?

3. A clergyman, 27 years of age, insures for \$10000, on the 20 year endowment plan. How much will he have paid the company at the end of the term?

4. A lawyer, 28 years of age, insures for \$15000 on the 10 year endowment plan. He dies after making the eighth payment. How much more would his estate have been worth if he had insured on the life plan?

5. A merchant takes a policy for \$8000, endowment participating profit plan. He is 37 years of age and takes the 30 year plan, receiving an annual dividend of \$9.60 per thousand. What has his insurance cost him at the end of the term?

6. A teacher, 29 years of age, takes a 10 year endowment policy for \$4000, premiums payable semi-annually. What does his insurance cost him, he having received a dividend of \$345.60 at the end of the term?

7. A banker, 32 years of age, considers two propositions on a \$50000 policy, being allowed a discount of 20% of first premium in either case. First, a 20 year endowment policy, premium payable semi-annually; second, a 20 year life plan policy, premium payable annually. He accepts the former, and finds that his premium is reduced by an annual cash dividend of \$7.65 per thousand. The banker dies when 40 years of age, having paid all premiums for the year. How much more would his estate have been worth if he had taken the latter proposition?

TAXES.

192. A **Tax** is a charge for the support of government.

193. **Property** is anything owned by one to the exclusion of others.

Property is divided into two classes, viz: Personal or movable, and real estate or immovable.

194. **Poll Tax** is a tax on male citizens.

195. An **Assessor** is an officer whose duty it is to list and value property.

196. A **Collector** is an officer whose duty it is to collect tax levies.

197. **Property Taxes** are at a certain per cent of the assessed value.

198. **Poll Taxes** are so much per head, or poll.

WRITTEN PRACTICE.

Find the taxes in the following, the assessed valuation and rate being given:

1. Assessed valuation \$8000, rate of taxation $\frac{1}{2}\%$.

SUGGESTION.—Multiply Valuation by the Rate.

2. Assessed valuation \$7500, rate of taxation $\frac{3}{8}\%$.

3. Assessed valuation \$12500, rate of taxation 5 mills on the dollar.

4. Assessed valuation \$9600, rate of taxation $4\frac{1}{3}$ mills on the dollar.

5. Assessed valuation of personal property \$8000, real property \$24000; rate of taxation 11 mills on the dollar.

6. Assessed valuation of personal property \$61000, real property \$3000000; rate of taxation $5\frac{1}{2}$ mills on the dollar.

7. Assessed valuation of personal property \$95000, real estate \$2500000; rate of taxation \$.75 per \$100.

8. Assessed valuation of personal property of a village is \$850000, real property \$5000000; rate of taxation \$.96 per \$100, and 1243 polls at \$1.50 each.

SUGGESTION.—Add Poll Tax to Property Tax.

9. A township assessor reports as follows: Personal property valued at \$120000, real estate \$2400000; the personal property is taxed on $\frac{3}{4}$ valuation, and real estate on $\frac{1}{2}$ valuation, there are 364 polls taxed \$1.75 each, the rate of taxation \$1.14 per \$100.

10. The return of assessment rolls of a certain town show, for the year ending 1896, the following valuations: Real estate \$3500000, personal property \$900000, and 1240 polls assessed \$1.25 each. To meet expenses, taxes are apportioned as follows on the basis of \$100: State tax \$.52, county tax \$.79 $\frac{3}{10}$, city and school tax \$.65, town tax \$.15, boulevard tax \$.20, park tax \$.45, bonded indebtedness \$.15, sanitary tax \$1.50, new sinking fund \$.25. There is a balance of \$250124.43 in the treasury. A collection fee of 1% is allowed. The state board of equalization added 19 per cent to personal property and 17 per cent to real property. Find the total tax, and also A's tax, who was assessed \$4000 on real estate and \$5000 on personal property and who pays for 3 polls.

Find the rate of taxation in the following:

11. Assessed valuation \$600000, tax levy \$3000.

SUGGESTION.—Divide Tax Levy by Valuation.

12. Assessed valuation \$4000000, tax levy \$300000.

13. Assessed valuation \$300000, tax levy \$2940, collector's commission being 2%.

SUGGESTION.—Tax levy is 98% of amount collected.

14. Assessed valuation \$200000, tax levy \$2406.40, collector's commission being 6%.

15. Assessed valuation \$250000, tax levy \$6080, collector's commission being 5%.

16. Assessed valuation of real property \$3500000, personal property \$500700, and there being 250 polls assessed \$1.25 each, collection charges 5%, tax levy \$95313.50.

SUGGESTION.—After finding added charge for collection, deduct poll tax before finding rate of taxation.

17. The assessed valuation of a certain town is, real property \$1000000, personal property \$500000, collection charges 2%, and there being 500 polls assessed \$1.50 each, the tax levy being \$24700.

18. A tax of \$6691.80 is to be raised in a certain village. The real property is valued at \$264000, which is assessed at $\frac{3}{4}$ valuation; the personal property is assessed \$54000, and there are 600 polls assessed \$2.50 each. What is the rate of taxation, the rate of collection being 5%, and what is A's tax whose real estate is valued at \$27000, personal property at \$5000, and who pays for 4 polls?

19. A certain school is supported by popular subscription to defray expenses, as follows: Salaries \$5000, rent \$1650, fuel \$150, gas \$125, janitor \$750, incidentals \$225. The enrollment numbered 325 pupils, of whom 20% missed 20% of the time, and 24% missed 5% of the time. The school year was for 40 weeks of 5 days each. What was A's tax who sent five pupils, one of them missing 20% of the time?

INTEREST.

199. Interest, Bank Discount, True Discount, Partial Payments, etc., are applications of percentage of the second class, and have the time element in addition to the usual percentage terms.

The base and the rate are the factors whose product gives the percentage; while in the above classes, the base, rate, and time are the factors whose product gives the percentage, or interest, or discount, etc.

200. Interest is the sum paid for the use of money or the detention of a debt.

201. The Principal is the sum of money on which the interest is computed.

202. The Rate of interest is the rate per cent per annum.

203. The Amount is the sum of the principal and interest.

204. Simple Interest is the interest on the principal alone for the given time.

205. Legal Interest is the rate established by law. The legal rate is used in all cases of implied contracts. The parties may agree upon any rate that is not forbidden by law.

206. Usury is the taking of a higher rate than the law allows.

207. Ordinary Interest is interest computed on the basis of 360 days to the year.

In this method, time is found by compound subtraction.

208. Exact Interest is interest computed on the basis of 365 days to the year.

In this method, time is found by counting the exact number of days.

TABLE OF INTEREST RATES.

	Legal.	Contract.		Legal.	Contract.
Alabama	8%	8%	Montana	10%	any %
Alaska	Nebraska	7	10
Arizona	7	any	Nevada	7	any
Arkansas	8	10	New Hampshire	6	6
California	7	any	New Jersey	6	6
Colorado	8	any	New Mexico	6	12
Connecticut	6	6	New York	6	6
Delaware	6	6	N. Carolina	6	8
Dist. of Columbia	6	10	N. Dakota	7	12
Florida	8	10	Ohio	6	8
Georgia	7	8	Oklahoma	7	12
Idaho	10	18	Oregon	8	10
Illinois	5	7	Pennsylvania	6	6
Indiana	6	8	Rhode Island	6	any
Iowa	6	8	S. Carolina	7	8
Kansas	6	10	S. Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	any	Utah	8	any
Maryland	6	6	Vermont	6	6
Massachusetts	6	any	Virginia	6	6
Michigan	6	8	Washington	10	any
Minnesota	7	10	W. Virginia	6	6
Mississippi	6	10	Wisconsin	7	10
Missouri	6	8	Wyoming	12	any

The student should be thoroughly familiarized with one or two methods, the selection being left with the teacher. One method well mastered is worth more than a mere knowledge of several—"We learn to do by doing."

MENTAL PRACTICE IN INTEREST.

Reduce the following time to years and fractions of a year:

1. 1 year 2 months.

SOLUTION.—In one year there are 12 months, 2 months are $\frac{2}{12}$ of 12 months, or $\frac{1}{6}$; therefore, 1 year 2 months is equal to $1\frac{1}{6}$ years.

2. 2 years 4 months.

7. 2 years 7 months.

3. 4 years 6 months.

8. 1 year 11 months.

4. 3 years 3 months.

9. 2 years 1 month.

5. 2 years 10 months.

10. 3 years 5 months.

6. 1 year 9 months.

11. 3 years 8 months 15 days.

At the following rates, what part of the principal equals the interest?

1. 2%

6. $7\frac{1}{2}\%$

11. $12\frac{1}{2}\%$

16. 4% for 5 years.

2. 4%

7. 8%

12. $14\frac{3}{4}\%$

17. 6% for 5 years.

3. 5%

8. 9%

13. $16\frac{2}{3}\%$

18. 3% for 10 years.

4. 6%

9. 10%

14. 20%

19. $4\frac{1}{2}\%$ for 4 years.

5. 7%

10. 12%

15. 25%

20. $7\frac{1}{2}\%$ for 2 years.

21. 8% for 5 years.

26. 10% for 2 years 6 months.

22. 3% for 2 years.

27. 5% for 6 years 8 months.

23. $2\frac{1}{2}\%$ for 10 years.

28. $7\frac{1}{2}\%$ for 6 years 6 months.

24. 7% for 5 years.

29. 8% for 2 years 3 months.

25. 9% for 2 years.

30. 10% for 5 years 6 months.

Find the interest on the following:

1. \$75 for 2 yr. at 10%.

11. \$320 for 3 yr. 3 mo. at 6%.

2. \$80 for 3 yr. at $12\frac{1}{2}\%$.

12. \$620 for 2 yr. 8 mo. at 10%.

3. \$120 for 4 yr. at 5%.

13. \$800 for 2 yr. 7 mo. at $7\frac{1}{2}\%$.

4. \$240 for 2 yr. at $8\frac{1}{2}\%$.

14. \$120 for 4 yr. 8 mo. at 5%.

5. \$72 for 5 yr. at 6%.

15. \$314 for 2 yr. 6 mo. at 10%.

6. \$84 for 3 yr. at 5%.

16. \$215 for 3 yr. 4 mo. at 8%.

7. \$150 for 2 yr. 6 mo. at 6%.

17. \$32 for 1 yr. 9 mo. at 5%.

8. \$240 for 3 yr. 9 mo. at 10%.

18. \$60 for 2 yr. 4 mo. at $6\frac{1}{2}\%$.

9. \$320 for 5 yr. 6 mo. at 4%.

19. \$75 for 1 yr. 4 mo. at 6%.

10. \$325 for 5 yr. 6 mo. at 4%.

20. \$1200 for 2 yr. 5 mo. at 9%.

Find the amount of

1. \$500 for 2 yr. 6 mo. at 6%.
2. \$450 for 3 yr. 6 mo. at 5%.
3. \$200 for 2 yr. 6 mo. at 10%.
4. \$150 for 6 mo. at 4½%.
5. \$36 for 9 mo. at 5%.
6. \$72 for 1 yr. 6 mo. at 6%.
7. \$600 for 3 yr. 4 mo. at 14½%.
8. \$120 for 2 yr. 6 mo. at 12½%.
9. \$350 for 3 yr. at 4%.
10. \$480 for 2 yr. 6 mo. at 8%.

How long will it take any principal to double itself at 10%? At 4%? At 12½%? At 14½%? At any rate per cent?

GENERAL RULE.

Multiply the interest for 1 year by the number of years, and take $\frac{1}{12}$ of a year's interest for each month, and $\frac{1}{365}$ of a month's interest for each day, after which, combine the several interest products.

Or, multiply the principal by the rate, and the time expressed in years and decimal of a year.

COMPUTING TIME.

209. Although there are 365 days in a year, yet it is customary in ordinary interest calculations to estimate the year as composed of 12 months of 30 days each, making 360 interest days.

Exact interest can only be obtained by counting 365 interest days in a year. However, the difference between the two methods is small, and 360 days being much more convenient, it has become sanctioned by common usage, and it is now the common method to count 12 months of 30 days each, as constituting the interest year.

The officers of the United States government compute exact interest in all transactions for the government, and the custom prevails also among many bankers.

EXAMPLES.

1. What is the interest on \$1080 for 7 months at 5%?

SOLUTION.

$$\$1080 \times .05 \times \frac{7}{12} = \$31.50.$$

EXPLANATION.—\$1080 multiplied by .05 gives the interest for 1 year, and the interest for 7 months is $\frac{7}{12}$ of the product.

2. What is the interest on \$1200 for 1 year 5 months 12 da. at 7%?

SOLUTION.

$$\begin{aligned} \$1200 \times .07 &= \$84.00 \\ \frac{5}{12} \text{ of } \$84 &= 35.00 \\ \frac{12}{360} \text{ of } (\$84 \div 12) &= 2.80 \\ \text{Total Int.} &= \$121.80 \end{aligned}$$

EXPLANATION.—7% interest on \$1200 for 1 year equals \$84, and for 5 months equals $\frac{5}{12}$ of \$84, and for 12 days equals $\frac{12}{360}$ of the interest for 1 month. Adding the interest obtained for years, months and days, gives the total interest.

WRITTEN PRACTICE.

Compute the interest on the following:

1. \$3600 for 3 yr. at 6%.
2. \$960 for 5 yr. at 8%.
3. \$1260 for 4 yr. 6 mo. at 5%.
4. \$324 for 3 yr. 3 mo. at 4%.
5. \$480 for 2 yr. 4 mo. at 6%.
6. \$320.50 for 4 yr. 8 mo. at 9%.
7. \$748.25 for 1 yr. 7 mo. at 10%.
8. \$263.84 for 6 mo. at 12%.
9. \$327 for 7 mo. at $6\frac{1}{2}\%$.
10. \$343.75 for 5 mo. at $4\frac{1}{2}\%$.
11. \$1263 for 4 mo. at $6\frac{3}{4}\%$.
12. \$725 for 3 mo. at $5\frac{1}{4}\%$.
13. \$46.30 for 8 mo. 15 da. at 6%.
14. \$426 for 10 mo. 18 da. at 8%.
15. \$1894 for 9 mo. 23 da. at 7%.

Find the time by compound subtraction, page 113.

16. Find the interest on \$1260 from Aug. 4, 1890, to Nov. 4, 1894, at 6%. At 9%.
17. Compute the interest on \$896.50, from March 3, 1890, to July 3, 1893, at $6\frac{1}{2}\%$. At $5\frac{1}{2}\%$.
18. What is the amount of \$3200, from May 1, 1891, to July 16, 1893, at 8%? At 12%?
19. On Jan. 15, 1890, I loaned \$3600. What sum should be returned to me Aug. 31, 1893, at 7%? At 9%?
20. On Aug. 4, 1895, I borrowed \$1260.50 at 6%. How much must I return Sept. 21, 1897, to pay my obligation?

THE SIX PER CENT METHOD.

210. The six per cent method consists of finding the interest on \$1, for the given time at 6%, and multiplying the same by the principal.

At six per cent the interest on \$1

For 1 yr., or 12 mo., is 6 cents, or .06 of the principal.

For 1 mo., or $\frac{1}{12}$ yr., is 5 mills. or .005 " " "

For 1 day, or $\frac{1}{360}$ mo., is $\frac{1}{72}$ mill, or .0001 " " "

RULE.—I. Take 6 cents for each year, 5 mills for each month, and $\frac{1}{6}$ mill for each day, as the interest on \$1 for the given time.

II. Multiply the interest on \$1 by the given principal.

The six per cent method may be used as a basis for any rate of interest, by increasing or decreasing the results. For interest at 1% take $\frac{1}{6}$ of the interest at 6%; at 2% take $\frac{1}{3}$, at 3% take $\frac{1}{2}$, at 4% subtract $\frac{1}{3}$ of the interest at 6%; at 4 $\frac{1}{2}$ % subtract $\frac{1}{4}$, at 5% subtract $\frac{1}{5}$, at 5 $\frac{1}{2}$ % subtract $\frac{1}{6}$; at 6 $\frac{1}{2}$ % add $\frac{1}{12}$, at 7% add $\frac{1}{6}$, at 7 $\frac{1}{2}$ % add $\frac{1}{4}$, at 8% add $\frac{1}{3}$, at 9% add $\frac{1}{2}$, at 10% divide by 6 and remove the decimal point one place to the right.

EXAMPLE.

What is the interest on \$3600, for 2 years 8 months 24 days, at 6 per cent?

SOLUTION.

Interest on \$1 for 2 years	at 6%	is \$.12
" " 1 " 8 months	" 6%	" .04
" " 1 " 24 days	" 6%	" .004
		<u>\$.164</u>

Interest on \$1 for 2 yrs. 8 mo. 24 da. at 6% is \$.164.

$3600 \times \$.164 = \590.40 , the interest for the required time.

WRITTEN PRACTICE.

Compute the interest on the following:

21. \$650 for 4 yr. at 6%.
22. \$1250 for 2 yr. 6 mo. at 6%.
23. \$375 for 3 yr. 4 mo. at 6%.
24. \$650 for 1 yr. 8 mo. 12 da. at 8%.
25. \$128.60 for 9 mo. 4 da. at 8%.
26. \$864.25 for 11 mo. 18 da. at 4%.
27. \$29.50 for 2 yr. 9 mo. 6 da. at 7 $\frac{1}{2}$ %.
28. \$360.50 for 4 mo. 15 da. at 7 $\frac{1}{2}$ %.
29. \$425 for 1 yr. 10 mo. 20 da. at 4%.
30. \$840.25 for 96 da. at 6%.
31. \$920 for 84 da. at 6%.
32. \$842.50 for 3 mo. 15 da. at 8%.
33. \$920.40 for 5 yr. 8 mo. 20 da. at 5%.
34. \$848 for 2 yr. 9 mo. 15 da. at 7%.
35. \$12850 for 8 yr. 23 da. 6 $\frac{1}{2}$ %.

THE CANCELLATION METHOD.

211. The Cancellation Method considers the months or days as fractional parts of a year, and abbreviates the work by cancellation.

PROCESS.—Write the principal above a horizontal line and point off two places, giving the interest for 1 year at 1 per cent; indicate multiplication by the rate expressed as an integer, giving the interest for 1 year at the rate named; write 360 below the horizontal line, indicating division, and giving the interest at the rate named for one day; write the given number of days above the line, indicating multiplication, and giving the required interest. Apply cancellation and complete the process indicated.

If the time does not include days, use twelve as the divisor and multiply by the number of months.

EXAMPLES.

1. What is the interest on \$800, for 7 mo. 15 da. at 8%?

SOLUTION.

$$\$800 \times .08 \times 225 \div 360 = \text{the interest.}$$

$$\frac{\$8.00 \times 8 \times 225}{360} = \$40, \text{ the interest.}$$

2. What is the interest on \$393, for 2 yr. 8 mo. at 5%?

SOLUTION.

$$\$393 \times .05 \times 32 \div 12 = \text{the interest.}$$

$$\frac{\$3.93 \times 5 \times 32}{12} = \$52.40, \text{ the interest.}$$

3. What is the interest on \$750, for 9 mo. 27 da. at 7½%?

SOLUTION.

$$\$750 \times .07\frac{1}{2} \times 297 \div 360 = \text{the interest.}$$

$$\frac{\$7.50 \times 15 \times 297}{2 \times 30 \times 12} = \$46.41, \text{ the interest}$$

Compute the interest on the following:

36. \$860 for 2 yr. 8 mo. at 7%.
37. \$940 for 3 yr. 9 mo. at 8%.
38. \$942.50 for 2 yr. 10 mo. at 6%.
39. \$365 for 11 mo. 20 da. at 6½%.
40. \$284.60 for 9 mo. 18 da. at 7½%.

41. \$1386 for 7 mo. 25 da. at $4\frac{1}{2}\%$.
42. \$8426.30 for 6 mo. 6 da. at 8% .
43. \$1265.75 for 7 mo. 24 da. at 9% .
44. \$3640 for 8 mo. 16 da. at 10% .
45. \$12640 for 4 mo. 5 da. at $6\frac{1}{4}\%$.

THE SIXTY DAY METHOD.

212. The Sixty Day Method first finds the interest at 6% for 60 days, and increases or diminishes this amount according to the time given in the problem.

PROCESS.—Moving the decimal point two places to the left gives 6% interest for 60 days; increasing or diminishing this interest gives the interest at 6% for the required number of days; increasing or diminishing the interest at 6% gives the interest at the required rate.

Since the interest for 1 year at 1% is obtained by pointing off two places in any principal; as, the interest on \$500 for 1 year at 1% is \$5; therefore, pointing off two places would give the interest for a less time at a higher rate.

Pointing off two places in any principal = Int. for 1 yr. at 1%.

Pointing off two places in any principal = Int. for 6 mo. at 2%.

Pointing off two places in any principal = Int. for 4 mo. at 3%.

Pointing off two places in any principal = Int. for 3 mo. at 4%.

Pointing off two places in any principal = Int. for 72 da. at 5%.

Pointing off two places in any principal = Int. for 60 da. at 6%.

The interest on \$750 for 1 year at 1% is \$7.50.

The interest on \$750 for 6 months at 2% is \$7.50.

The interest on \$750 for 4 months at 3% is \$7.50.

The interest on \$750 for 3 months at 4% is \$7.50.

The interest on \$750 for 72 da. at 5% is \$7.50.

The interest on \$750 for 60 da. at 6% is \$7.50.

EXAMPLES.

1. What is the interest on \$350 for 60 days at 6%?

SOLUTION.

\$350 pointed off as
hundredths = \$3.50.

EXPLANATION.—6% interest on any sum for 60 days is 1%, or 1 hundredth of it. 1% of any number is obtained by pointing off two decimal places from the right of the number.

2. What is the interest on \$1500 for 93 days at 6%?

SOLUTION.

\$15|00 = Int. for 60 da.

7|50 = Int. for 30 da.

75 = Int. for 3 da.

\$23|25 = Int. for 93 da.

EXPLANATION.—Interest for 30 days is $\frac{1}{2}$ that for 60, and for 3 days is $\frac{1}{10}$ that for 30 days. By uniting the partial interest products the entire interest is obtained.

3. What is the interest on \$250 for 3 mo. 15 da. at $7\frac{1}{2}\%$?

SOLUTION.

\$2|50 = 60 da.

1|25 = 30 da.

625 = 15 da.

4|375 = 105 da. at 6%

1|094 = 105 da. at $1\frac{1}{2}\%$

5|469 = 105 da. at $7\frac{1}{2}\%$

EXPLANATION.—3 months 15 days equal 105 days, equal to 60 days plus 30 days plus 15 days. The interest on \$250 for 60 days is \$2.50, divided by two is \$1.25, the interest for 30 days, which amount divided by two gives \$.625, the interest for 15 days. The entire interest at 6 per cent increased by $\frac{1}{4}$ of itself is the interest at $7\frac{1}{2}\%$.

DRILL ON THE SIXTY DAY METHOD.

	6od.	3od.	2od.	15d.	12d.	10d.	6d.	5d.	3d.	1d.	9od.	12od.	6d.
\$2400	\$24	\$12	\$8	\$6	\$4.80	\$4	\$2.40	\$2	\$1.20	\$.40	\$36	\$48	\$2.40
\$1200	?	?	?	?	?	?	?	?	?	?	?	?	?
\$3600	?	?	?	?	?	?	?	?	?	?	?	?	?
\$360	?	?	?	?	?	?	?	?	?	?	?	?	?
\$7200	?	?	?	?	?	?	?	?	?	?	?	?	?

WRITTEN PRACTICE.

Compute the interest on the following at 6%:

46. \$840 for 63 da. (60 and 3).

47. \$960 for 75 da. (60 and 15).

48. \$320 for 96 da. (60, 30, and 6).

Moving the decimal point 3 places gives the interest for 6 days at 6%.

49. \$1260 for 123 da. (2 60's and 3).

50. \$2480 for 85 da. (60, 20, and 5).

51. \$1244 for 54 da. (60 less 6).

52. \$2360 for 59 da. (60 less 1).

53. \$9200 for 36 da. (6 6's).

54. \$1125 for 42 da. (7 6's).

55. \$480.50 for 18 da. (3 6's).

Compute the interest on the following at 8%:

56. \$720 for 73 da.

57. \$460 for 63 da.

58. \$920.60 for 72 da.

59. \$843.50 for 87 da.

60. \$1264 for 8 mo. 20 da.

Find the amount of the following:

61. \$450 for 65 da. at 6%. At 7%.

The amount is found by adding the interest and principal.

62. \$860 for 1 mo. 20 da. at 5%. At $6\frac{1}{2}\%$.

63. \$320 for 3 mo. 15 da. at 4%. At 8%.

64. \$1265 for 4 mo. 18 da. at 8%. At 7%.

65. 496.50 for 5 mo. 24 da. at $7\frac{1}{2}\%$. At $4\frac{1}{2}\%$.

66. \$1460.25 for 1 yr. 6 mo. 23 da. at 6%. At 8%.

67. \$2375 for 2 yr. 3 mo. 20 da. at 5%. At 7%.

68. \$8000 for 1 yr. 4 mo. 6 da. at $5\frac{1}{2}\%$. At $6\frac{1}{2}\%$.

69. \$6200 for 3 yr. 2 mo. 18 da. at $5\frac{1}{4}\%$. At $6\frac{3}{4}\%$.

70. \$425 for 1 yr. 4 mo. 14 da. at 8%. At 9%.

71. \$960.75 for 3 mo. 21 da. at 4%. At 6%.

72. \$896.43 for 2 mo. 16 da. at 9%. At 12%.

73. \$2640 for 4 mo. 25 da. at $4\frac{1}{2}\%$. At $7\frac{1}{2}\%$.

74. \$1230 for 7 mo. 16 da. at 8%. At 9%.

75. \$4261 for 2 yr. 8 mo. 21 da. at 4%. At 5%.

Find the time in the next five problems by compound subtraction.

76. \$800 from Jan. 15, 1890, to June 18, 1890, at 8%.

77. \$2400 from Feb. 11, 1891, to June 3, 1892, at $4\frac{1}{2}\%$.
At $7\frac{1}{2}\%$.

78. \$650 from June 17, 1892, to Jan. 21, 1894, at 7%. At
9%.

79. \$565.25 from Jan. 29, 1890, to Jan. 1, 1893, at 6%.
At $6\frac{1}{2}\%$.

80. \$940 from June 15, 1891, to Jan. 15, 1894, at $6\frac{1}{2}\%$.
At $7\frac{1}{2}\%$.

Find the time in the next five problems by counting the exact number of days in each month—a method that is sometimes used, although a year is counted 360 days. This method is sometimes called the **banker's method**.

81. \$1400 from May 1, 1890, to Sept. 25, 1890, at 6%.
At 7%.

82. \$560 from Jan. 24, 1891, to Dec. 16, 1891, at 7%.
At 5%.

83. \$840.50 from Feb. 2, 1892, to Oct. 4, 1892, at $5\frac{1}{2}\%$.
At $6\frac{1}{2}\%$.

84. \$375 from April 3, 1890, to May 1, 1891, at 6%.
At 9%.

85. \$484.60 from June 4, 1890, to July 16, 1893, at 8%.
At 9%.

86. If I borrow \$25000 for 2 years, interest at 5%, and immediately loan \$15000 for the same time at $7\frac{1}{2}\%$, and the balance for 1 year and 10 months at $6\frac{3}{4}\%$, what do I gain?

87. A broker placed the following loans for a capitalist on a commission of $\frac{1}{4}\%$: \$1500 Jan. 1, 1890, at 6%; \$3500 Aug. 1, 1890, at 4%; \$1200 Sept. 4, 1890, at 3 per cent; \$1400 Sept. 5, 1890, at 3%; and \$15000 Oct. 14, 1890, at 4%. The loans were all called in July 1, 1893. What sum did the capitalist realize on the loans? (Compound subtraction.)

88. What is the interest on £424 5s. 6d. for 2 years, 10 months, 15 days, at 6%?

89. What is the amount of £560 8s. 8d. for 3 years, 6 months, 18 days, at $7\frac{1}{2}\%$?

90. A man bought a farm for \$12000 upon the following terms: Cash, $\frac{1}{4}$; for the balance, 3 equal notes for 6 months, 1 year, and 1 year 6 months respectively, with interest at 8%. What was the total amount paid?

91. Ellis, Parks & Co. bought of Marshall Field & Co. the following bills of goods, on time, paying 6% interest, viz.: Jan. 1, 1893, \$450; Mar. 6, 1893, \$600; Apr. 16, 1893, \$750; Aug. 24, 1893, \$1260; and Oct. 14, 1893, \$1005. They remit total amount due per draft Jan. 1, 1894. What was the face of the draft? (c. s.)

92. Find the total interest on the following at 6%:

\$600 for 6 da.	\$300 for 30 da.
\$720 for 12 da.	\$640 for 36 da.
\$840 for 9 da.	\$320 for 42 da.
\$360 for 18 da.	\$480 for 48 da.
\$240 for 24 da.	\$120 for 54 da.
\$500 for 21 da.	\$800 for 3 da.

SUGGESTION.—By pointing off three places in the principal, you will have the interest for 6 days at 6 per cent.

93. Find the total interest on

\$720 for 6 mo. 3 da. at 6%.	\$840 for 123 da. at 4%.
\$560 for 5 mo. 20 da. at 7%.	\$360 for 246 da. at 5%.
\$840 for 8 mo. 23 da. at $4\frac{1}{2}\%$.	\$590 for 125 da. at 8%.
\$320 for 7 mo. 24 da. at 3%.	\$144 for 87 da. at $4\frac{1}{2}\%$.
\$495 for 9 mo. 16 da. at 8%.	\$396 for 214 da. at $7\frac{1}{2}\%$.

94. Find the total interest on

\$1260 for 2 yr. 6 mo. 15 da. at 5%.	\$260 for 8 mo. 9 da. at 6%.
\$2120 for 1 yr. 6 mo. 20 da. at 6%.	\$120 for 6 mo. 3 da. at 4%.
\$3140 for 2 yr. 8 mo. 14 da. at $6\frac{1}{2}\%$.	\$963 for 4 mo. 25 da. at 3%.
\$1250 for 1 yr. 7 mo. 23 da. at $5\frac{1}{2}\%$.	\$145 for 9 mo. 15 da. at 9%.

95. Find the total interest on

\$675.25 for 3 mo. 18 da. at 6%.	\$642.15 for 6 mo. 18 da. at 5%.
\$362.48 for 6 mo. 20 da. at 7%.	\$324.61 for 8 mo. 15 da. at 6%.
\$125.63 for 7 mo. 18 da. at 5%.	\$145.73 for 3 mo. 20 da. at 4%.
\$347.62 for 4 mo. 23 da. at 8%.	\$736.24 for 1 mo. 24 da. at 3%.

96. Find the total amount of

\$360 for 125 da. at 6%.	\$364 for 142 da. at $4\frac{1}{2}\%$.
\$840 for 246 da. at 7%.	\$576 for 316 da. at $5\frac{1}{4}\%$.
\$920 for 315 da. at 8%.	\$384 for 283 da. at $6\frac{3}{4}\%$.
\$370 for 217 da. at 5%.	\$496 for 436 da. at $7\frac{1}{2}\%$.
\$960 for 193 da. at 4%.	\$324 for 162 da. at $6\frac{1}{2}\%$.

INTEREST PROBLEMS.

213. GIVEN.—Interest, Time, and Rate, to find Principal.

RULE.—Divide the given interest by the interest on one dollar for the given time at the given rate; the quotient will be the principal.

$$\text{Formula—}\frac{I}{R \times T} = P$$

EXAMPLE.

What sum of money at interest for 1 yr. 6 mos. at 6% will gain \$36.72?

Int. on \$1 for 1 yr. 6 mos. = \$.09.

$\$36.72 \div .09 = \$408.$

OR

$$\frac{\$36.72}{1\frac{1}{2} \times .06} \text{ or } \frac{\$36.72 \times 2}{.06 \times 3} = \$408.$$

EXPLANATION.—Since \$.09 is the interest on \$1 for the given time, \$36.72 is the interest on as many dollars as .09 is contained times in \$36.72, or 408 times.

WRITTEN PRACTICE.

What sum of money will yield

97. \$1094.40 in 2 yr. 6 mo. 12 da. at 6%?

98. \$597.60 in 1 yr. 4 mo. 18 da. at 12%?

99. \$174.40 in 1 yr. 9 mo. 24 da. at 4%?

100. \$284.60 in 1 yr. 2 mo. 4 da. at 9%?

101. \$96.48 in 3 mo. 15 da. at 5%?

102. \$120.30 in 9 mo. 12 da. at 8%?

103. \$246.85 in 11 mo. 27 da. at 9%?

104. \$423.60 in 3 yr. 4 mo. 21 da. at 4%?

105. \$520.60 in 3 yr. 6 mo. 21 da. at 6½%?

106. \$360 in 2 yr. 4 mo. 18 da. at 5½%?

107. On Jan. 1, 1896, a merchant borrowed a certain sum of money at 6%; on Oct. 1, 1896, he paid the principal and interest. What sum did he borrow if the interest was \$288? (c. s.)

214. GIVEN.—Amount, Time, and Rate, to find Principal.

RULE.—*Divide the given amount by the amount of one dollar for the given time and rate; the quotient will be the principal.*

$$\text{FORMULA.} \quad \frac{\text{Amt.}}{1 + (R \times T)} = P.$$

EXAMPLE.

What sum of money at interest at 6% will amount to \$690 in 2 yr. 6 mo.?

SOLUTION.

$\$1.00 + .15 = \$1.15.$

$\$690 \div 1.15 = \$600.$

EXPLANATION.—6% interest on \$1 for 2 yr. 6 mo. is \$.15, and the amount of \$1 is \$1.15. \$690 is the amount of as many dollars at interest as \$1.15 is contained times in \$690, or 600 times.

WRITTEN PRACTICE.

What sum of money will amount to

108. \$8294.40 in 2 yr. 6 mo. 12 da. at 6%?
 109. \$4027.50 " 2 " 4 " 15 " " 5%?
 110. \$3125.00 " 2 " 9 " 18 " " 8%?
 111. \$9265.17 " 4 " 2 " 18 " " $7\frac{1}{2}\%$?
 112. \$1275.60 " 10 " 20 " " $5\frac{1}{4}\%$?
 113. \$ 384.71 " 7 " 15 " " 6%?
 114. \$1047.80 " 2 " 8 " " 9%?
 115. \$ 975 " 1 " 3 " " 10%?
 116. \$4264.16 " 4 " 9 " 9 " " 5%?
 117. \$1068 " 1 " 11 " 15 " " 6%?

118. On Jan. 1, 1896, a merchant borrowed a certain sum of money at $7\frac{1}{2}\%$; on Dec. 16, 1896, he cancelled the principal and interest by paying \$4768.50. What was the sum loaned?

215. GIVEN.—Principal, Interest, and Time, to find Rate.

RULE.—*Divide the given interest by the interest on the principal for the given time at 1% ; the quotient will be the rate.*

$$\text{Formula.} - \frac{I}{(1\% \text{ of } P) \times T} = R$$

EXAMPLE.

At what rate will \$250 produce \$37.50 in 2 yr. 6 mo.?

SOLUTION.

$$\$250 \times .01 \times 2\frac{1}{2} = \$6.25.$$

$$\$37.50 \div 6.25 = 6 \text{ times, or } 6\%.$$

EXPLANATION.—Since the in-

terest on \$250 for 2 yr. 6 mo. at 1% is \$6.25, \$37.50 is as many per cent as \$6.25 is contained times in \$37.50, or 6 times, or 6%.

OR

$$\frac{37.50}{250 \times 2\frac{1}{2}} = \frac{37.50 \times 2}{250 \times 5} = .06$$

WRITTEN PRACTICE.

At what per cent will

119. \$960 produce \$88.32 in 1 yr. 6 mo. 12 da.?
 120. \$7200 produce \$1159.20 in 2 yr. 3 mo. 18 da.?
 121. \$4250 produce \$811.75 in 3 yr. 2 mo. 6 da.?
 122. \$720 produce \$53.20 in 2 yr. 1 mo. 20 da.?
 123. \$1000 produce \$192.67 in 3 yr. 2 mo. 16 da.?
 124. \$1260 produce \$133.98 in 2 yr. 7 mo. 27 da.?

125. \$380 produce \$21.09 in 7 mo. 12 da.?

126. \$596 produce \$13.91 in 4 mo. 20 da.?

127. \$430.50 produce \$105.47 in 4 yr. 1 mo.?

128. \$263.85 produce \$31.66 in 1 yr. 4 mo.?

129. On Feb. 28, 1896, a merchant borrowed \$600, and on Dec. 17, 1896, he paid the interest and principal with \$643.35. What was the rate per cent? (c. s.)

216. GIVEN.—Principal, Interest, and Rate, to find Time.

RULE.—*Divide the given interest by the interest on the principal at the given rate for 1 year; the quotient will be the time in years or decimal of a year.*

$$\text{Formula.}—\frac{I}{P \times R} = T$$

EXAMPLE.

In what time will \$300 gain \$52.50 at 7%?

$$\$300 \times .07 = \$21$$

$$\$52.50 \div 21 = 2\frac{1}{2}$$

$$2\frac{1}{2} \text{ yr.} = 2 \text{ yr. } 6 \text{ mo.}$$

OR

$$\frac{52.50}{300 \times .07} = 2\frac{1}{2}, \text{ or } 2 \text{ yr. } 6 \text{ mo.}$$

EXPLANATION.—One year's interest on \$300 at 7% is \$21. \$52.50 is as many years' interest as \$21 is contained times in \$52.50, or $2\frac{1}{2}$ times, equal to 2 years 6 months.

WRITTEN PRACTICE.

In what time will

130. \$630 produce \$56.70 at 6%?

131. \$7500 produce \$365 at $6\frac{1}{2}\%$?

132. \$1200 produce \$97.65 at $7\frac{1}{2}\%$?

133. \$420 produce \$42 at 4%?

134. \$940 produce \$126.85 at $5\frac{1}{4}\%$?

135. \$3260 produce \$365.80 at 4%?

136. \$5000 produce \$1260 at 7%?

137. \$4200 produce \$420 at 5%?

138. \$3200 produce \$48 at 6%?

139. \$840 produce \$216 at 8%?

140. A merchant borrows \$900 on Aug. 1, 1896, at $6\frac{3}{4}\%$, and on the date of maturity he paid \$944.55 for the note and interest. Find the time and date of payment.

PROMISCUOUS WRITTEN EXERCISE

Find the required terms.

	Principal.	Rate.	Time.			Interest.	Amount
141.	\$ 720.00	6%	2 yr.	6 mo.		?	?
142.	\$ 460.00	4½%	1 "	3 "	6 da.	?	?
143.	\$1260.00	7½%	3 "	2 "	18 "	?	?
144.	\$ 324.00	5%	2 "	4 "	15 "	?	?
145.	\$ 162.75	7%	1 "	3 "	24 "	?	?
146.	\$ 640.00	6%		?		\$ 86.40	?
147.	\$ 960.00	5%		?		\$114.00	?
148.	\$2400.00	4%		?		\$192.00	?
149.	\$3120.00	3%		?		\$325.60	?
150.	\$1260.25	4½%		?		\$142.16	?
151.	\$ 370.00	?	1 yr.	6 mo.	15 da.	\$ 34.23	?
152.	\$ 480.00	?	2 "	3 "	12 "	\$ 76.72	?
153.	\$ 125.60	?	4 "	8 "		\$ 52.75	?
154.	\$2360.00	?	1 "	7 "	9 "	\$227.74	?
155.	\$4150.00	?		11 "	27 "	\$329.23	?
156.	?	6%	1 "	6 "	24 "	\$338.40	?
157.	?	7%	2 "	3 "	18 "	\$450.00	?
158.	?	5%	4 "	1 "	20 "	\$642.50	?
159.	?	4%	1 "	7 "	9 "	\$316.42	?
160.	?	8%	2 "	8 "	6 "	\$125.60	?
161.	?	6%	2 "	6 "	12 "	?	\$5184.00
162.	?	9%	3 "	7 "	27 "	?	\$3195.20
163.	?	3%	4 "	3 "	2 "	?	\$4160.50
164.	?	4½%	2 "	2 "	15 "	?	\$ 360.00
165.	?	7½%	1 "	1 "	9 "	?	\$3240.00
166.	\$3200.00	?	2 "	6 "	6 "	?	\$3683.20
167.	\$1240.00	?	1 "	4 "	12 "	?	\$1392.52
168.	\$2125.00	?		11 "	24 "	?	\$2250.38
169.	\$1260.00	?	2 "		15 "	?	\$1375.76
170.	\$ 496.00	?	3 "	3 "	3 "	?	\$ 592.97
171.	\$1440.00	6%		?		?	\$2100.00
172.	\$ 960.00	7%		?		?	\$1260.50
173.	\$1246.00	5%		?		?	\$1575.00
174.	\$3400.00	8%		?		?	\$4260.25
175.	\$6200.00	9%		?		?	\$6496.00

EXACT INTEREST.

217. Exact Interest is interest computed on the basis of 365 days to the year.

Exact interest, for any time less than one year, counts $\frac{1}{365}$ of a year's interest for each day. Exact interest and ordinary interest are the same for any number of years. Ordinary interest for periods less than one year may be changed approximately to exact interest by subtracting $\frac{1}{73}$ of the interest. Exact interest may be changed to ordinary interest by adding $\frac{1}{73}$ of the interest. The exact time between dates should be found. (Page 114.)

EXAMPLE.

Compute the exact interest on \$1200 from Aug. 1, 1894, to Jan. 1, 1895, at 6%.

SOLUTION.

153 days = exact time.
 $\frac{\$1200 \times .06 \times 153}{365} = \$30.18 +$
 \$30.18 = exact interest.

OR

\$12 00 = Int. for 60 da.			
24	=	"	120 "
6	=	"	30 "
60	=	"	3 "
<hr style="border: none; border-top: 1px solid black; margin: 2px 0;"/>			
30 60 = Int. for 153 da.			
\$30.60 = ordinary Int.			
\$30.60 \div 73 = \$.42.			
\$30.60 - .42 = \$30.18, exact Int			

WRITTEN PRACTICE.

Compute the exact interest on the following:

1. \$1260 for 332 da. at 6%.
2. \$375 for 185 da. at 5%.
3. \$960.75 for 243 da. at $7\frac{1}{2}$ %.
4. \$346.25 for 146 da. at 7%.
5. \$960 for 225 da. at 8%.
6. Compute the exact interest on \$420 from July 5, 1893, to Dec. 6, 1893, at $6\frac{1}{2}$ %.
7. What is the exact interest on \$3600 from Sept. 4, 1893, to Aug. 13, 1894 at 5%.
8. Find the exact interest on a \$5000 U. S. bond from Jan. 1, 1893, to Oct. 1, 1893, at 5%.

9. Find the difference between the exact and ordinary interest on \$25000, for 194 days at 5%.

10. Find the total exact interest on

\$360 for 92 da. at 6%.

\$120 for 90 da. at 6%.

\$520 for 36 da. at $6\frac{1}{2}\%$.

\$340 for 83 da. at 7%.

\$720 for 25 da. at $5\frac{1}{2}\%$.

\$160 for 75 da. at 8%.

11. Find the total exact interest on

\$960 for 116 da. at 4%.

\$320 for 62 da. at 9%.

\$480 for 314 da. at 3%.

\$145 for 37 da. at 4%.

\$125 for 124 da. at 7%.

\$500 for 16 da. at 6%.

12. Find the total ordinary interest on

\$3200 for 120 da. at 6%.

\$460 for 92 da. at 5%.

\$1260 for 142 da. at 7%.

\$1236 for 36 da. at 4%.

\$3120 for 314 da. at 4%.

\$3840 for 84 da. at 9%.

\$1460 for 215 da. at $4\frac{1}{2}\%$.

\$1450 for 39 da. at 6%.

13. Find the difference between the total exact and total ordinary interest on the following:

\$1200 for 123 da. at 6%.

\$365 for 219 da. at 5%.

\$960 for 146 da. at 5%.

\$496.50 for 180 da. at 3%.

\$320 for 95 da. at 7%.

\$324.60 for 30 da. at 4%.

\$480 for 292 da. at 8%.

\$189.70 for 85 da. at 6%.

\$560 for 165 da. at 9%.

\$320 for 120 da. at $6\frac{1}{2}\%$.

\$940 for 73 da. at $4\frac{1}{2}\%$.

\$490.20 for 40 da. at $6\frac{3}{4}\%$.

COMPOUND INTEREST.

218. The parties to a note may agree that all interest accruing upon it is to be added periodically to the principal, the sum to be taken as the principal for the succeeding period; the interest for this period is again added, the sum comprising the principal for the next period, and so on. The total interest which accrues on an obligation of this kind is known as **Compound Interest.**

The interest is usually added to the principal or "compounded" annually, semi-annually, or quarterly; but compound interest may not be collected unless there is a specific agreement to that effect in the note. In some states the form of this "compound interest clause" is prescribed by the statutes.

EXAMPLE.

Find the compound interest of \$3600 for three years at 6%.

SOLUTION.

\$3600.	= Principal.
<u>216.</u>	= Int. for 1st year.
3816.	= 1st amount.
<u>228.96</u>	= Int. for 2d year.
4044.96	= 2d amount.
<u>242.70</u>	= Int. for 3d year.
4287.66	= 3d amount.
<u>3600.</u>	Subtract principal.
\$687.66	= Compound interest.

WRITTEN PRACTICE.

1. What is the amount of \$1340 for three years at 5% compound interest?
2. Required the compound interest on \$3500 for 6 years at ~~7½%~~ ^{7%}.
3. Find the amount of \$4000 for 4 years 6 months at 4½% compound interest.

Compound Interest is generally computed by means of Interest Tables, giving the amount of \$1 for various times and rates. The table on another page gives the amount of \$1 at compound interest for any number of years from 1 to 50 inclusive, at the principal rates from 4% to 10%.

The student is expected to compute the amounts in the blank spaces, thereby showing how these tables are made.

COMPOUND INTEREST TABLE.

Showing the amount of \$1 at compound interest for any number of years, from 1 to 50, inclusive, at rates given.

Years	4 per ct.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	10 per ct.
1	1.0400 0000	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	
2	1.0816 0000	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	
3	1.1248 6400	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	
4	1.1698 5856	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	
5	1.2166 5290	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	
6		1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	
7		1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	
8		1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	
9		1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	
10		1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	
11		1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	
12		1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	
13		1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	
14		1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	
15		1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	
16		2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	4.5949 730
17		2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	5.0544 703
18		2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	5.5599 173
19		2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	6.1159 390
20		2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	6.7275 000
21		2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	
22		2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	
23		2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	
24		2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	
25		3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	
26		3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	
27		3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	
28		3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	
29		3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	
30		3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	
31	3.3731 3341	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	
32	3.5080 5875	4.0899 8104	4.7649 415	6.4533 867	8.7152 703	11.7370 830	
33	3.6483 8110	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	
34	3.7948 1634	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	
35	3.9460 8899	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	
36		4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	30.9126 805
37		5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	34.0039 486
38		5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	37.4043 434
39		5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 377	41.1447 778
40		5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	45.2592 256
41		6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	
42		6.3516 1548	7.7615 876	11.5570 327	17.1442 563	25.3394 319	
43		6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	
44		6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	
45		7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 434	
46		7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	
47		7.9152 6349	9.9059 711	15.4659 167	24.0457 070	37.2320 122	
48		8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	
49		8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	
50	7.1066 8335	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	117.3908 529

WRITTEN PRACTICE.

Use the interest table in the following:

4. What is the compound interest on \$25000 for 24 years at 5%?

5. A son, ten years of age, by bequest, is left \$20000 which is immediately placed at compound interest at 8%, payable semi-annually. What will be the amount of the bequest when the son is of age?

6. What sum will amount to \$6866.72 in 25 years at 6%, compounded annually?

7. What sum of money will amount to \$17599.97 in 40 years at 5%, compounded annually?

8. What sum of money will produce \$17191.86 in 30 years at 7%, compounded annually?

9. What sum must be invested at the age of 20 years that will amount to \$25000 at the age of 50 years at 4%, compounded annually?

3.24334731

ANNUAL INTEREST.

219. When, by special agreement, interest is allowed on the unpaid simple interest which is assumed to be due and payable periodically, the total accrued interest is known as **Annual Interest**. In annual interest the amount of the principal is not increased, but the interest on the unpaid simple interest payments is computed separately.

EXAMPLE.

Find the amount due on the following note at maturity, no payments having been made:

\$2000.00.

Chicago, Ill., Jan. 1, 1894.

Five years after date, we promise to pay W. I. Staley, or order, Two Thousand Dollars with interest at 7 per cent per annum, payable yearly without grace.

King & Kipp.

SOLUTION.

\$ 700 = Int. on principal for 5 years at 7%.

98 = Int. on annual interest for 10 years at 7%.

798 = Amount of interest due.

2000 = Principal.

\$2798 = Amount due at maturity.

EXPLANATION.—The simple interest on \$2000 for 5 years at 7% is \$700. The interest for the first year is \$140; if this is not paid until maturity, it would be 4 years past due; the second interest would be 3 years past due, etc.; in all 10 years' interest on \$140, or \$98, would be the amount produced on the simple interest, which added to the simple interest equals \$798, total interest, plus the principal \$2000, equals \$2798, the amount due at maturity.

WRITTEN PRACTICE.

Find the amount due on the following at maturity:

	Principal.	Rate.	Time.	Interest Payable.
1.	\$7200	6%	4 years	yearly
2.	\$4500	7%	3 years	yearly
3.	\$9260	5%	6 years 6 mo.	yearly
4.	\$3400	4½%	3 years	semi-yearly
5.	\$7500	7½%	4 years	semi-yearly
6.	\$9000	8%	2 years 6 mo.	semi-yearly
7.	\$4000	4%	5 years	semi-yearly
8.	\$2150	10%	4 years	quarterly
9.	\$4680	6%	3 years 6 mo.	quarterly
10.	\$2100	8%	6 years 9 mo.	quarterly

In computing the interest on the following, find the interest on the interest at 5 per cent, the legal rate in Illinois, no grace.

11.

\$1200.00

Springfield, Ill., Jan. 1, 1894.

Five years after date, I promise to pay, N. A. Barrett, or order, Twelve Hundred Dollars with interest at 7 per cent per annum, payable semi-annually.

Ed. H. Coe.

12.

\$500.00.

Quincy, Ill., Jan. 15, 1894.

Six years after date, I promise to pay W. N. Ferris, or order, Five Hundred Dollars with interest at the rate of 6 per cent per annum, payable quarterly.

C. E. Wessel.

13.

I deposit \$10000 in a savings bank that allows 4% interest, compounded quarterly. The banker immediately invests this sum in city bonds that yield 6% annually. He loans the annual interest at short terms at $7\frac{1}{2}\%$, averaging 8 loans of 30 days each, per year. How much does the banker make, all these investments being converted into cash at the end of 2 years? (No grace.)

PARTIAL PAYMENTS.

220. Partial Payments are payments in part of a note, bond, or debt, and made at different times.

221. Indorsements are acknowledgments of such payments written on the back of the note or bond, stating the date of the payment and the amount paid.

There have been no legal enactments establishing a method of computing interest in the case of partial payments. The two leading rules, or methods, are based, the one on the decisions of the Federal Court, the other on the usage of merchants.

THE UNITED STATES COURT RULE.

I. Find the amount of the principal to the time when the payment, or the sum of two or more payments, equals or exceeds the interest due; subtract the payment or the sum of the payments from the amount.

II. Consider the remainder as a new principal, and proceed in the same manner with other payments to the time of the settlement.

The principle on which this rule is based, is that money paid must first apply on the interest accrued, and the remainder, if any, applies on the principal; and that the interest unpaid cannot be added to the principal so as to compound the interest.

EXAMPLE.

Find the amount due Jan. 1, 1894, on the following note:
\$800.00.

Chicago, June 29, 1892.

One year after date, I promise to pay to The C. E. Marble Printing Co., or order, Eight Hundred Dollars, value received, with interest at 6 per cent.

N. A. Barrett.

On the back of this note are written the following indorsements: Nov. 1, 1892, \$100; Jan. 1, 1893, \$50; June 16, 1893, \$150; Sept. 21, 1893, \$250.

SOLUTION BY U. S. RULE.		6%=RATE.
92- 6-29.....	\$800	= Principal and date.
4- 2.....	16	266= Interest for 4 mo. 2 da.
	816	266= Amount of principal.
92-11- 1.....	100	= First payment and date.
2	716	266= New principal.
	7	162= Interest for 2 mo
	723	428= Amount of new principal.
93- 1- 1.....	50	= Second payment and date
5-15.....	673	428= New principal.
	18	519= Interest for 5 mo. 15 da.
	691	947= Amount of new principal.
93- 6-16.....	150	= Third payment and date.
3- 5.....	541	947= New principal.
	8	571= Interest for 3 mo. 5 da.
	550	518= Amount of new principal.
93- 9-21.....	250	= Fourth payment and date.
3-10.....	300	518= New principal.
	5	008= Interest for 3 mo. 10 da.
94- 1- 1.....	\$305	526= Balance due Jan. 1, 1894.

PROOF OF TIME.

ADD PARTIAL DIFFERENCES.

94-1- 1= Date of settlement.
 92-6-29= Date of note.
 1-6- 2= Total time.

4 mo. 2 da.
2 " 0 "
5 " 15 "
3 " 5 "
3 " 10 "
1 yr. 6 mo. 2 da.

EXPLANATION.—The principal and indorsements should be placed in a vertical column, allowing three spaces between the items for: 1. Principal, or difference brought down, 2. Interest, and 3. Amount, before deducting the payment. Subtract each date at the left from the following date, to find time for which the principal will draw interest before the next payment.

WRITTEN PRACTICE.

2.

\$2000.00.

Indianapolis, Ind., July 1, 1893.

One year after date, we jointly promise to pay William M. Whigam, or order, Two Thousand Dollars for value received, with interest at the rate of 8 per cent until paid.

J. V. Farwell & Co.

On this note were the following indorsements:

March 1, 1894	\$500	Jan. 17, 1895.....	\$600
Aug. 15, 1894	\$450		
Find balance due July 1, 1896.			

3.

\$980.00.

Phoenix, Ariz., May 1, 1890.

One year after date, I promise to pay F. S. McDaniel, or order, Nine Hundred Eighty Dollars for value received, with interest at the rate of 20 per cent per annum.

E. M. Lamson.

The following were the payments:

June 16, 1891.....	\$250
Sept. 14, 1891.....	\$200
Nov. 16, 1891	\$175

Find the balance which was paid Jan. 2, 1892.

4.

\$3200.00.

Minneapolis, Minn., Aug. 4, 1893.

On or before three years after date, we promise to pay Curtiss & Chapman, or order, Thirty-two Hundred Dollars, with interest at the rate of 8 per cent per annum.

Ed. J. Luce & Co.

On this note were indorsed the following payments:

Oct. 2, 1893.....	\$200.	Nov. 6, 1894.....	\$350.
March 3, 1894.....	260.	March 15, 1896	650.
Aug. 5, 1894.....	500.		

Find the amount due on the date of maturity.

SUGGESTION.—Date of note, plus time of note and 3 days, equals date of maturity.

5. A note of \$6500, at $7\frac{1}{2}\%$, dated March 1, 1890, bore the following payments:

Sept. 13, 1890.....\$500.	Oct. 17, 1891.....\$500.
April 15, 1891..... 100.	June 12, 1892..... 200.
Aug. 18, 1891..... 150.	

What was the balance due Sept. 1, 1892?

6. What was the amount due Aug. 1, 1891, on a note of \$1560.00, dated Burlington, Iowa, November 1, 1888, bearing the following indorsements?

Dec. 2, 1888.....\$75.	April 14, 1890\$200.
Aug. 30, 1889..... 30.	July 2, 1890..... 20.
Oct. 7, 1889..... 40.	Sept. 23, 1890..... 25.
Feb. 16, 1890..... 50.	March 5, 1891 750.

7. A note of \$2400, at $4\frac{3}{4}\%$, dated May 14, 1890, bore the following payments:

Oct. 20, 1890.....\$500.	Dec. 15, 1891.....\$60.
Aug. 16, 1891 50.	March 3, 1892 50.
Nov. 4, 1891..... 60.	

What was due Nov. 1, 1892?

8. A mortgage of \$5000, dated June 1, 1889, bore the following indorsements:

Dec. 1, 1889\$150.	June 1, 1891.....\$150.
June 1, 1890 150.	Dec. 1, 1891..... 150.
Dec. 1, 1890 150.	June 1, 1892..... 150.

What sum would pay off the claim Dec. 16, 1892, at 6%?

9. Find the amount due Aug. 1, 1894, on a note of \$3500, dated Sept. 4, 1892, and indorsed as follows:

Dec. 19, 1892.....\$300.	April 4, 1893\$ 100.
Feb. 14, 1893..... 450.	July 3, 1894 1000.

At $4\frac{1}{2}\%$. At $7\frac{1}{2}\%$.

10. On Oct. 15, 1892, I borrowed a certain sum of money secured by mortgage at 6% interest. The following partial payments have been made:

Nov. 24, 1892.....\$200.	Aug. 18, 1893.....\$750.
April 15, 1893..... 300.	

Jan. 31, 1894, I still owe \$4099.789. Find the sum borrowed.

11.

\$2000.00.

Chicago, Ill., Feb. 12, 1896.

One year after date I promise to pay L. E. Goodyear, or order,
Two Thousand Dollars, value received, without grace.

H. S. Lehr.

On this note were the following indorsements:

Feb. 12, 1897.....\$500. Sept. 20, 1897.....\$400.

Aug. 5, 1897..... 200. Jan. 1, 1898..... 500.

Find the balance due Feb. 12, 1898.

12.

\$2000.00.

Chicago, Jan. 1, 1896.

Six months after date I promise to pay James Spencer, or order,
Two Thousand Dollars, value received, at the Prairie State National
Bank, without grace.

A. B. James.

Indorsed as follows:

Sept. 6, 1896.....\$300. Oct. 14, 1897.....\$100.

Jan. 15, 1897..... 50. Nov. 15, 1897..... 500.

What is the balance due Jan. 1, 1898?

13.

\$5000.00.

Jacksonville, Ill., Jan. 1, 1896.

Two years after date, we promise to pay The Granite Cement Co.,
or order, Five Thousand Dollars, value received, with interest at 5
per cent until due, after maturity to draw 7 per cent interest. (No
grace.)

Brown & Davis.

Indorsed as follows:

March 16, 1897.....\$1000. Feb. 20, 1898.....\$900.

Nov. 15, 1897..... 1200. July 16, 1898..... 800.

What is the balance due Nov. 1, 1899?

THE MERCHANTS' RULE.

For one year or less.

RULE.—Find the amount of the principal to the date of settlement;
find the amount of each payment from the date of payment to the date
of maturity; take the sum of these amounts from the amount of the
principal; the remainder is the balance due.

For more than one year.

RULE.—Find the balance due at the end of one year, as in the preceding rule, and treat this as a new principal; proceed in the same manner for each entire year that follows, and for the portion of a year that may occur before the time of settlement.

The Merchants' Rule is generally used in computing interest on open accounts.

EXAMPLE.

Find the balance due June 29, 1893, of an account of \$800 contracted June 29, 1892, with interest at 6%.

Payments have been made as follows: Nov. 1, 1892, \$100; Jan. 1, 1893, \$50; June 16, 1893, \$15.

SOLUTION BY MERCHANTS' RULE.

93-6-29	Date of settlement.	6%
92-6-29	\$800
1	48
92-11-1	... \$100	848
7-28	... 3	966
93-1-1	... 50	
5-28	... 1	483
93-6-16	... 15	
13	... 032	170 481
		\$677 519

EXPLANATION. — Arrange the principal and indorsements in the order of their dates in two vertical columns, the right for the principal, and the left for the indorsements, allowing one space to intervene between the items. Subtract each date from the settlement date, to find interest period for each item, insert the interest beneath each item, and from the amount of principal and interest, subtract the sum of the indorsements

and interest. The difference is the required balance due.

WRITTEN PRACTICE.

1. A note due July 1, 1892, for \$2500, had the following indorsements:

Sept. 3, 1892\$200. March 19, 1893.....\$300.

Jan. 16, 1893..... 500. June 25, 1893..... 400.

What was the amount due July 1, 1893, at 6%? At 12%?

2. A note for \$3250, dated Aug. 16, 1890, was indorsed as follows:

Oct. 14, 1890.....\$200. April 1, 1891... ..\$1300.

Jan. 2, 1891..... 375. July 3, 1891..... 750.

Find balance due Aug. 10, 1891, at $5\frac{1}{2}\%$. At $6\frac{1}{2}\%$.

3. Find the amount due Oct. 1, 1893, on a note of \$2450,

dated Oct. 4, 1891, at 5% interest, which bore the following payments:

Nov. 1, 1891, \$1350; Jan. 14, 1892, \$100; March 4, 1892, \$900.

4. A note for \$6000, dated Feb. 1, 1890, for 3 months, without interest, was indorsed as follows: (No grace.)

June 16, 1890.....\$1000. Sept. 14, 1892....\$760.

Aug. 19, 1890..... 600. Jan. 20, 1893.....850.

March 24, 1891..... 350.

Find amount due March 15, 1893, at 4%. At 8%.

5. A note dated July 1, 1888, due in 6 months without grace, for \$5000, with interest after maturity at 8%, has indorsements of \$375 every three months, beginning with April 1, 1889, as the date of the first payment. What amount remained after the last payment, which was made Oct. 4, 1892?

TRUE DISCOUNT.

222. Discount is an allowance made for the payment of a debt before it is due.

223. True Discount is the difference between the amount of a debt due at some future time, and its present valuation.

224. The Present Worth of a debt is its present valuation. It is such a sum that if put at interest on the date of discount, will amount to the face of the debt on the due date.

In true discount the debt due is treated as an amount, therefore the solution would be the same as the solution of interest problems under Art. 214.

Days of grace will not be counted in working the problems *in true discount*.

PROMISCUOUS WRITTEN EXERCISE.

	Face.	Rate.	Time.	Discount.	Pres. Wth.
1.	\$784.	6%	2 yr.	?	?
2.	787.40	9%	3 yr.	?	?
3.	750.40	6%	1 yr. 6 mo. 12 da.	?	?
4.	920.70	6%	4 mo. 18 da.	?	?
5.	?	7½%	1 yr. 6 mo.	\$ 40.50	?
6.	?	3%	1 yr. 8 mo. 18 da.	26.78	?
7.	?	7%	6 mo. 6 da.	260.40	?

PROMISCUOUS WRITTEN EXERCISE.

	Face.	Rate.	Time.	Discount.	Pres. Wth.
8.	?	9%	21 da.	\$5.25	?
9.	?	8%	5 mo. 24 da.	39.68	?
10.	?	?	1 yr. 6 mo.	64.80	\$ 720
11.	?	?	2 yr. 4 mo. 12 da.	127.80	900
12.	?	?	1 yr. 6 mo. 18 da.	59.52	640
13.	?	?	3 mo 21 da.	74.	3000
14.	?	6%	?	13.95	606.05
15.	?	8%	?	17.50	607.50
16.	?	9%	?	113.90	736.10
17.	?	7½%	?	8.05	451.95
18.	\$650	6%	?	12.05	?
19.	960	9%	?	26.24	?
20.	520	6%	?	74.36	?

WRITTEN PROBLEMS.

21. What is the present worth of a debt of \$448 due in 2 yr., money being worth 6%?

22. Find the present worth of a debt of \$465.50 due in 2 yr. 6 mo., money being worth 9%.

23. What sum will pay a debt of \$787.80 due in 3 yr. 6 mo. 12 da., at 6%?

24. I owe \$660 due in 2 yr. 6 mo. What sum of money will pay the debt now, money being worth 8%?

25. I bought a bill of dry goods amounting to \$978 on 4 mo. credit. One month after date of purchase I paid the present worth at 7½%. What sum did I pay?

26. A dealer purchased a bill of groceries amounting to \$874.64 on 6 mo. credit, or \$835 cash. Which is the better offer and how much, money being worth 8%?

27. Brown & Co. paid a bill of \$2007, 21 days before it was due. What was the true discount, money being worth 6%?

28. A farm was offered for cash \$12000, or for \$5000 due in 4 mo., and \$8000 due in 1 yr. 6 mo. Which offer is the better and how much, money being worth 8%?

29. I bought 1200 bushels of wheat at \$1.25 per bushel on 8 months credit, and immediately shipped same to my agent in

Chicago, who sold and remitted me draft for \$1762.50. I paid for my purchase 2 mo. 15 days before due, less true discount at 6%. How much did I gain by the transaction?

30. A merchant bought 600 barrels of potatoes at \$4.75 per barrel, 10% off if paid in cash. He accepted the cash offer and borrowed the money at bank, paying the interest in advance for 2 months at 7%. How much did he gain?

31. On Mar. 1, 1896, I bought one bill of goods amounting to \$1246.25 on 2 mo. credit, and one of \$896.40 on 3 months credit. On April 1, 1896, I paid the present worth of both bills. What sum did I pay, money being worth 6%? (c. s.)

32. A offers flour at \$8 per barrel on 8 mo. credit, and B offers the same grade of flour at \$7.50 per barrel on 6 mo. credit. Which offer is the better and how much, money being worth 6%?

33. A real estate dealer sold a section of land for \$16000 on the following terms: Cash \$4000; \$4000 in 6 months; \$3000 in 1 year; and the balance in 2 years, all payments secured by mortgage. Two months after date the purchaser finds that he will be allowed a true discount of 10%. To accept this offer, he must borrow the required sum for 2 years at $7\frac{1}{2}\%$ interest, payable in advance. He accepts the offer, does he gain or lose and how much?

34. Bought a bill of goods amounting to \$504 on 10 months credit. How much money will pay the debt at the time of purchase at 6%?

35. Find the true discount on a bill of groceries amounting to \$3640.50, of which \$1250 is to be paid in 4 months, \$1436 to be paid in 8 months, and the remainder in 1 year 8 months, money being worth 6%.

36. Bought a bill of jewelry amounting to \$3640, less a trade discount of 25% and 20%, on a credit of 4 months. Money being worth $7\frac{1}{2}\%$, how much should be allowed as an additional discount if the bill is paid 1 month after the date of sale?

37. A commission merchant bought a consignment of 960 bushels of wheat at \$.87 $\frac{1}{2}$ per bushel on 4 months credit; he immediately sold it at a profit of $8\frac{1}{4}\%$, and then paid his pur-

chase price less a true discount of 6%. How much did he gain by this transaction?

38. On closing business Jan. 1, 1896, I find the following sales outstanding: Dec. 1, 1895, a bill amounting to \$960.40 on 60 days credit; Dec. 4, 1895, a bill amounting to \$564.75 on 2 months credit; Dec. 15, 1895, a bill amounting to \$436.50 on 30 days credit, and on Dec. 24, 1895, a bill of \$550.40 on 60 days credit. On Jan. 1, 1896, I offer these debtors a true discount of 6%. They immediately accept, how much do I receive? (c. s.)

BANK DISCOUNT.

225. A **Bank** is an institution for the deposit, discount, or circulation of money.

226. A **Note** is written evidence of debt, coupled with a promise to pay.

227. The **Parties** to a note are maker and payee.

228. The **Maker** is the party promising.

229. The **Payee** is the one to whom the promise is made.

230. A **Draft** is an order on one person to pay to another.

Time drafts are drawn a certain number of days after sight or date.

Sight drafts are drawn "at sight."

231. The **Parties** are drawer, drawee, and payee.

232. The **Drawer** is the one who writes the draft. He asks that the draft be paid.

233. The **Payee** is the one to whom the draft is given.

234. The **Drawee** is the one on whom the draft is drawn. He is asked to pay the draft.

235. An **Indorsement** is a writing upon the back of commercial paper.

A. It may be for the purpose of transferring one's interest.

B. It may be to acknowledge a part payment.

C. It may be for the purpose of changing the original contract.

236. An **Acceptance** is an acknowledgment by the drawee of a draft that he will pay when due.

It is generally in writing upon the face.

237. An **Indorser** is one who writes his name on the back of commercial paper.

238. **Days of Grace** are three days usually allowed for the payment of paper after the expiration of the time expressed.

The law relating to days of grace is not uniform in the different states.

The maturity of a note is the expiration of the time mentioned in the note, and the days of grace.

A note is nominally due at the expiration of the time stated on the face of the note.

A note is legally due at the expiration of the days of grace.

239. The **Face** is the sum named in the note.

240. **Bank Discount** is simple interest computed upon a sum due at a future date, and paid in advance.

241. The **Proceeds** is the face or amount less the discount.

242. The **Term of Discount** is the time from the date of discount to the date of maturity.

The term of discount includes the days of grace.

There seems to be no uniformity adopted by the banks in regard to the finding of the term of discount. Some use compound subtraction, and 360 days to the year; others count the exact number of days, and use either 360 or 365 days to the year.

In this work we have adopted the method that seems to be in general use. The term of discount is found by counting the exact number of days, excluding the day of discount and including the last day of maturity, and calling 360 days a year.

AMOUNT TO BE DISCOUNTED.

In non-interest bearing notes the face is the sum to be discounted.

In interest bearing notes the face plus the interest due at maturity is the sum to be discounted.

BANK DISCOUNT COMPARED WITH PERCENTAGE.

Face (if no interest) = Base.

Face + Interest = Base.

Rate of Discount = Rate, or Rate Per Cent.

Discount = Percentage.

Proceeds = Difference, or Percentage.

243. To find the **Term of Discount**, the date of discount and date of maturity being known.

In estimating the date of maturity, add months or days according to the term given in the note. In estimating term of discount, count the exact days between date of discount and date of maturity.

EXAMPLE.

Find the date of maturity and the term of discount of a note at 4 mo., dated Jan. 4, 1896, and discounted on same date.

SOLUTION.

Jan. 4 + 4 mo. 3 days = May 7, date of maturity.

27 days left in Jan.

29 " " Feb.

31 " " Mar.

30 " " Apr.

7 " " May.

124, total number of days, the term of discount.

OR,

4 mo. 3 days = 123 days.

Add 2 " extra in Jan. and Mar.

125

less 1 day for Feb.

124 days, term of discount.

WRITTEN PRACTICE.

Find the date of maturity and the term of discount of the following paper, discounted on date of issue:

	Date of Note.	Time.		Date of Note.	Time.
1.	Feb. 16, 1892,	90 da.	5.	Dec. 29, 1896,	2 mo.
2.	Mar. 11, 1892,	3 mo.	6.	Oct. 31, 1896,	120 da.
3.	May 31, 1892,	6 mo.	7.	Dec. 30, 1896,	2 mo.
4.	July 31, 1892,	60 da.	8.	Dec. 31, 1896,	2 mo.

Find the date of maturity and term of discount of the following:

	Date of Note.	Time.	Date of Discount.
9.	April 16, 1895	60 days	April 16, 1895
10.	May 3, 1895	3 mo.	May 3, 1895
11.	May 16, 1896	90 days	May 16, 1896
12.	June 14, 1896	6 mo.	June 14, 1896
13.	July 25, 1896	4 mo.	Aug. 4, 1896
14.	Aug. 20, 1896	75 days	Sept. 1, 1896
15.	Sept. 24, 1896	90 days	Oct. 15, 1896
16.	Oct. 29, 1892	8 mo.	Dec. 24, 1892
17.	Nov. 1, 1895	120 days	Nov. 30, 1895
18.	Dec. 15, 1895	9 mo.	Mar. 15, 1896

244. To find the Discount and Proceeds, the face, rate, and time being known, on notes without interest.

RULE.—I. *Compute the discount, like interest on the face of the note for the term of discount and at the rate named.*

II. *Subtract the discount from the face, to find the proceeds.*

On notes bearing interest.

RULE.—I. *Compute interest on the face of the note to maturity, and add the interest to the face of the note, to find amount due.*

II. *Compute the discount on this amount for the term of discount.*

III. *Subtract the discount from the amount due at maturity, to find the proceeds.*

Formulas.

$$F \times R \times T = D \dots \dots F - D = P$$

$$(F + I) \times R \times T = D \dots \dots F + I - D = P$$

Discounting paper on date of issue.

TIME NOTE IN DAYS. WITHOUT INTEREST.

EXAMPLE.

\$650.00.

Detroit, Mich., Apr. 3, 1896.

Ninety days after date we promise to pay Marshall Field & Co., or order, Six Hundred Fifty Dollars. Value received.

Earnest & Faithful.

Find proceeds discounted on date of issue at 6%.

SOLUTION.

\$650.00 = Face of note.

10.075 = Discount for 93 days at 6 per cent.

\$639.925 = Proceeds.

EXPLANATION.—Allowing days of grace, the note matures in 93 days. The interest, or bank discount, for 93 days on \$650 at 6% is \$10.075, which taken from \$650 leaves \$639.925, the proceeds.

TIME NOTE IN MONTHS. WITHOUT INTEREST.

EXAMPLE.

\$650.00. *Detroit, Mich., Apr. 3, 1896.*

Three months after date we promise to pay Marshall Field & Co., or order, Six Hundred Fifty Dollars. Value received, without interest. Earnest & Faithful.

Find proceeds, discounted on date of issue at 6%.

SOLUTION.

Apr. 3+3 mo. 3 da.=July 6, date of maturity.

Exact time from Apr. 3 to July 6=94 days, term of discount.

\$650.00 = Face of note.

10.18 = Discount (Interest) for 94 days at 6%.

\$639.82 = Proceeds.

WRITTEN PRACTICE.

Find the proceeds of the following:

Use days of grace unless otherwise instructed.

	Date of Note.	Date of Discount.	Time.	Face.	Rate of Disc.
19.	Mar. 4, 1896	Mar. 4, 1896	90 da.	\$ 720	6 %
20.	Apr. 15, 1896	Apr. 15, 1896	80 da.	\$ 980	7½ %
21.	Dec. 31, 1896	Dec. 31, 1896	60 da.	\$ 925	6¾ %
22.	Dec. 28, 1896	Dec. 28, 1896	60 da.	\$1260	4½ %
23.	Jan. 14, 1897	Jan. 14, 1897	75 da.	\$ 760	5¼ %
24.	June 3, 1896	June 3, 1896	3 mo.	\$2400	6½ %
25.	July 5, 1896	July 5, 1896	4 mo.	\$5680	5½ %
26.	Sept. 3, 1896	Sept. 3, 1896	8 mo.	\$ 420.75	4¾ %
27.	Dec. 28, 1896	Dec. 28, 1896	2 mo.	\$ 860.50	7 %
28.	Dec. 30, 1896	Dec. 30, 1896	2 mo.	\$ 924	5¾ %

SUGGESTION.—Write out above notes.

Discounting paper on date later than date of issue.

JOINT NOTE. WITHOUT INTEREST.

EXAMPLE.

\$750.00.

Chicago, June 15, 1895.

Three months after date, we promise to pay O. E. Grant, or order, Seven Hundred Fifty Dollars. Value received.

W. M. Whigam,

E. M. Lamson.

Find proceeds, discounted July 15, 1895, at 6 per cent.

SOLUTION.

June 15 + 3 mo. 3 days = Sept. 18, date of maturity.

Exact time from July 15 to Sept. 18 = 65 days.

\$750.00 = Face of note, amount to be discounted.

8.125 = Discount for 65 days.

\$741.875 = Proceeds of note.

EXPLANATION.—Allowing days of grace, the note matures on Sept. 18. The exact number of days from July 15 to Sept. 18 is 65. The interest or bank discount for 65 days on \$750 at 6% is \$8.125, which taken from \$750 leaves \$741.875, the proceeds.

WRITTEN PRACTICE.

Find the proceeds of the following:

	Date of Note.	Date of Discount.	Time.	Face.	Rate of Disc't	
29.	May 3, '95	June 4, '95	72 da.	\$ 840	6 %	43
30.	July 5, '95	July 16, '95	65 da.	\$1300	5 %	55
31.	June 16, '95	Aug. 1, '95	95 da.	\$ 750	7 %	52
32.	Aug. 14, '95	Sept. 3, '95	83 da.	\$ 860	5½ %	46
33.	Aug. 31, '96	Sept 30, '96	75 da.	\$ 680	6½ %	43
34.	Sept. 30, '96	Oct. 15, '96	3 mo.	\$ 500	7½ %	71
35.	Aug. 31, '96	Sept. 30, '96	6 mo.	\$1275	6¾ %	157
36.	Oct. 15, '96	Dec. 20, '96	8 mo.	\$5280	5¾ %	158
37.	Nov. 24, '96	Jan. 2, '97	7 mo.	\$3400	4½ %	170
38.	Dec. 30, '96	Feb. 28, '97	5 mo.	\$2100	5¼ %	27

SUGGESTION.—Write out above notes.

Discounting paper having rate of brokerage.

Brokerage is computed on the face.

JOINT AND SEVERAL NOTE. WITHOUT INTEREST.

EXAMPLE.

\$960.00.

New York, Aug. 16, 1896.

Four months after date we, or either of us, promise to pay S. S. Packard or order, Nine Hundred Sixty Dollars. Payable at Prairie State National Bank.

A. B. Jones.

M. V. Bemis.

Find proceeds, discounted Sept. 3, 1896, discount $6\frac{1}{2}$ per cent, brokerage one-quarter per cent.

SOLUTION.

Aug. 16+4 mo. 3 da.=Dec. 19, date of maturity.

Exact time from Sept. 3 to Dec. 19=107 days.

\$960.00=Face of note.

18.55=Discount for 107 days at $6\frac{1}{2}\%$.

2.40=Brokerage on \$960 at $\frac{1}{4}\%$.

20.95=Total charges.

\$939.05=Proceeds of note.

EXPLANATION.—Allowing days of grace the note matures Dec. 19, 1896. The exact number of days from Sept. 3, to Dec. 19, is 107, the term of discount. The discount for this time is \$18.55 The brokerage of one-quarter per cent charged on the face is \$2.40, which added to the discount, \$18.55, makes a total charge of \$20.95. The face, \$960, less the total charge \$20.95, equals \$939.05, the proceeds.

WRITTEN PRACTICE.

Find the proceeds of the following:

	Date of Note.	Date of Disct.	Time.	Face.	Rate of Disct.	Rate of Bro.
39.	Feb. 16, '96	Feb. 16, '96	90 da.	\$1600	5 %	$\frac{1}{4}$ %
40.	Mar. 14, '96	Apr. 1, '96	3 mo.	\$ 750	6 %	$\frac{1}{8}$ %
41.	May 1, '96	May 1, '96	65 da.	\$ 980	$5\frac{1}{4}$ %	$\frac{1}{8}$ %
42.	Apr. 1, '96	July 12, '96	6 mo.	\$ 575.60	7 %	$\frac{1}{8}$ %
43.	Jan. 20, '96	Jan. 20, '96	2 mo. 15 da.	\$ 840.50	$7\frac{1}{2}$ %	$\frac{1}{10}$ %
44.	Feb. 24, '96	Mar. 20, '97	1 yr. 6 mo.	\$ 380.70	8 %	$\frac{1}{4}$ %
45.	July 16, '96	July 16, '96	3 mo. 18 da.	\$ 746	$6\frac{3}{4}$ %	$\frac{1}{8}$ %
46.	Aug. 20, '96	Sept. 1, '97	1 yr. 3 mo.	\$6400	5 %	$\frac{1}{8}$ %
47.	Oct. 15, '96	Oct. 15, '96	9 mo.	\$1350	$5\frac{1}{2}$ %	$\frac{1}{4}$ %
48.	Sept. 1, '96	Oct. 15, '96	60 da.	\$2450	$4\frac{1}{2}$ %	$\frac{1}{10}$ %

SUGGESTION.—Write out above notes.

Discounting interest-bearing paper.

TIME NOTE. WITH INTEREST.

EXAMPLE.

\$760.75.

Detroit, Mich., Dec. 16, 1896.

Seventy-five days after date, we promise to pay Harper Bros. & Co. or order, Seven Hundred Sixty and $\frac{75}{100}$ Dollars. Value received, with interest at the rate of $5\frac{1}{4}$ per cent per annum.

Smith, Jones & Co.

Find proceeds, discounted on date of issue at 7%.

SOLUTION.

75 days + 3 days = 78 days.

\$760.75 = Face of note.

8.65 = Interest on face for 78 days at $5\frac{1}{4}\%$.

\$769.40 = Amount to be discounted.

11.67 = Discount for 78 days at 7%.

\$757.73 = Proceeds.

WRITTEN PRACTICE.

Find the proceeds of the following:

	Date of Note.	Date of Disc.	Time.	Face.	Rate of Int.	Rate of Dis.
49.	May 18, '96	May 18, '96	5 mo.	\$ 890.	6%	7%
50.	June 1, '96	June 1, '96	90 da.	\$ 765.25	$6\frac{1}{2}\%$	$7\frac{1}{2}\%$
51.	Aug. 24, '96	Aug. 24, '96	7 mo.	\$1275.	5%	6%
52.	July 16, '96	July 31, '96	85 da.	\$ 625.	$6\frac{1}{4}\%$	$6\frac{3}{4}\%$
53.	Sept. 17, '96	Oct. 20, '96	10 mo.	\$1365.	$5\frac{1}{2}\%$	$6\frac{1}{2}\%$
54.	Nov. 20, '96	Nov. 20, '96	75 da.	\$2400.	5%	7%
55.	Dec. 5, '96	Jan. 2, '97	125 da.	\$1455.	$5\frac{3}{4}\%$	$6\frac{3}{4}\%$
56.	Sept. 4, '96	Sept. 4, '96	8 mo.	\$3200.	4%	6%
57.	Oct. 1, '96	Oct. 20, '96	1 yr.	\$4000.	$4\frac{1}{2}\%$	$5\frac{1}{2}\%$
58.	Nov. 16, '96	Nov. 16, '96	9 mo.	\$3500.	5%	8%

Discounting interest-bearing paper through a broker.

Brokerage is computed on the face.

EXAMPLE.

\$9500.00.

San Francisco, July 2, 1896.

Eight months after date I promise to pay Hinman & Smith, or order, Nine Thousand Five Hundred Dollars. Value received, with interest, at the rate of $6\frac{3}{4}$ per cent per annum until paid.

A. R. Kip

Find proceeds, discounted Sept. 1, 1896, at 8 per cent, brokerage one-tenth per cent.

SOLUTION.

July 2+8 mo. 3 da.=Mar. 5, date of maturity.

Exact time from Sept. 1 to Mar. 5=185 days.

\$9500.00=Face of note.

432.84=Interest for 8 mo. 3 da. at $6\frac{3}{4}\%$.

9932.84=Amount to be discounted.

408.35=Discount for 185 days at 8%.

9.50=Brokerage on \$9500 at $\frac{1}{10}\%$.

417.85=Total charges.

\$9514.99=Proceeds of note.

WRITTEN PRACTICE.

Find the proceeds of the following:

	Date of Note.	Date of Disc.	Time.	Face.	Rate Int.	Rate Dis.	Rate Bro.
59.	Aug. 13, '96	Aug. 13, '96	2 mo.	\$3200	4%	6%	$\frac{1}{8}\%$
60.	Oct. 3, '96	Nov. 2, '96	90 da.	\$1225	6%	7%	$\frac{1}{5}\%$
61.	July 2, '96	July 2, '96	6 mo.	\$3400	$5\frac{1}{2}\%$	$6\frac{1}{2}\%$	$\frac{1}{4}\%$
62.	Apr. 21, '96	May 20, '96	100 da.	\$5000	$5\frac{1}{2}\%$	$6\frac{1}{2}\%$	$\frac{1}{5}\%$
63.	May 15, '96	June 25, '96	7 mo.	\$2100	$6\frac{1}{2}\%$	6%	$\frac{1}{4}\%$
64.	Mar. 28, '96	Mar. 28, '96	90 da.	\$1400	$7\frac{1}{2}\%$	7%	$\frac{1}{4}\%$
65.	June 16, '96	July 18, '96	8 mo.	\$1350	7%	$6\frac{3}{4}\%$	$\frac{1}{4}\%$
66.	Sept. 21, '96	Sept. 21, '96	125 da.	\$1860	$6\frac{3}{4}\%$	$7\frac{1}{4}\%$	$\frac{1}{8}\%$
67.	Oct. 14, '96	Oct. 30, '96	9 mo.	\$1720	$6\frac{1}{4}\%$	$6\frac{3}{4}\%$	$\frac{1}{10}\%$
68.	Dec. 16, '96	Jan. 3, '97	60 da.	\$4365	$4\frac{1}{2}\%$	$5\frac{1}{2}\%$	$\frac{1}{8}\%$

PROMISCUOUS WRITTEN PROBLEMS.

69.

\$2000.00.

Cincinnati, June 18, 1896.

On December 1, 1896, I promise to pay Geo. E. King, or order, Two Thousand Dollars, for value received with interest at the rate of $6\frac{1}{2}\%$ per cent per annum.

Sam'l B. Willey.

Find proceeds, discounted Aug. 1, 1896, at 7%.

70. Wishing to negotiate a loan of \$5000 for 6 months from July 1, 1892, I secure two offers, one at a discount of $6\frac{1}{2}\%$, 365 days to the year, and the other at a discount of 6%, 360 days.

to the year, with a collection of $\frac{1}{4}\%$. Which is the better offer and how much?

71. I offered at a bank the following paper which was discounted and proceeds placed to my credit Sept. 12, 1896, at $6\frac{3}{4}\%$:
Note dated Aug. 16, 1896, Time 3 mo., Int. 6 %, Face \$ 890.50

"	Sept. 4, 1896,	" 90 da.,	" $5\frac{1}{2}\%$,	" 1260.00
Draft accep.	Aug. 24, 1896,	" 60 da.,	"	1250.75
"	Sept. 3, 1896,	" 4 mo.,	"	564.25
" maturing	Nov. 15, 1896,	"	"	1000.00

What amount was placed to my credit?

72. My bank account is overdrawn \$3164.75, and I offer the following discounts, which are passed and placed to my credit Jan. 4, 1896, at 6%:

Note dated Nov. 1, 1895, for \$2500, Time 6 mo., Int. $5\frac{1}{2}\%$

Note dated Nov. 24, 1895, for 4800, Time 3 mo., Int. 4 %

Draft accept. Dec. 5, 1895, for 1000, Time 60 da.

Find my present credit balance with the bank.

73.

\$10000 00.

Chicago, January 1, 1897.

At 60 days sight pay W. M. Whigam, or order, Ten Thousand Dollars, and charge to the account of

To Illinois Steel Co., S. Chicago Crane Bros. & Co.

Find proceeds, discounted Jan. 20, 1897, at 6%.

The following was written across the face: "Accepted Jan. 5, 1897, Illinois Steel Co., per H. S. Lehr, Pres."

Above draft is due 63 days after the date of acceptance.

74.

\$25000.00.

New York, February 1, 1896.

At sixty days after date pay James B. Brown Mfg. Co., or order. Twenty-five Thousand Dollars and charge to the account of

Metropolitan Elevated R. R. Co. Pullman Palace Car Co.,
Chicago, Ill.

For proceeds, discounted March 1, 1896, at 6%.

The following was written across the face: "Seen Feb. 8, 1896, Metropolitan Elevated R. R. Co., per A. B. Jones, Treas."

Above draft is due 63 days after date of paper.

75.

\$8000.00.

Boston, Mar. 1, 1895

Five years after date I promise to pay A. B. Jamieson, or order, Eight Thousand Dollars with interest at the rate of 6 per cent per annum.

S. A. Goodfellow.

On this note were the following indorsements:

June 16, 1895, \$1200.

Aug. 1, 1897, 2000.

Oct. 25, 1898, 2500.

This note was discounted at bank July 1, 1899, at $7\frac{1}{2}\%$.

Find proceeds.

76.

\$600.00.

Chicago, Aug. 1, 1896.

At ninety days after sight pay to A. N. Palmer, or order, Six Hundred Dollars and charge same to the account of

A. B. Manning & Co.

To Grant & Snyder, San Francisco, Cal.

Find proceeds discounted on Aug. 1, 1896, at bank at $7\frac{1}{2}\%$, allowing 6 days for acceptance, and same time for return of collection.

77. Sold goods at $33\frac{1}{3}\%$ above cost price, for \$5500 on 3 months credit; the buyer offers to pay cash if allowed 10% discount; seller does not accept offer, but takes buyer's note and has it discounted at bank at $\frac{1}{2}\%$ a month. How much more did he gain by the latter than by the former transaction, and what was his total gain?

245. To find the face of a note, the proceeds, term of discount, and rate of discount being given.

RULE.—I. Subtract the discount for the given time at the given rate from \$1.00, the result is the proceeds of \$1.00.

II. Divide the given proceeds by the proceeds of \$1.00, the result is the face.

EXAMPLE.

78. Wishing to borrow \$1200, for what sum must I draw my note, payable at bank in 60 days, so that when discounted at 6%, I shall receive the desired amount?

SOLUTION.

\$1.00 = Face of \$1.

.0105 = Discount on \$1 for 63 da. at 6%.

.9895 = Proceeds of \$1.

\$1200 = Proceeds of note.

\$1200 ÷ .9895 = \$1212.73, face of note.

79. I purchased goods amounting to \$750 on 6 mo. credit, 5% discount being offered for cash. I immediately accept the cash offer, and have my note for 60 days discounted at bank at 6% for just enough to pay the account. What did I gain or lose?

WRITTEN PRACTICE.

	Proceeds.	Time of note.	Rate of Discount.
80.	\$ 720.	120 days	8%
81.	\$1600.	3 mo.	7½%
82.	\$ 950.	4 "	4½%
83.	\$ 668.10	126 days	5%
84.	\$2956.	96 "	5½%
85.	\$1451.25	6 mo.	6½%
86.	\$5856.50	123 days	7%
87.	\$ 862.75	63 "	8%
88.	\$ 881.60	92 "	9%

SUGGESTION.—Write out above notes.

89. Owing \$960, I have my note discounted at a bank at 6% for 90 days, for such a sum that the proceeds will just pay the debt. What was the face of the note?

90. A man owes \$12000, and to pay this has a note of \$7500, maturing in 45 days, discounted at 6%, and draws a second note payable in 60 days for such a sum that when discounted at the same rate the proceeds of both notes will enable him to pay the debt. Find the face of the second note.

STOCKS AND BONDS.

246. A **Corporation** is an association of individuals transacting business as a single person under rights and limitations granted by statute, or charter.

247. A **Statute** is a general enactment of a state, or government.

248. A **Charter** is a special enactment, granting the rights and defining the nature of the business of the corporation.

249. The **Capital Stock** of a corporation is the money or capital invested and represented by the shares.

250. A **Share** is one of the equal parts into which the capital stock is divided.

A share is usually \$100, but may be any sum agreed upon. Shares of \$50 and \$25 are sometimes designated as half-stock and quarter-stock.

251. A **Stock Certificate** is written evidence of the holder's title to a described share or interest in stock.

252. The **Gross Earnings** are the total receipts from the business before any expenses have been deducted.

253. The **Net Earnings** are the profits after all the expenses have been paid.

254. A **Dividend** is an apportionment of a certain part of the earnings; it is usually declared at a certain per cent.

255. An **Assessment** is a sum levied, usually at a certain per cent, upon the stock to meet expenses.

A corporation that declares a dividend is generally prosperous; its stock is usually above par.

A corporation that levies an assessment is generally not in a prosperous condition; its stock is generally quoted below par.

256. The **Par Value**, or **Face Value**, is the sum for which the certificate of stock calls.

257. The **Market Value** is what the stock brings in the market when sold.

Stock is at par when sold for face value, or 100%; it is below par when it sells for less than 100%; it is above par when it sells for more than 100%.

258. Preferred Stock is stock that is entitled to a share of the profits before the regular stock. It does not share in the general dividends.

259. Watered Stock is an inflation of the capital stock by the issuance of stock for which no payment is made.

It is issued to cover up large per cent dividends, and also whenever the charter limits the per cent of dividend.

260. Bonds are obligations issued by corporations, to pay a certain sum of money.

They are similar in effect to notes, the main difference being that the bond is under seal while the note is not.

261. Government Bonds are bonds issued by the U. S. Government. They usually take their name from the rate and date they bear, as $4\frac{1}{2}$'s of '91, means $4\frac{1}{2}\%$ bonds payable in 1891.

Stock quotations are market prices quoted at a certain sum per share, as $112\frac{1}{2}$ means $\$112\frac{1}{2}$ per share of $\$100$, a premium of $12\frac{1}{2}\%$. $94\frac{1}{2}$ means $\$94.75$ per share of $\$100$, a discount of $5\frac{1}{2}\%$.

Brokerage is computed on the par value. Stocks bought at $112\frac{1}{2}$ with $\frac{1}{8}\%$ brokerage, cost $1.12\frac{1}{2} + .00\frac{1}{8} = 1.12\frac{5}{8}$, or $\$112\frac{5}{8}$ per share of $\$100$. Stocks bought at $54\frac{1}{2}$ with $\frac{1}{8}\%$ brokerage, cost $.54\frac{1}{2} + .00\frac{1}{8} = .54\frac{5}{8}$, or $\$54\frac{5}{8}$ per share of $\$100$.

As compared with percentage: Par Value=Base; Per cent of Premium, Discount, Assessment, or Dividend=Rate per cent; Premium, Discount, Assessment, or Dividend=Percentage; Market Value=Percentage, sometimes called amount, or difference.

WRITTEN PRACTICE.

1. What is the premium on 12 shares of stock bought at $112\frac{1}{2}$? What is the market value?

SUGGESTION.— $12\frac{1}{2}\%$ of $\$1200$ is the premium.

2. What is the discount on 15 shares of stock sold at $94\frac{1}{2}$? What is the market value?

3. A speculator holding 85 shares of stock receives a dividend of $\$637.50$. What was the rate of dividend?

SUGGESTION.— $\$637.50$ is what per cent of $\$8500$?

4. A gentleman holds 36 shares of mining stock and meets an assessment of $\$153$. What was the rate of assessment?

5. A merchant holding railroad stock receives a dividend

of \$320. How many half-shares does he hold, 4% being the rate of dividend?

6. A business man meets an assessment of \$83.25, which is levied at $2\frac{1}{4}\%$ on his stock. How many quarter-shares does he hold?

7. What is paid for $7\frac{1}{2}\%$ bonds that produce an income of 6% on money invested?

SUGGESTION.—\$7 $\frac{1}{2}$ income on one share, is 6% of what number?

8. What is the quotation of 8% stock that produces an income of $12\frac{1}{2}\%$?

9. What rate per cent on the investment is realized by investing in 6% stock at 75?

SUGGESTION.—\$6 income on 1 share, is what per cent of \$75?

10. Which is the better investment, and how much, City 5's at 84, or Government 6's at 124?

11. A 7% dividend is declared on 50 shares of Chicago City Railway stock. What is the amount of the dividend?

12. A business venture is unsuccessful and levies an assessment of $9\frac{1}{2}\%$. How much does A pay who holds 24 shares?

13. A banker sold through a broker 74 shares of C. C. R. R. stock at $16\frac{1}{2}\%$ premium, paying $\frac{1}{8}\%$ brokerage. Find the net proceeds of the sale.

14. B bought 140 shares C. & N. W. preferred stock at $106\frac{1}{4}$, and sold the same at 108. What was his net profit, after allowing brokerage at $\frac{1}{8}\%$ for buying and the same for selling?

15. C bought through his broker 240 shares of W. U. Telegraph at $142\frac{1}{4}$. He sold 130 shares of the same at $148\frac{3}{4}$, and the remainder at $143\frac{1}{2}$, allowing $\frac{1}{8}\%$ for brokerage on the purchase, and the same for brokerage on the sale. What was his net profit?

16. A speculator sold 240 shares of irrigation stock for \$42090, losing by the sale \$600. What was his buying quotation?

17. My broker bought steamboat stock for me at $124\frac{3}{4}$, and immediately sold same at $143\frac{1}{4}$, remitting as net proceeds \$1341. How many shares did he buy, a brokerage of $\frac{1}{8}\%$ being allowed on buying and the same on selling?

18. A speculator bought stock at a discount of $7\frac{1}{2}\%$, and sold one-half at a premium of $3\frac{1}{2}\%$, and the remainder at par. Allowing a brokerage of $\frac{1}{8}\%$ each way, how many shares did he hold if he netted \$6900?

19. A corporation having net earnings of \$5665 wishes to declare a dividend after reserving a working fund of \$2000. The corporation organized with a capital stock of 250 shares, but has since issued 90 shares of preferred stock, secured at 6%. What rate of dividend can they declare?

20. Paid $109\frac{3}{4}$ for W. U. Telegraph and $\frac{1}{8}\%$ brokerage. What per cent will I receive on my investment, if I collect a dividend of 5% semi-annually?

21. If money is worth 6%, how much can I offer to pay for bonds bearing 5% interest, after allowing the usual brokerage of $\frac{1}{8}\%$?

22. Which is the better investment, and how much, 120 shares of railroad bonds paying $2\frac{1}{2}\%$ semi-annually, bought at 75; or the same number of shares bearing 8% annually, bought at 125?

23. A broker receives \$80 brokerage at $\frac{1}{8}\%$ for purchasing some bonds at 125. How much does the owner invest?

24. I purchased 50 shares of stock in each of five corporations, paying 130, 125, 105, 85, and 70 respectively, the corresponding dividends being 8%, $7\frac{1}{2}\%$, 6%, 5%, and 4% annually. Would my investment have been better, and how much, in bonds at par yielding 6%, not counting brokerage in the transaction?

25. A retired army officer has real estate that produces an annual rental of \$1200. He sells the property for \$15000 and invests the money in registered bonds at 90, that pay $7\frac{1}{2}\%$ dividends. Did he increase, or diminish his yearly income, and how much?

26. I sent my broker \$5000 with instructions to make the following purchases: 75 shares American Sugar; 80 shares American Sugar pf.; 40 shares C., M. & St. P. R. R.; 20 shares C., M. & St. P. R. R. pf.; 20 shares Texas Pacific; 20 shares D. L. & W.; 20 shares Norfolk & Western pf. He purchases them in the order named at the following quotations: 125 $\frac{1}{2}$,

102 $\frac{3}{4}$, 76 $\frac{7}{8}$, 147 $\frac{1}{2}$, 8 $\frac{1}{2}$, 159 $\frac{1}{4}$, and 5 $\frac{3}{4}$ respectively, charging brokerage at $\frac{1}{8}\%$. My banker has been instructed to receipt for stock and charge balance to my account. How much does the broker receive from my banker?

27. A certain corporation whose capital stock is represented by 3000 shares, has on hand undivided profits amounting to \$61250. The stockholders wish to announce the dividend at 12 $\frac{1}{2}\%$ and retain a reserve balance of \$15000. How many shares of watered stock must they issue, to bring the rate of dividend down to the required rate?

EXCHANGE.

262. **Exchange** is the method of paying accounts by means of written orders called bills of exchange.

263. **Bills of Exchange** are divided into two classes, viz.: Domestic and Foreign.

264. **Domestic Exchange** is one in which the drawer and drawee reside in the same country or state.

265. **Foreign Exchange** is one in which the drawer and drawee reside in different countries or states.

266. A **Sight Draft** or bill of exchange is one payable at sight, or presentation.

267. A **Time Draft** is one drawn after sight or date. Time Drafts are accepted, and all drafts are entitled to grace according to common law.

The rate of exchange between two places depends upon the condition of trade between them. If equal amounts are to be remitted by each to the other, the cost of exchange will be very nearly the same as the face of the exchange, or very nearly par. If large sums are to be remitted by one country which has few collections to make, the rate of exchange will be above par, or at a premium; and in the country having the balance of trade in its favor, the rate of exchange on the first country will be below par.

The premium will never exceed the cost of transporting money safely; for, if it did, merchants would send the money instead of the exchange.

WRITTEN PRACTICE.

1. What is the cost of a sight draft on Boston for \$1600 at $\frac{1}{8}\%$ premium?

SUGGESTION.— $\$1600 \times 1.00\frac{1}{8} = \text{Cost.}$

2. How much must be paid for \$1000 exchange on Baltimore at $\frac{1}{4}\%$ discount?

SUGGESTION.— $\$1000 \times .99\frac{3}{4} = \text{Cost.}$

3. What is the cost of a draft on Charleston for \$960, payable 60 days after sight, at $\frac{1}{8}\%$ premium; the current rate of interest being 6%?

SOLUTION.

$\$.0105 = \text{Dis. on } \$1 \text{ for } 63 \text{ da. at } 6\% (63 \div 6).$

$\$1 - .0105 = \$.9895$, proceeds of \$1.

$\$.9895 + .00\frac{1}{8} = \$.99075$, cost of exchange.

$\$960 \times .99075 = \951.12 , cost of draft.

4. How much must be paid for a bill of exchange at 60 days on New York for \$1260 at $\frac{1}{4}\%$ discount?

5. What is the face of a sight draft that can be bought for \$802, at $\frac{1}{4}\%$ premium?

SUGGESTION.— $\$802 \div 1.00\frac{1}{4} = \text{Face.}$

6. An agent having \$238.80 due his employer, is instructed to remit by sight draft. What is the face of the draft, exchange being at $\frac{1}{2}\%$ discount?

7. How large a draft, payable in 30 days must I draw, in order that, when sold, it will produce \$796.60, the exchange being at a premium of $\frac{1}{8}\%$, and interest being 6%?

$\$.0055 = \text{Dis. on } \$1 \text{ for } 33 \text{ da. at } 6\% (33 \div 6).$

$\$1 - .0055 = \$.9945$, proceeds of \$1.

$\$.9945 + .00\frac{1}{8} = \$.99575$, cost of exchange.

$\$796.60 \div .99575 = \800 , face of draft.

8. A commission merchant wishes to remit to his principal \$594.10 by draft at 30 days. What is the face of the draft which he can buy with the sum, exchange being at $\frac{1}{4}\%$ discount, and current rate of interest at 8%?

Find the cost of the following sight drafts:

	Face.	Exchange.		Face.	Exchange.
9.	\$3200	$\frac{1}{4}\%$ premium.	12.	\$ 960	$\frac{1}{8}\%$ discount.
10.	750	$\frac{1}{2}\%$ discount.	13.	4000	25¢ discount per \$1000.
11.	2000	$\frac{1}{8}\%$ premium.	14.	2400	40¢ premium per 1000.

Find the cost of the following time drafts, 6% interest:

Face.	Time.	Exchange.	Face.	Time.	Exchange.
15. \$1200	30 da.	$\frac{1}{4}\%$ premium.	18. \$2400	1 mo.	$\frac{1}{4}\%$ premium.
16. 960	60 da.	$\frac{1}{8}\%$ discount.	19. 1260	2 mo.	$\frac{1}{4}\%$ discount.
17. 350	90 da.	$\frac{3}{10}\%$ premium.	20. 2500	90 da.	$\frac{1}{10}\%$ premium.

Find the face of the following sight drafts:

Value.	Exchange.	Value.	Exchange.
21. \$498.75	$\frac{1}{4}\%$ discount.	24. \$450.45	$\frac{1}{10}\%$ premium.
22. 944.70	$\frac{1}{2}\%$ premium.	25. 648.70	$\frac{1}{5}\%$ discount.
23. 619.23	$\frac{1}{8}\%$ discount.	26. 5236.40	$\frac{7}{10}\%$ premium.

Find the face of the following time drafts, 6% interest:

Value.	Time.	Exchange.	Value.	Time.	Exchange.
27. \$413.49	60 da.	$\frac{1}{2}\%$ premium.	30. \$535.95	2 mo.	$\frac{3}{10}\%$ premium.
28. 545.60	30 da.	$\frac{1}{4}\%$ discount.	31. 607.91	3 mo.	$\frac{1}{10}\%$ discount.
29. 605.96	90 da.	1% premium.	32. 318.24	1 mo.	at par.

33. A Chicago merchant bought a sight draft on New York for \$1200, at $\frac{1}{8}\%$ discount, and one on New Orleans for \$1620 at $\frac{1}{4}\%$ premium, paying for same by check. What was the face of the check?

34. An Omaha wholesale dealer passed his banker the following order for drafts: One on New York at 30 days for \$1260; one on Baltimore at 60 days for \$750; two on San Francisco at sight for \$800 each; and one on Salt Lake City at 60 days for \$1500. The banker charged him $\frac{1}{4}\%$ premium on eastern exchange, and allowed $\frac{1}{4}\%$ discount on western exchange. For what sum must he fill his check, money being worth 6%?

35. My agent sells a consignment of wheat for \$1260, on a commission of 2%, and pays other charges amounting to \$76.40. I instruct him to remit by New York exchange at 60 days purchased at $\frac{1}{4}\%$ discount. What is the face of the draft that I receive, money being worth 6%?

36. A speculator remits a bill of exchange at 60 days for \$3000, with instructions to discount the same and buy 200 shares of Drainage Canal 6's, and draw at sight on him for the balance. The broker buys 120 shares of the bonds at $124\frac{1}{4}$, and the remainder at $120\frac{3}{8}$, charging a brokerage of $\frac{1}{8}\%$. The broker draws as instructed. What is the face of the draft, money being worth 6%?

SAVINGS BANKS.

268. A **Savings Bank** is a bank of deposit for the accumulation of small savings belonging to the industrious and thrifty. Interest is allowed on deposits and is credited as a deposit at the end of the interest term. The depositor is supplied with a book in which he is credited with amounts deposited, and in which he is debited as he withdraws.

269. The **Interest Term** is the term that a certain sum or balance must be in the bank to draw interest.

Checks cannot be drawn against an ordinary savings bank account. The depositor must take his bank book with him and have the amount entered, that he withdraws. He generally signs a receipt for the money taken out. The interest terms are not uniform; with some the term is one year, with some six months, and with others it is three months.

The regular time for crediting interest is: If semi-annual, Jan. 1, and July 1; if quarterly, Jan. 1, April 1, July 1, and Oct. 1, of each year.

Banks do not usually allow interest on fractional parts of a dollar. There is no general uniformity in the method of finding the interest.

We will explain two methods of estimating the interest on deposits.

In the first method, the interest term will be six months, but interest will be allowed on the smallest balance for each quarter.

In the second method, the interest term will be six months, but interest will be allowed on the smallest average monthly balance.

Interest is not counted on fractional parts of a dollar.

EXAMPLE.

What is the balance of the following savings bank account on Jan. 1, 1897, interest being allowed at 6%? Balance in bank Jan. 1, 1896, \$150. Deposits: Mar. 15, \$75; Apr. 1, \$125; July 5, \$160; Sept. 14, \$96; Nov. 12, \$84; and on Dec. 9, \$50.

SOLUTION BY FIRST METHOD.

DATE. 1896	DEPOSITS.	WITH- DRAWALS.	BAL- ANCES.	SMALLEST QR. BAL.	Int. on each qr. at 6%=1½%
Jan. 1			\$150		\$150 for 3 mo.= \$2.25
Mar. 15	\$75		225	150	\$350 for 3 mo.= 5.25
Apr. 1	125		350	350	Total Int. 6 mo. \$7.50
July 1	Cr. Int. 7 50		357 50		
July 5	160		517 50	357 50	\$357.00 for 3 mo.= \$ 5.36
Sept. 14	96		613 50		613.00 for 3 mo.= 9.19
Nov. 12	84		697 50		Total Int. 6 mo. \$14.55
Dec. 9	50		747 50	613 50	
Jan. 1	Cr. Int. 14 55		762 05	Bal. Due.	

SOLUTION BY SECOND METHOD.

DATE. 1896.	DEPOSITS.	WITH- DRAWALS	BAL- ANCES.	SMALLEST BAL. EACH MONTH.	AVERAGE MONTHLY BALANCE.
Jan. 1			\$150 00	Jan. \$150	
Mar. 15	\$ 75 00		225 00	Feb. 150	\$1500 total ÷ 6 = \$250,
April 1	125 00		350 00	Mar. 150	average balance.
				Apr. 350	Int. on \$250 for 6
				May 350	mo. at 6% = \$7.50
				June 350	
July 1	Cr. Int. \$7 50		\$357 50	July \$357 50	
				Aug. 517 50	\$3317 ÷ 6 = \$552, av-
July 5	160 00		517 50	Sept. 517 50	erage balance.
Sept. 14	96 00		613 50	Oct. 613 50	Int. on \$552 for 6
Nov. 12	84 00		697 50	Nov. 613 50	mo. at 6% = \$16.56.
Dec. 9	50 00		747 50	Dec. 697 50	
Jan. 1	Cr. Int. \$16 56		\$764 06	Bal. due.	

WRITTEN PRACTICE.

Solve the following problems by each of the above methods:

1. On January 1, 1895, my bank balance was \$350, interest allowed at 6%. During the year I made the following deposits:

Jan. 14, \$125; Feb. 2, \$74.50; Mar. 24, \$92; Apr. 16, \$75; Apr. 24, \$12; May 15, \$64; June 6, \$100; July 7, \$25; Aug. 23, \$124.75; Sept. 23, \$86; Oct. 12, \$45; Nov. 30, \$150; Dec. 15, \$90.

What was my balance Jan. 1, 1896?

2. A depositor's balance was \$420 on Jan. 1, 1894. He made the following deposits during the year:

Feb. 16, \$350; Apr. 4, \$450; June 5, \$300; June 27, \$280; Aug. 3, \$800; Oct. 17, \$375; Dec. 16, \$260.

During the year of 1895, no deposits were made. What was his balance Jan. 1, 1896, interest being allowed at 4%?

3. On January 1, 1895, a merchant has a balance in bank of \$600, and during the year he makes the following deposits and withdrawals:

Deposits: Jan. 4, \$200; Jan. 24, \$250; Mar. 3, \$275; Apr. 16, \$275; May 5, \$196; June 5, \$300.

Withdrawals: Jan. 5, \$125; Jan. 24, \$130; Feb. 16, \$75; Mar. 3, \$140; Apr. 3, \$150; June 4, \$50; June 16, \$75.

What was the balance due July 1, 1896, at 6%?

4. Find the balance due upon the following savings bank account Jan. 1, 1897, interest at 3%:

Balance in bank Jan. 1, 1896, \$396.75.

Deposits: Jan. 4, \$125; Jan. 6, \$320; Feb. 4, \$243.75; Feb. 16, \$125; Mar. 5, \$327.60; Mar. 16, \$142.64; Mar. 30, \$325.72; Apr. 15, \$243.70; May 25, \$327.80; July 5, \$378.20.

5. A merchant checks his surplus cash each month from his regular bank, and deposits the same in a savings bank that allows 4% interest. He deposits as follows:

Jan. 1, 1896, \$1200, Jan. 31, \$900; Feb. 28, \$500; Mar. 31, \$2500; Apr. 30, \$1500; May 31, \$1800; June 30, \$400; Sept. 30, \$1600; Oct. 31, \$3000; Nov. 30, \$1400.

Meeting with an unexpected loss on Dec. 24, he withdraws \$10000. What is his balance Jan. 1, 1897? How much interest did he lose by not being able to let the balance in bank stand to Jan. 1, 1897?

EQUATION OF ACCOUNTS.

270. Equation of Accounts is the process of finding the date upon which several items of an account due on different dates, may be cancelled by one payment without loss or gain to either party.

271. An Account is an item or a collection of items under one distinct head.

272. The Term of Credit is the time to elapse before a debt falls due.

273. The **Average Term of Credit** is the time allowed for the payment of the sum of several debts due at different dates.

274. The **Focal Date** is the date assumed, with which all the other dates are compared. Any date may be taken as a focal date.

275. A **Debit** is an item in an account which shows that the title is owing a given amount on a given date.

276. A **Credit** is an item in an account which shows that the title is being owed a given amount on a given date.

GENERAL PRINCIPLES.—1. If a debt is paid before it is due, the payer loses and the payee gains the use of the sum for the time before the due date.

2. If a debt is paid after it is due, the payer gains and the payee loses the use of the sum for the time after the due date.

3. The payment of a sum of money before it is due, is balanced by holding an equal sum of money an equal length of time past the due date.

SIMPLE EQUATIONS.

277. A **Simple Equation** is the equation of an account consisting of all debits or all credits.

MENTAL PRACTICE.

1. If I work 10 days for a neighbor, how many men of equal ability should he send to work for me 2 days, in return for my services?

2. If I use \$5 of A's money for 3 months, how much of my money should A use for one month, in return for the accommodation?

3. If I gave a certain amount for the use of \$100 for 6 months, for how long a time should I have the use of \$200 for the same consideration?

4. The use of \$20 for 5 months is equivalent to the use of what sum for 1 month?

5. If I am entitled to 30 days credit on \$200, how long a term of credit should I have on the remainder if I pay \$100 at once?

6. A owes B two payments of equal amount, one due in 4 months, and the other due in 6 months. In what time may he pay both with one payment?

7. C owes \$200 due in 2 months, and \$400 due in 5 mo. In what time may he pay both sums with one payment?

The use of

8. \$300 for 4 mo. = \$600 for what time?

9. \$600 for 2 mo. = \$300 for what time?

10. \$800 for 3 mo. = \$1200 for what time?

11. \$1200 for 2 yr. = \$800 for what time?

12. \$800 for 3 yr. = the use of what sum for 2 yr.?

13. \$500 for 4 yr. = the use of what sum for 5 yr.?

EXAMPLE.

A merchant sold goods as follows: \$500 on 7 months credit; \$600 on 3 months credit; \$700 on 5 months credit; \$800 on 2 months credit. Find the equated time for the payment, all these purchases having been made on the same date.

SOLUTION.

\$500 for 7 mo. = \$3500 for 1 mo.

600 for 3 mo. = 1800 for 1 mo.

700 for 5 mo. = 3500 for 1 mo.

800 for 2 mo. = 1600 for 1 mo.

\$2600 for ? mo. = \$10400 for 1 mo.

$\$10400 \div 2600 = 4$ times, or 4 mo.

The average term of credit or equated time is 4 mo.

WRITTEN PRACTICE.

1. A wholesale dealer sold merchandise as follows: \$600 on 3 mo. credit, \$800 on 3 mo. credit, \$800 on 4 mo. credit, and \$100 on 6 mo. credit. Find the equated term of credit.

2. On Aug. 1, 1896 an agent received a consignment of 400 barrels of flour which he sold as follows: 75 bbl. on 30 da., 50 bbl. on 60 da., 25 bbl. on 10 da., 120 bbl. on 30 da., and the remainder on 20 days. What is the equated date?

SUGGESTION.—Aug. 1 + no. of days of credit = equated date of payment.

3. An importer sold the following bill of goods Sept. 1, 1896 \$750 on 10 days, \$600 on 20 days, \$300 on 30 days, \$800

on 30 days, \$900 on 60 days, and \$1000 on 90 days. What is the equated date of payment?

4. On Feb. 10, 1896, a wholesale dealer made purchases as follows: \$600 on 10 days, \$750 on 20 days, \$625 on 30 days, \$675 on 30 days, \$725 on 60 days, and \$850 on 90 days. He wishes to give his note for the entire amount. Find the due date of same.

EXAMPLE.

In the following, the purchases are made on different dates: I bought of Marshall Field & Co. the following invoices of goods:

Jan. 6, 1896,	an invoice amounting to \$960.			
Jan. 24,	"	"	"	840.
Feb. 8,	"	"	"	600.
Feb. 16,	"	"	"	520.
Mar. 4,	"	"	"	320.

What is the average date of purchase?

SOLUTION.—PRODUCT METHOD.

1896.

Jan. 0	= Focal date.
Jan. 6	6 960 5760
" 24	24 840 20160
Feb. 8	39 600 23400
" 16	47 520 24440
Mar. 4	64 320 20480
	<u>3240</u> <u>94240</u>

$$94240 \div 3240 = 29$$

Jan. 0 + 29 = Jan. 29, the required date.

SUGGESTION.—Any date may be assumed as the focal date. Count the exact time from the focal date to the date of the item, to find the credit on each item.

By assuming the payment to be made on Jan. 0 (an earlier date than any item) we will lose the time between that date and the date of the item, i. e., 6 days in the first, 24 days in the second, etc.

We find by averaging our payment on the focal date Jan. 0, it would produce a loss of the use of the total, \$3240 for 29 days, consequently the focal date is extended 29 days, or Jan. 29, the equated date.

SOLUTION.—INTEREST METHOD.

1896.

Jan. 0 = Focal date.

Jan. 6	6	960	96 = Interest on \$960 for 6 da. at 6%
Jan. 24	24	840	3 36 = " " 840 " 24 " " 6%
Feb. 8	39	600	3 90 = " " 600 " 39 " " 6%
Feb. 16	47	520	4 07 = " " 520 " 47 " " 6%
Mar. 4	64	320	3 41 = " " 320 " 64 " " 6%
		3240	15 70 = " " 3240, required No. of days.

Int. on \$3240 for 1 day at 6% = \$.54

 $15.70 \div .54 = 29$

Jan. 0 + 29 days = Jan. 29

SUGGESTION.—The interest method is short and practical, at the same time giving the student an excellent review of interest.

WRITTEN PRACTICE.

5. J. V. Farwell & Co. sold to H. E. Jones:

1896.

Jan. 1 a bill of \$375.

Jan. 16 " " " 260.

Feb. 20 " " " 175.

Feb. 28 " " " 360.

Mar. 15 " " " 800.

May 20 " " " 240.

What is the equated date of payment?

6. A sold to B the following bills of goods:

1895.

Mar. 16 a bill amounting to \$840.

Apr. 24 " " " 375.

May 21 " " " 250.

May 30 " " " 264.

June 25 " " " 365.

July 23 " " " 480.

What is the equated date of sale?

7. On Apr. 1, 1896, a commission merchant received for account and risk of shipper, 600 bbl. of flour, which he sold as follows:

On Apr. 10,	150 bbl.	at	\$6.25.
Apr. 24,	145 "	" "	6.50.
May 16,	75 "	" "	6.60.
May 28,	125 "	" "	6.75.
June 4,	Balance "	" "	5.50.

What is the equated date of sale?

8. Hibbard, Bartlett, Spencer & Co. made the following sales:

1895					
Aug. 20,	bill amounting to	\$ 360	less	10%	
" 30,	" " "	" 1260	" "	12½%	
Sept. 15,	" " "	" 850	" "	2-10's	
" 24,	" " "	" 600	" "	20% and 25%	
Oct. 15,	" " "	" 320	" "	33½%	
" 24,	" " "	" 480	" "	10%, 5% and 5%	

Find the equated date.

9. The John Spry Lumber Co. made the following sales:

Apr. 4,	12400 ft. flooring at	\$21	per	M
" 16,	3200 " " "	18	"	M
" 24,	2400 " " "	16	"	M
May 15,	4870 " " "	18.50	"	M
June 20,	3260 " " "	14.75	"	M
" 28,	4500 posts, round at	25.00	"	C

Find the equated date of payment.

10. L. E. Goodyear bought of Brown & Co.:

Aug. 11,	1896, Mdse. amounting to	\$465.
Aug. 23,	" " "	924.
Sept. 10,	" " "	840.
Oct. 16,	" " "	379.
Oct. 27,	" " "	540.

What is the equated date of payment, each item being allowed a credit of 30 days?

SUGGESTION.—Extend each date 30 days, or add 50 days to each term of credit; i. e., Aug. 0 to Aug. 11 is 11 days, plus credit of 30 days is 41 days, term of credit for first item; or, find the equated date, as before, and extend it 30 days, since all have the same term of credit.

11. W. J. Durand bought of E. C. Lawson & Bro.:

Sept. 20, 1896, an invoice amounting to \$324.60.

Oct. 23, " " " " 425.36.

Nov. 4, " " " " 364.52.

Nov. 25, " " " " 243.75.

Dec. 16, " " " " 364.26.

Find the equated date, a special credit of 60 days being allowed on each item.

SUGGESTION.—In computing the interest or finding the product, use only the amount expressed as dollars; counting a fractional amount of 50c. or more, as an additional dollar; i. e., count the first item \$325, the second \$425, etc.

12. Marshall Field & Co.'s sales to A. A. Brewer are as follows: (Use first date as focal date.)

Feb. 20, 1896, a bill of \$364.75

Mar. 12, " " " 236.50

Mar. 28, " " " 243.24

Apr. 15, " " " 426.35

June 1, " " " 324.36

June 29, " " " 425.64

A credit of 90 days is allowed on each item. Find equated date of payment.

13. A merchant bought merchandise, as follows:

Apr. 4, 1896, \$400 at 30 da.

Apr. 25, 300 at 60 da.

May 23, 250 at 45 da.

June 25, 500 at 30 da.

Find the equated date of payment.

SUGGESTION.—Add the special credit to the time of each item from focal date; i. e., Apr. 0 to Apr. 4 is 4 days plus 30 days, is 34 days, the credit for the first item.

14. A merchant sold, as follows:

May 5, 1896, \$500 at 60 da.

May 20, 600 at 30 da.

July 4, 300 at 20 da.

July 27, 475 at 60 da.

What is the equated date of payment?

15. J. S. Gridley & Co., bought goods as follows:
1896.

Apr. 4, a bill of \$426 at 30 days credit.

" 14, " " " 320 " 60 " "

May 14, " " " 460 " 30 " "

July 5, " " " 570 " 90 " "

Aug. 1, " " " 300 " 60 " "

" 15, " " " 250 " 30 " "

What is the equated date of payment?

16. Brown & Davis in account with Strauss & Smith, Dr.:
1896.

Mar. 4, as per bill rendered \$276.74 at 30 da.

" 16, " " " " 642.30 " 60 "

Apr. 20, " " " " 324.75 " 90 "

May 4, " " " " 264.63 " 45 "

" 27, " " " " 425.37 " 60 "

June 21, " " " " 360.00 " 2 mo.

July 24, " " " " 420.60 " 4 "

What is the equated date of payment?

SUGGESTION.—In adding the special credit term given in months, count the extra days for 31 day months; i. e., 2 months from June 20 is Aug. 20, or 61 days, the special time to add.

17. On July 16, 1896, a merchant sold a bill of dry goods amounting to \$2400, one-third to be paid in 60 days, one-fourth in 3 months, one-fourth in 4 months, and the balance in 4 months. Find the date when the entire bill could be paid without loss to either party.

COMPOUND EQUATIONS.

278. A Compound Equation is the equation of an account having both debits and credits:

EXAMPLES.

1 Find the equated date of the following accounts:

Dr.		E. C. LAMSON.				Cr.	
1895.							
Aug. 4	Mdse.	420 00	Aug. 20	Cash		280 00	
" 24	"	360 00	Sept. 5	"		200 00	
Sept. 3	"	225 75	Oct. 10	"		50 25	

EQUATION OF ACCOUNTS

Dr. SOLUTION (INTEREST METHOD). *Cr.*

Aug. 0 = Focal date.									
Aug.	4	4	420		28	Aug.	20	20	280
	24	24	360		1 44	Sept.	5	36	200
Sept.	3	34	225	75	1 28	Oct.	10	71	50 25
			1005	75	3 00				530 25
			530	25	2 72				
			475	50	28				

Int. on \$475.50 for 1 da. at 6% = \$.079.

\$.28 ÷ .079 = 4 times, or 4 days.

Aug. 0 + 4 da. = Aug. 4, the equated date.

Dr. SOLUTION (PRODUCT METHOD). *Cr.*

Aug. 0 = Focal date.									
Aug.	4	4	420		1680	Aug.	20	20	280
"	24	24	360		8640	Sept.	5	36	200
Sept.	3	34	225	75	7684	Oct.	10	7	50 25
			1005	75	18004				530 25
			530	25	16350				
			475	50	1654				

1654 ÷ 475.50 = 4 times, or 4 days.

Aug. 0 + 4 = Aug. 4, the equated date.

2. When in equity should the balance be paid?

Dr. H. E. IVES. *Cr.*

1896.									
Mar. 4			Mdse.	750		Apr. 16		Cash	300

Dr. SOLUTION (PRODUCT METHOD). *Cr.*

Mar. 0 = Focal date.									
Mar.	4	4	750	3000		Apr.	16	47	300
			300						14100
			450						3000
									11100

11100 ÷ 450 = 25 days.

Mar. 0 - 25 days = Feb. 4, the equated date of payment.

SUGGESTION.—If the balances both fall on the same side, count forward from the focal date, to find the equated date; if they fall on opposite sides, count back from the focal date.

It is evident from the above solution that the payment of the balance could not be made on the equated date, it being earlier than either debit or credit item.

The equated date furnishes the date upon which payment, if made, would produce neither loss nor gain to either party; therefore, it furnishes the date from which interest should be computed, when the payment of the balance is made at a later date.

If payment is made before the equated date, we are allowed discount.

NOTE.—The teacher should designate the focal date and method of solution.

WRITTEN PRACTICE.

18.

<i>Dr.</i>	W. J. LEFFRING.	<i>Cr.</i>
1895.	1895.	
Jan. 1.....	Mdse. \$425.00	Feb. 2.....Cash, \$590.50
Feb. 16.....	" 327.60	

19.

<i>Dr.</i>	A. B. JAMES & Co.	<i>Cr.</i>
1895.	1895.	
Mar. 14.....	Mdse. \$562.50	Mar. 25.....Cash, \$426.00
Mar. 29.....	" 436.25	Apr. 15....." 250.75

20.

<i>Dr.</i>	O. E. GRANT & BRO.	<i>Cr.</i>
1895.	1895.	
Apr. 1.....	Mdse. \$325.95	Apr. 20.....Cash, \$200
May 4.....	" 275.80	June 25....." 100

21.

<i>Dr.</i>	CARL ZUETTEMEISTER.	<i>Cr.</i>
1896.	1896.	
Apr. 24.....	Mdse. \$425.00	May 15.....Cash, \$425
May 20.....	" 375.50	June 10....." 200
June 30.....	" 269.00	

22.

<i>Dr.</i>	J. S. GRIDLEY.	<i>Cr.</i>
1895.	1895.	
June 20.....	Mdse. \$425.40	June 24.....Cash, \$225.40
July 5.....	" 247.65	July 16....." 324.75
Aug 24.....	" 364.70	Aug. 1....." 100.00
Sept. 10.....	" 248.35	" 31....." 200.00

23.

<i>Dr.</i>	GEO. BAUMANN.	<i>Cr.</i>
1895.	1895.	
Aug. 24, Mdse. 30 da.	\$240.75	Aug. 30, Cash, \$200
Sept 10, " 60 "	360.25	Sept. 16, Note 1 mo. 250
Oct. 20, " 90 "	365.00	Nov. 30, Acceptance, 30 da. 400

24.

<i>Dr.</i>		J. W. BUTLER PAPER CO.		<i>Cr.</i>
1895.			1895.	
Aug. 1,	Mdse. 1 mo.	\$324.50	Sept. 20, Cash,	\$300.00
" 30,	" 2 "	247.30	Oct. 14, Draft 10 da.	100.00
Sept 20,	" 3 "	463.40	Nov. 1, Note 2 mo.	250.00
Oct. 10,	" 2 "	247.80	Dec. 4, Cash,	348.50

25.

<i>Dr.</i>		FRED. J. MAGERSTADT.		<i>Cr.</i>
1895.			1895.	
Mar. 3,	Mdse.	\$480.75	Mar. 9, Cash,	\$420.75
Apr. 16,	" 30 da.	325.60	May 10, "	300.00
May 1,	" 20 "	248.36	June 1, Note 60 da.	240.00
" 31,	" 60 "	426.42	July 5, Draft 2 mo.	500.00
June 24,	" 10 "	364.00	Aug. 4, Check dated 8-20,	200.00
Aug. 20,	" 30 "	249.65	Sept. 19, Acceptance 3 da.	248.90
Sept. 4,	" Net,	624.35	Oct. 20, Note 2 mo.	365.20
Oct. 31,	" 60 da.	387.63	Nov. 24, Cash	387.63
Dec. 24,	" 60 "	426.38	Dec. 20, Cash,	150.00
1896.			1896.	
Jan. 5,	" 30 "	263.21	Jan. 9, Cash,	200.00
" 31,	" Net,	427.95	Feb. 5, Cash,	300.00

26. A. N. PALMER CO., LTD.

In account with WELLS & JONES, Dr.

Apr. 16, 1896,	Mdse. 30 da.	\$360.50 less 20%.
" 24,	" 60 "	480.25 net.
May 14,	" 20 "	325.50 less 2-10's.
" 29,	" 2 mo.	524.30 " 33 $\frac{1}{3}$ %.
July 5,	" 1 "	426.00 " 25% and 20%.
What is the equated date?		

27. A. B. BROWN & CO.,

Bought of FRENCH & POTTER.

Mar. 16, 1896,	300 bbl. Salt at \$1.25.
Mar. 26,	20 bbl. Sugar, each 209 lb. @ 4 $\frac{1}{4}$ ¢.
Apr. 16,	40 firkins Butter, each 52 $\frac{1}{2}$ lb. @ 16 $\frac{3}{4}$ ¢.
May 15,	124 bbl. Apples @ \$1.75.
June 5,	140 bu. Potatoes @ 25¢.
What is the equated date?	

28. A. B. Jones bought of Kingman & Co. June 1, 1896, 400 bu. of Oats at 25¢, on 30 da. credit; June 30, 460 bu. Corn at 26¢, on 60 da. credit; July 5, 125 bu. Wheat at 85¢, on 3 mo. credit; Aug. 20, 165 bu. Rye at 64¢, on 4 mo. credit; Sept. 5, 34 tons Timothy Hay at \$14.50, on 3 mo. credit. On the date of the last purchase, Jones offers his note for 3 mo., drawing 7% interest for the whole amount of the purchase, dated on the equated date. Kingman & Co. accept the note and immediately discount it at bank at 6%. What amount do they realize?

29. W. J. NIXON,

Bought of POPE MFG. CO.

Jan. 4, 1896, Mdse. 30 da. \$ 500 less 30%.
 Feb. 3, " 60 " 1500 " 33½%.
 Feb. 24, " 45 " 1250 " 20% and 10%.
 Mar. 3, " 30 " 975 " 40%.

Nixon wishes to meet this account in two payments of equal amounts, with an interval of 60 da. What are the dates of payment?

30. On Mar. 1, 1896, Marshall Field & Co. sold Thorsen & Cassidy a bill of goods amounting to \$2147.60, on 3 mo. credit. On Apr. 1, 1896, Thorsen & Cassidy paid \$900; on Apr. 30, 1896, they paid \$500. What is the date of an interest bearing note that they should give for the balance?

31. I hold five notes against Carson, Pirie, Scott & Co., dated as follows: Apr. 3, 1896, payable in 6 mo. for \$648; May 5, 1896, payable in 4 mo. for \$572.60; May 24, 1896, payable in 3 mo. for \$826.45; June 6, 1896, payable in 4 mo. for \$562; June 20, 1896, payable in 60 days for \$325.48. On July 1, 1896, they paid me \$1200, I agreeing to cancel their five notes and take an interest bearing note for 3 mo. for the balance. What is the date of the new note?

32. Find the cash balance of the following accounts:

<i>Dr</i>		A. B. JONES.		<i>Cr.</i>	
1895		1895			
Mar. 8	Mdse.	460 75	Apr. 2	Cash	300 00
" 24	"	375 46	May 5	Sight Draft	200 00

The above account was settled Sept. 1, 1895, at 6%.

SUGGESTION.—Find the equated date as before, and count the exact time between the equated date and settlement date. If the settlement date is later, add the interest; if earlier, subtract the interest, called discount

Or, since the settlement date is given, count the exact time from each date to the settlement date as the credit for that item, i. e., from Mar. 8 to Sept. 1, is 177 da. The balance of the debit and credit interest to be added or subtracted will produce the amount due at the settlement date.

33.

<i>Dr.</i>				L. E. GOODYEAR.				<i>Cr.</i>	
1895				1895					
Apr. 16	Mdse.	562	75	May 4	Cash	372	24		
May 24	"	374	25	June 10	"	276	83		
June 5	"	247	38	July 14	"	427	38		
July 2	"	426	83	Aug. 5	"	327	91		

Settled Aug. 20, 1895, at $7\frac{1}{2}\%$.

34.

<i>Dr.</i>				EMIL DAVIS.				<i>Cr.</i>	
1896				1896					
May 16, Mdse.	60 da.	\$324.60		June 4,	Cash	\$400.00			
May 24,	" 30 "	147.83		June 23,	Note 90 da.	250.00			
June 5,	" 45 "	465.00		Aug. 1,	Acceptance 1 mo.	147.83			
July 6,	" 60 "	328.25		Sept. 5,	Cash	600.00			
Aug. 4,	"	476.80							

Settled Oct. 1, 1896, at 6%.

35.

<i>Dr.</i>				HINZ & HOEKENDORF.				<i>Cr.</i>	
1896				1896					
June 4, Mdse.	4 mo.	\$842.55		July 1,	Note 3 mo.	\$842.55			
July 1,	" 3 "	372.60		July 31	Accept. 2 mo.	372.60			
" 29,	" 2 "	427.36		Aug. 20,	Cash	100.00			
Aug. 4,	" Net	328.63		Sept. 1,	Mdse. returned	426.41			
" 31,	" 2 mo.	426.45		Oct. 4,	Check dated 10-20	250.00			
Sept. 5	" 60 da.	375.23							
" 30,	" 30 "	167.83							

Balance paid Nov. 1, 1896, at 8%

<i>Dr.</i>				36.	<i>Cr.</i>			
				SMITH & MILLER.				
1895				1895.				
Sept 24,	Mdse.	30 da.	\$265.74	Oct. 4,	Cash	\$100.00		
Oct. 10,	"	60 "	375.28	Nov. 24,	Note 3 mo.,	246.10		
Nov. 24,	"	10 "	284.36	Dec. 1,	Draft 60 da.	145.80		
Dec. 27,	"	Net	865.72	Dec. 15,	Cash	100.00		
1896.				1896.				
Jan. 14,	"	60 "	324.36	Jan. 4,	Cash	550.00		
Feb. 21,	"	30 "	146.25	Feb. 20,	Note 10 da	240.75		
Mar. 16,	"	10 "	236.24	Feb. 23,	Cash	146.25		
Apr. 4,	"	10 "	246.95	Mar. 10,	Cash	250.00		
Settled June 10, 1896, at 5%.								

<i>Dr.</i>				37.	<i>Cr.</i>			
				TRIPP BROS. & Co.				
1896.				1896.				
Oct. 10,	Mdse.	30 da.	\$175.80	Nov. 16,	Cash	\$175.80		
Oct. 29,	"	40 "	362.40	Dec. 12,	Draft 10 da.	200.00		
Nov. 5,	"	10 "	360.00					
Dec. 6,	"	{ \$240.	***					
		{ Less 10 & 10%.						
1897.				1897.				
Jan. 5,	"	{ \$475.60	*** **	Jan. 19,	Cash	450		
		{ Less 20 & 10%.		Jan. 24,	Cash	250		
Jan. 10,	"	{ \$472.50.	*** **					
		{ Less 5 & 2%.						

The account was settled Feb. 1, 1897, at $7\frac{1}{2}\%$.

38. Find equated date for settling the net proceeds of the following account sales:

CHICAGO, Apr. 30, 1896.

Account Sales of Flour.

Sold for account of WELLS & WENTWORTH,
ALTA, IOWA.

1896.									
Mar.	8	124 bbl.	Superfine Flour @ \$6.10 net		*** **				
"	8	48 "	do @ 6.20 less $2\frac{1}{2}\%$						
"	28	120 "	do @ 5.95 30 da.						
Apr.	2	60 "	do @ 6.20 30 da.						
"	12	80 "	do @ 6.15 10 da.						
CHARGES.									
Mar.	4	Freight I. C. R. R.		124	50				
"	20	Advertising		18	25				
"	30	Advanced on Flour		450					
Apr.	1	Cooperage		13					
"	14	Storage		25	80				
***	**	Commission 3%		***	***	***	***	***	***
***	**	Net Proceeds							

SUGGESTION.—Equate the sales, credit side, to find the date of commission; then equate the average date of sale commission date, with charges, debit side, to find date of net proceeds.

PARTNERSHIP.

279. A **Partnership** is an association of persons for the purpose of conducting some lawful business.

280. A **Partner** is one of the persons conducting the partnership.

281. The **Capital** is that which is invested, money, real estate, experience, skill, or anything of value.

282. A **Resource** is anything of value belonging to the business.

283. A **Liability** is a debt due by the business. Liabilities are sometimes divided into two classes; outside and inside liabilities. In a settlement the outside liabilities must be paid first; i. e., debts due outside parties must be paid before partners are allowed to withdraw investments.

The excess of the resources over the liabilities is, at the beginning, the net investment; at the end, the present worth.

284. An **Inventory** is a list of goods on hand together with cost price; it is generally taken once a year.

285. A **Net Gain** is an excess of the gains over the losses.

286. A **Net Loss** is an excess of the losses over the gains.

The losses and gains are usually divided according to agreement; if no agreement is made, the law divides them equally.

287. An **Insolvency** is an inability to pay one's debts; the liabilities are in excess of the resources.

288. The **Net Insolvency** is the excess of the liabilities over the resources.

MENTAL PRACTICE.

1. If A and B invest the same sum, what part of the gain should each have?

2. If B and C invest the same sum, what part of the loss should each share?

3. If C invests 2 times as much as D, what part of the gain should each have?

4. If D invests $\frac{1}{2}$ as much as E, what part of the gain should each have?

5. If E invests 3 times as much as F, what part of the gains and losses should each have?

6. If F invests \$2 as often as G invests \$3, what part of the gains and losses should each have?

7. If G invests \$3 as often as H invests \$4, what part of the gains and losses should each have?

8. If H invests \$5 as often as I invests \$4, what part of the gains and losses should each have?

9. If I invests for 1 year and J invests the same amount for 2 years, what part of the gains and losses should each have?

10. If J invests \$2000 for 1 year and K invests \$3000 for 2 years, what part of the gains and losses should each have?

11. If K invests \$300 for 4 months and L invests \$400 for 3 months, what part of the gains and losses should each have?

12. If L invests \$600 for 5 months and \$1000 for 4 months, and M invests \$5000 for 1 month, what part of the gains and losses should each have?

13. Two men investing equally in business gained \$3000 during the year. What part of the gain belongs to each man? How much belongs to each?

14. During a year in business, three partners, having equal investments, gained \$7500. What part of the gain belongs to each?

15. A and B formed a partnership, each investing \$3000; at the end of the year they have gained \$4500. To what sum is each one entitled?

16. C and D engaged in business for 1 year, each investing \$500. At the end of the year their books show the following gains: On merchandise \$3000, on real estate \$4000, and on shipments \$1500. What is each one's share of the gains?

17. E and F engage in business, each investing \$4750; at the end of the year their gains and losses are as follows: On merchandise they gained \$4100, on real estate they gained \$2200, on shipments they gained \$1200; and it cost them \$2000

to pay salaries and expenses. What was each partner's share of the net gain?

18. Two men, G and H, engage in business for two years, and invest as follows: G invests cash \$3000, H invests merchandise invoiced at \$2000. At the end of the time, they found a gain of \$5000 in merchandise account and a gain of \$2500 in real estate. What is each partner's share of the gain, the same being divided in proportion to his investment?

19. I and J engage in business for 1 year, agreeing to share gains and losses in proportion to amount invested by each. I invested \$2000 for 3 months and \$3000 for 2 months; J invests \$4000 for 2 months and \$2000 for 2 months. They gained \$4200 on merchandise and \$800 on grain; the losses being \$600. What is each one's share of the net gain?

20. Three men hire a pasture for \$45. The first put in 24 cows for 5 weeks, the second put in 20 cows for 4 weeks, and the third put in 100 sheep for 2 weeks. How much should each pay, if the pasturage of one cow is equal to that of two sheep?

WRITTEN PRACTICE.

1. A and B commenced business with a cash capital of \$12000, and at the end of one year they sold out for \$14560. What was the gain?

2. B and C commenced business with a capital of \$14500, and at the end of two years had but \$12300. What was the average loss per year?

3. C and D commenced business with the following resources and liabilities: Resources—cash \$6000, real estate \$4900, bills receivable \$2300. Liabilities—bills payable \$6860. At the end of the year they sold out for \$11500, how much did they gain? What is each one's share of the gain?

4. D and E commenced business with an indebtedness of \$5600. At the end of one year they had a capital of \$4860. How much did they gain?

5. E and F commenced business with cash \$1200, merchandise \$3200, bills receivable \$1465; they owed on personal accounts \$3600, and on bills payable \$7500. At the end of two

years they have cash \$4600, merchandise \$3560, real estate \$4760, and bills receivable \$6400. What was each one's share of the gain?

6. H and I commenced business Jan. 1, 1895. H invested cash \$2000, real estate \$4000. I invested merchandise \$4500, bills receivable \$3000. Jan. 1, 1896, their resources were as follows: Cash \$6000, merchandise \$7500, real estate \$4000, bills receivable \$4780. At the same date they were owing on bills payable \$3750, personal accounts \$2140. What was the gain or loss for the year? What was each partner's worth at closing?

7. J and K commenced the general merchandise business Jan. 1, 1896. J invested $\frac{1}{3}$ and K $\frac{2}{3}$ of the capital. Jan. 1, 1897, their resources are as follows: Merchandise \$6000, cash \$3200, real estate \$2150, bills receivable \$3124, interest receivable \$147.50. The loss during the year was \$878.50. How much did each one invest?

8. K and L formed a partnership and invested respectively \$7000 and \$8000. The profits for the year were, on merchandise \$3600, on real estate \$6900. What was each one's share of the gain, it being divided in proportion to their investment?

SUGGESTION.—K is entitled to $\frac{7}{15}$ and L $\frac{8}{15}$ of the gain.

9. L, M, and N invested \$4500, \$5500, and \$6500 respectively in business, agreeing to share gains and losses in proportion to their investments. The net gain for the year was \$15510. What was each one's share?

10. M, N, and P contract to erect an office building for \$120000. M furnished material worth \$27500, N furnished material worth \$30000, and P paid for labor and supervision \$45000. The gain being divided proportionately, how much did each one receive?

11. N, P, and Q engage in business investing respectively \$12000, \$15000, and \$13000. At the end of a year they have the following resources: Cash \$21000, real estate \$24000, bills receivable \$14000, and personal accounts due them \$15360. At the same date they owe on bills payable \$14750, and for interest on same \$450. Q is to receive a salary of \$2400 for managing

the business. What is each one's share of the gain, divided according to the investment?

12. P, Q, and R conduct a general merchandise business with a joint capital of \$26000. How much did each invest if, at the end of one year, their gains were respectively \$3000, \$4000, and \$6000?

13. Q and R formed a co-partnership for the purpose of conducting a renting and general real estate business, investing as follows: Q invested Jan. 1, 1896, \$5000, and on June 1, 1896, \$3000. R invested Jan. 1, 1896, \$7500, and on Aug. 1, 1896, \$6000. Q withdrew on July 1, 1896, \$3000, and Sept. 1, 1896, R withdrew \$2500. How much is each one's share of a gain of \$46710, the business being conducted for one year, and profits being divided in proportion to amount invested by each partner? What is each partner's present worth?

SOLUTION.

Q's investments.

Jan. 1, \$5000 for 12 mo. = \$60000 for 1 mo.

June 1, \$3000 for 7 mo. = 21000 for 1 mo.

\$81000 for 1 mo.

Q's withdrawal.

July 1, \$3000 for 6 mo. = \$18000 for 1 mo.

Q's net investment = \$63000 for 1 mo.

R's investments.

Jan. 1, \$7500 for 12 mo. = \$90000 for 1 mo.

Aug. 1, \$6000 for 5 mo. = 30000 for 1 mo.

\$120000 for 1 mo.

R's withdrawal.

Sept. 1, \$2500 for 4 mo. = \$10000 for 1 mo.

R's net investment = \$110000 for 1 mo.

\$63000 + \$110000 = \$173000, total net investment for 1 mo

$\frac{63000}{173000} = \frac{63}{173}$ Q's share of the gain.

$\frac{110000}{173000} = \frac{110}{173}$ R's share of the gain.

$\frac{63}{173}$ of \$46710 = \$17010, Q's gain.

$\frac{110}{173}$ of \$46710 = \$29700, R's gain.

$\$5000 + 3000 - 3000 = \5000 , Q's investment.

$\$7500 + 6000 - 2500 = \11000 , R's investment.

$\$5000 + 17010 = \22010 , Q's Present Worth.

$\$11000 + 29700 = \40700 , R's Present Worth.

14. R and S are partners in the boot and shoe business. R invests \$4000 for 3 months, \$7500 for 6 months, and \$2000 for $1\frac{1}{2}$ months. S invests \$5000 for 10 months and \$1000 for 2 months. The gains are to be shared in proportion to average investment of each. The gains for the year are \$8400. Find the gain and present worth of each partner.

PROMISCUOUS WRITTEN PROBLEMS.

15. Three men, A, B, and C, form a partnership for the purpose of buying and selling coal. A invests \$2000, B invests \$3000, C invests \$2500. Their profits amount to \$3000. What is each partner's share of the gain, distributed according to investment?

16. D, E, and F form a partnership. D invests \$9000, E invests \$6000, F invests \$500. They purchase a stock of lumber for \$18000, and lose $\frac{1}{3}$ of it by fire and $\frac{1}{6}$ of the remainder in transit. They sell the remainder of the lumber at a profit of 125 per cent of its cost. Their expenses are \$750. What is the net gain of each partner, divided according to investment?

17. G and H engage as partners in buying and shipping stock, sharing gains and losses equally. G pays for sheep \$8750, and freight \$750, shipping them to H. H sells the sheep for \$11250, sending G \$5000 of the cash received. H pays \$6325 for cattle which he ships to G, paying for freight and other expenses of shipping \$825. G sells the cattle at an advance of $33\frac{1}{3}$ per cent of their entire cost, receiving the money. How shall the partners settle with each other?

18. H and I invest in business, each \$10000. At the end of three months H draws out \$1000, and I invests \$1000. At the end of 9 months H invests \$3000, and I invests \$1000. They gain during the year \$3600. What is each partner's present worth, after dividing the gain according to their average investments?

19. L and M are partners, sharing equally in gains and losses. L invests \$3000 Jan. 1, 1896. M invests \$2500 Feb. 1, 1896. Each partner is to receive 6% interest on his capital invested and to pay interest at the same rate on all sums withdrawn. L draws out \$500 and M invests \$1500 June 1; L invests \$1000 and M draws out \$500 Sept. 1. On the first of January, 1897, their books show the following: Resources, ledger accounts \$6000, 5% of which are uncollectible; notes receivable \$3300; cash \$2200; merchandise unsold \$683. Liabilities: Ledger accounts \$1500; notes payable \$2500. What is each partner's interest, after the division of the gain or loss?

20. N and O enter into partnership to build a house. N works 10 days and furnishes two hands who each work 10 days; O works 20 days, and also spends three days in buying materials for the building. A day's work of each is counted of equal value. At the completion of the job they receive \$1200 in payment. They owe for lumber \$550; stone, lime, and sundries \$220; mason work \$125. O has paid for hardware used \$27.50. What is due each partner from the proceeds?

21. P, Q, and R rent a pasture for three months, paying \$225, of which each one advances \$75, with the understanding that sheep are to be estimated at one-half the price of cattle and horses. P puts in 25 head of cattle, Q puts in 20 horses, and R puts in 100 sheep. After one month, P puts in 15 head of cattle, and R removes 50 sheep. After two months, P removes 10 head of cattle, and Q removes 5 horses. How shall they settle at the close of the three months?

22. S, T, U, and V engage in partnership in the milling business. S and T run the mill, devoting their entire time to the business, instead of investing capital. U is a provision merchant, and sells the breadstuffs and feed, ground at the mill, for which the firm allows him a commission of 10%. U and V invest \$10000 each. At the end of one year, S and T have each drawn out \$800, and V has loaned the firm \$3000, which has been at interest for three months at 6%. U has sold breadstuffs for the firm to the amount of \$12500. They sell the mill and stock on hand for \$24000; they have uncollected accounts amounting to \$2450, which V agrees to take at 80%

of their face. The firm have on hand, cash \$2750, notes receivable \$1550. They owe farmers on account for grain \$2125. What is each partner's interest, after the gain or loss is divided?

23. Five partners in business invest equal amounts and share gains and losses equally. At the end of two years they close out the business, collecting and paying all notes and accounts; each partner being debited for all amounts collected by him, and credited for all amounts paid by him. The following are the results of the partnership accounts, including their investments:

A—Dr. \$5296.26, Cr. \$9625.48. B—Dr. \$7623.84, Cr. \$4287.35.
C—Dr. \$4823.15, Cr. \$3643.28. D—Dr. \$1985.34, Cr. \$4949.25.
E—Dr. \$10271.41, Cr. \$7494.64. How shall they settle with each other?

24. G. and H. begin business, agreeing to share losses and gains, in proportion to amount invested by each. G invests \$7000 and H invests \$8000. They buy a section of land for \$25000, paying cash \$9000 and giving a mortgage to secure the balance due in one year with interest at 6%. They immediately subdivide the land into lots, paying for surveying, platting, and grading \$2500. One-fifth of the lots are sold at a profit of 20%, $\frac{1}{2}$ of the remainder at a profit of 75%, and the balance at a profit of $33\frac{1}{3}\%$. What is each one's share of the gain, after paying the mortgage and interest at the end of one year?

ADDITIONAL WRITTEN PROBLEMS.

The following additional problems in Partnership and the topic, "Duties and Customs," will be inserted in the main text in the Revised Edition.

ADDITIONAL PARTNERSHIP PROBLEMS.

25. A firm made the following investment at the beginning of the year: Assets: Cash on hand and in bank \$3,000, mdse. \$4 000, bills receivable \$3,500, accrued interest on bills receivable \$240. Liabilities: Accounts payable \$3.125, bills payable \$1.750. At the close of the year the resources were: Cash \$2,780, mdse. \$4,560, bills receivable \$4.120, accrued interest \$175, personal accounts receivable \$4,125. Liabilities: Personal accounts payable \$3,740, bills payable \$850, accrued interest on bills payable \$110. The firm consists of A, B & C. They share gains as follows: A 42 per cent, B 38 per cent and C 20 per cent. Find the amount of gain due each partner.

26. The assets and liabilities of a firm at closing are as follows: Cash \$7,500, bills receivable \$3,250, personal accounts receivable \$5,120, merchandise \$12,000, insurance unexpired \$75, accrued interest on bills receivable \$300. Personal accounts payable \$2,100, bills payable \$3,000. The firm decides to write off 5 per cent of personal accounts receivable and bills receivable. The wages unpaid amount to \$450. The firm consists of A and B, A having invested \$8,500, and B having invested \$7,500. Find each one's share of the gain or loss according to his investment.

27. A, B and C form a partnership for one year, beginning January 1, 1904. Each one is to receive interest on his investment at the rate of 4 per cent per annum and to be charged interest at the rate of 6 per cent per annum on all withdrawals. On January 1st A invests \$6,000, B invests \$7,000,

and C invests \$7,500. On March 1st A invests \$1,000, B withdraws \$1,200, and C invests \$500. C withdraws \$500 at the end of each three months. On June 1st A withdraws \$1,200, B withdraws \$900, and C withdraws \$400. The loss and gain account shows the following: Merchandise a gain of \$10,540, interest a gain not including partners' accounts of \$1,256.40, expense a loss of \$3,160. The firm decides to reserve \$2,000 from the profits for a special fund, and decides to write off 10 per cent of the bills receivable account, amounting to \$4,000. The net profit is to be divided as follows: A 45 per cent, B 30 per cent, and C 25 per cent. What is each one's net profit and net worth at closing?

28. D, E and F are partners sharing gains and losses according to average investment of each. They are to be charged and credited with interest on their account at the rate of 5 per cent per annum. Their accounts are as follows:

D		1904.	
1904.		1904.	
Feb. 15, withdrew,	\$ 600 00	Jan. 1, invested,	\$12000 00
Mar. 15, " "	700 00	Mar. 15, " "	3000 00
June 20, " "	350 00	April 24, " "	2560 00
Aug. 16, " "	1000 00	July 21, " "	840 00
Sept. 18, " "	800 00		
Dec. 20, " "	350 00		

E		1904.	
1904.		1904.	
April 1, withdrew,	\$ 500 00	Jan. 1, invested,	\$10000 00
May 15, " "	300 00	May 28, " "	2000 00
July 25, " "	600 00	July 5, " "	1000 00
Oct. 20, " "	350 00	Sept. 24, " "	2500 00
Nov. 21, " "	124 35		

F		1904.	
1904.		1904.	
Feb. 1, withdrew,	\$ 200 00	Feb. 1, invested,	\$ 2000 00
Mar. 1, " "	200 00	Aug. 15, " "	1000 00
April 1, " "	200 00		
May 1, " "	200 00		
June 1, " "	200 00		
July 1, " "	200 00		
Aug. 1, " "	200 00		
Sept. 1, " "	200 00		
Oct. 1, " "	200 00		
Nov. 1, " "	200 00		
Dec. 1, " "	200 00		

ASSETS AND LIABILITIES EXCLUSIVE OF PARTNERS' ACCOUNTS.

Cash,	\$ 4560 00	Bills Payable,	\$2100 00
Bills Receivable,	7420 00	Per. Acct. Payable,	5700 00
Accounts Receivable,	6325 00	Accrued Interest,	250 00
Real Estate,	12000 00		
Bonds,	7500 00		
Bond Premium,	400 00		
Mdse.,	26000 00		
Suspense,	840 00		

Prepare special interest account to December 31, the close of the year, and enter interest in each partner's account. Determine each one's net loss or gain and carry the same to his account. Balance each partner's account.

29. X, Y and Z form a partnership for one year. Losses and gains to be shared as follows: 40 per cent, 37 per cent, and 23 per cent. X is to have a salary of \$3000 for managing the business, which sum is to be credited at the end of each six months. Interest at $4\frac{1}{2}$ per cent is to be allowed on average net credit, and charged at $7\frac{1}{2}$ per cent on average net withdrawal. The following is a Trial Balance at the end of the year:

X, Net Capital,	-	-	-	\$ 12,000 00	
Y, Net Capital,	-	-	-	14,000 00	
Z, Net Capital,	-	-	-	15,500 00	
Mdse.,	-	-	-	\$120,000 00	144,650 00
Real Estate,	-	-	-	10,000 00	
Shipment Account,	-	-	-	9,765 00	8,960 00
Cash,	-	-	-	12,060 00	
Bills Receivable,	-	-	-	9,700 00	
Personal Accounts Receivable,	-	-	-	14 600 00	
Personal Accounts Payable,	-	-	-		3,620 00
Bills Payable,	-	-	-		5,260 00
Expense,	-	-	-	5 420 00	
Interest and Discount,	-	-	-	524 00	360 00
Cash Discount,	-	-	-	351 00	824 00
Plant and Machinery,	-	-	-	15,604 00	
Wages,	-	-	-	7 150 00	
				<u>\$205,174 00</u>	<u>\$205,174 00</u>

Additional information given:

Of the notes on hand, one for \$1200 has been drawing interest for eleven months at 6 per cent, and one for \$800 is non-interest bearing and has nine months yet to run; money is worth 5 per cent.

Of the notes outstanding, one for \$960 bears interest at 6 per cent for one year, it is dated six months ago, and one for \$1000 is non-interest bearing and was issued four months ago at a discount of $4\frac{1}{2}$ per cent, the note being for one year.

The mdse. on hand was invoiced to the firm for \$25164. It is decided to advance the cost 2 per cent to cover transportation charges. Items charged to expense but still on hand valued at \$850. Wages not paid, \$750. The entering of the salary due X was overlooked. Five per cent of the accounts and bills receivable is to be carried to doubtful account. There is a \$100 bill in the cash drawer that is a counterfeit. The value of the real estate remains fixed. You are to write off $7\frac{1}{2}$ per cent depreciation to plant and machinery. Goods shipped and not sold valued at \$3160.

ANALYSIS OF PARTNERS' ACCOUNTS.

X			
1904.		1904.	
May 1,	withdrew,	Jan. 1,	invested,
Sept. 1,	"	July 1,	"
	\$2500 00		\$10000 00
	1500 00		6000 00
Y			
1904.		1904.	
Mar. 1,	withdrew,	Jan. 1,	invested,
June 1,	"	Oct. 1,	"
Nov. 1,	"		
	\$1400 00		\$12000 00
	1000 00		7000 00
	2000 00		
Z			
1904.		1904.	
Feb. 1,	withdrew,	Jan. 1,	invested,
July 1,	"	June 1,	"
Oct. 1,	"		
	\$1000 00		\$12000 00
	1500 00		6500 00
	500 00		

What is the net profit or loss of the business? What is the balance of each partner's account?

DUTIES AND CUSTOMS.

The Constitution of the United States provides that:

"The congress shall have power to lay and collect taxes, duties, imposts, and excises, to pay the debts and provide for the common defense and general welfare of the United States."

Duties are assessments levied on imported goods. There are two classes of duties, viz: ad valorem duties and specific duties.

Ad valorem duties are levied on imported goods at a certain per cent of the foreign valuation.

Specific duties are levied on imported goods according to weight or bulk.

Tare is allowance made for weight of material used in packing and casing.

A tariff is a list of the goods subject to import tax, and the rate of charge for each class of goods.

A Custom House is a government office established for the transaction of business relating to duties and the collection of the same.

A Bonded Warehouse is a warehouse under the control of government customs officials where imported goods are stored until the duty is paid.

Internal revenue is the proceeds of taxes levied by the government on certain goods manufactured in the United States. Such goods are frequently stored in bonded warehouses until sold or exported.

Drawback is money refunded for duties paid on imported goods which are afterwards exported.

WRITTEN PROBLEMS.

1. A imported 1000 bu. of wheat bran at 30c a bu. What is the duty at 20 per cent?

2. A merchant imported 1200 yds. of Wilton carpets invoiced at £1 per yd. Find the total cost to the merchant, the specific duty being 60c per sq. yd. and ad valorem duty 40 per cent, the carpet being 24 inches wide.

NOTE. £1 = \$4.8665.

3. The following were imported from France:

1 gross cotton embroidered gloves at 10 fr. per doz., duty 60 per cent.

6 doz. India rubber gloves at 45 fr. per doz., duty 30 per cent.

1 gross woolen gloves, weight 14 lbs., at 20 fr. per doz., duty 44c per lb. and 60 per cent.

$\frac{1}{2}$ gross gloves, 17 in. long, at 120 fr. per doz., duty \$4.50 per dozen pair.

Find the duty and also the total cost to the importer, including freight of \$24.60.

NOTE. 1 fr. = \$.193.

4. Brown & Co. import from Germany merchandise invoiced at 3125 marks, a discount of 5 per cent being allowed. The goods weigh 1200 pounds. Transportation charges, \$175. Specific duty, 8c per lb. and ad valorem duty 45 per cent. What is the total cost to the importer?

NOTE. 1 mark = \$.2385.

5. A merchant imported from Sheffield:

1 gross pen knives at 2s. per dozen, duty 40 per cent.

1 gross pruning knives at 6s. per doz., duty 5c apiece and 40 per cent.

2 gross scissors, 8s. per doz., duty 75c per doz. and 25 per cent.

1 gross razors, 15s. per doz., duty \$1.00 per doz. and 15 per cent.

Find the total duty and cost of goods to the merchant.

6. An importer of tobacco receives an importation of cigars weighing 960 lbs. invoiced at \$1.75 per lb. This price was raised 10 per cent by the custom officials. What duties must be paid by the importer, specific duty \$4.50 per lb., and ad valorem 25 per cent?

7. Find the duty on an importation of 250 gals. of liquors invoiced at 5 fr. per gal., 2 per cent being allowed for leakage, duty \$1.50 per gal. Also find the total cost to the importer, including charges amounting to \$24.60.

8. Find the total duty to the importer:

11240 lbs. of pig iron, duty \$4 per ton.

8360 lbs. of pig lead, duty $2\frac{1}{8}$ c per lb.

2146 lbs. of pig zinc, duty $1\frac{1}{2}$ c per lb.

NOTE. The long ton of 2240 lbs. is used at the Custom House.

9. Invoice of carpets received by Lyons & Co., Chicago, Ill.:

Marks	Q	Articles	Quan.	Price	Duty	Cost
L. C.	61	Aubusson -----	600 yds.	9s, 4d	60c sq. yd. 40 per cent	-----
L. C.	62	Axminster -----	752 "	5s, 6d	60c sq. yd. 40 per cent	-----
	62	Brussels -----	1240 "	8s, 6d	44c sq. yd. 40 per cent	-----
	62	Moquette -----	450 "	12s,	60c sq. yd. 40 per cent	-----
L. C.	63	Tapestry -----	620 "	5s,	28c sq. yd. 40 per cent	-----
L. C.	64	Wilton -----	800 "	£1,	60c sq. yd. 40 per cent	-----

Cases ----- \$18.00

What is the duty -----

What is the total cost -----

NOTE. Above Carpets 18 inches in width.

10. The goods imported as per description in problem nine were exported to the City of Mexico, less the Wilton carpets. What is the amount of drawback the importer is entitled to, deducting the one per cent withheld by the government?

11. A merchant imported 250 feet of figured linoleum at 3 s. per foot. What was the duty at 8c per sq. yd. and 15 per cent, the width of the linoleum being 10 feet? Also find total cost to the merchant, including charges of \$16.25.

ANSWERS TO ADDITIONAL PROBLEMS.

PARTNERSHIP.

25 A's gain, \$2181.90; B's gain, \$1974.10; C's gain, \$1039
26 A's gain, \$3334.39; B's gain, \$2942.11. **27** A's net profit, \$2501.51, A's net worth, \$8532.84; B's net profit, \$1667.67, B's net worth, \$6756.17; C's net profit, \$1389.72, C's net worth, \$7257.39. **28** D's net profit, \$14322.99, D's net worth, \$29654.51; E's net profit, \$11199.47, E's net worth, \$25397.10; F's net profit, \$1078.83, F's net worth, \$1943.39. **29** Total net gain, \$33594.43; X's net worth, \$28894.02; Y's net worth, \$26866.19; Z's net worth, \$23802.97.

DUTIES AND CUSTOMS.

1 \$60. **2** Total cost, \$8655.72. **3** Duty, \$90.48; Total cost, \$375.63. **4** \$1297.67. **5** Total duty, \$64.80; Total cost, \$178.68. **6** \$4782. **7** Duty, \$367.50; Total cost, \$633.35. **8** \$229.91. **9** Total duty, \$5498.25; Total cost, \$16411.38. **10** Retained by Government, \$37.01; Drawback, \$3653.96. **11** Total duty, \$569.70; Total cost, \$4235.83.

Appendix.

The following pages include a treatment of several topics of advanced arithmetic that ordinarily would not be presented in a book of this character, but which are given here for the convenience of teachers who may wish to have their classes take up these topics, and without making use of another text.

EXTRACTION OF THE CUBE ROOT.

A Cube, or Third Power, is the product obtained by multiplying together any three equal factors. Thus the cube of 2 equals $2 \times 2 \times 2$, or 8. The cube of $\frac{2}{3}$ equals $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, or $\frac{8}{27}$, and the cube of .4 equals $.4 \times .4 \times .4 = .064$.

Conversely, the Cube Root of a number is one of its three equal factors. Thus, the cube root of 8 is 2; the cube root of $\frac{8}{27}$ is $\frac{2}{3}$; and the cube root of .064 is .4.

The Cube Root is indicated by the sign ($\sqrt[3]{}$); thus, $\sqrt[3]{125} = 5$.

NOTE.—Comparatively few numbers are perfect powers, that is, numbers that are separable into equal and ascertainable factors. Thus, the number 36 is separable into the two equal factors, 6×6 ; the number 343 into the three equal factors $7 \times 7 \times 7$, and the number 81 into the four equal factors, $3 \times 3 \times 3 \times 3$; but the numbers 20, 46, 57, 72, etc., are not thus separable and are therefore not exact powers, and their roots can only be found approximately.

The root of a number that is not a perfect power in the degree of the root required, is called a *surd*. Thus the square roots of 12, 15, or 20, and the cube roots of 25, 38, or 100, and the fourth roots of 27, 40, or 75, are *surds*, and can only be found approximately.

Before proceeding further, let the student commit to memory *thoroughly* the following table of perfect cubes and their roots:

$1^3 = 1$	$7^3 = 343$
$2^3 = 8$	$8^3 = 512$
$3^3 = 27$	$9^3 = 729$
$4^3 = 64$	$10^3 = 1000$
$5^3 = 125$	$100^3 = 1,000,000$
$6^3 = 216$	$1000^3 = 1,000,000,000$

From an inspection of the foregoing table it is evident:

1. That the cube root of any number consisting of three figures, or less, consists of *one figure*.
2. That the cube root of any number consisting of more than three figures and less than seven, will consist of *two figures*.
3. That the cube root of any number consisting of more than six figures and less than ten, will consist of *three figures*, and so on.

Hence, to find the number of figures in the cube root of any number:—

Separate the figures into groups of three figures each. The number of groups thus formed will equal the number of figures in the cube root.

EXAMPLE.—The cube root of 32|768 contains two figures (32); and the cube root of 12|812|904 consists of three figures (234).

EXERCISE.

State the number of figures in the cube root of each of the following numbers:

1674	21764321
42798	1207658247
3479160	24765872647962

PROCESS OF FINDING THE CUBE ROOT.

It may be shown universally by algebra and geometry, and specifically by arithmetic, that the *cube of the sum* of any two numbers is equal to four united quantities, viz.:

- (1.) The cube of the first number.
- (2.) Three times the square of the first number, multiplied by the second number.
- (3.) Three times the first number, times the square of the second number.
- (4.) The cube of the second number.

We may illustrate this by the following arithmetical operation: Required the cube of $5+4$.

$$\begin{array}{r}
 5+4 \\
 5+4 \\
 20+16 \\
 25+20 \\
 25+40+16=81 \text{ (the square of } 5+4) \\
 5+4 \\
 \hline
 100+160+64 \\
 125+200+80 \\
 125+300+240+64=729 \text{ (=the cube of } 5+4)
 \end{array}$$

Summary: $125=5^3$
 $300=3 \times (5^2 \times 4)$
 $240=3 \times (5 \times 4^2)$
 $64=4^3$
 $729=(5+4)^3$

Every number consisting of two figures may be considered as the *sum of its tens and its units*.

Thus: $25=(20+5)$, $12=(10+2)$, $79=(70+9)$, etc.

If, therefore, we represent the tens by the letter t and the units by the letter u , we may formulate the cube of every such number as follows:

$(t+u)^3=t^3+3t^2u+3tu^2+u^3$. That is, the cube of every number consisting of tens and units is separable into the four quantities, viz.:

- (1.) The cube of the tens.
- (2.) Three times the square of the tens, times the units.
- (3.) Three times the tens, times the square of the units.
- (4.) The cube of the units.

Thus, in the case of the number 25, the cube may be found as follows:

$$\begin{array}{r}
 t^3(20 \times 20 \times 20) = 8000 \\
 3t^2u(3 \times (20 \times 20) \times 5) = 6000 \\
 3tu^2(3 \times 20 \times 5 \times 5) = 1500 \\
 u^3(5 \times 5 \times 5) = 125 \\
 25^3 = \overline{15625}
 \end{array}$$

EXERCISE.

Using the foregoing method, find the cubes of the numbers: 16, 27, 38, 56, 95.

We will now apply this formula to the reverse process of finding the *cube root* when the cube is given.

Required the cube root of 42875.

$$\begin{array}{r}
 t^3 + 3t^2u + 3tu^2 + u^3 = 42 \overline{)875} \quad (30 + 5 \\
 t^3 = 27 \\
 3t^2 = 2700 \overline{)15} \quad 875 \\
 3tu = 450 \quad 15 \quad 875 \\
 u^2 = 25 \\
 \hline
 3175
 \end{array}$$

EXPLANATION.—First, we perceive that the number contains two “cube root periods,” and that its root therefore contains two figures (tens and units).

Since the cube of tens consists of thousands, the largest cube contained in the thousands (27) is the cube of the tens of the root, and the cube root of the 27 (thousands) is 3 the tens of the root. This root we place at the right, as the first figure of the required root, and subtract its cube (27) from the thousands, obtaining a remainder of 15, to which we bring down the next period.

We now know that this remainder (15875) comprises the remaining three parts of the cube formula ($3t^2u + 3tu^2 + u^3$). And since the first of these terms ($3t^2u$) usually comprises the larger part of the remaining number, we may *assume* that it comprises the *whole* of it, and obtain the value of u by *dividing* the remainder, 15875, by $3t^2 (3 \times 30 \times 30) = 2700$. Performing the division by this “trial divisor” we obtain 5 for the units figure.

It now remains to complete the divisor by adding to the trial divisor the omitted values of t and u in the third and fourth parts of the formula. These are; $3tu (3 \times 30 \times 5) = 450$, and $u^2 = 25$. The sum of these gives the complete divisor, 3175, which multiplied by the unit figure of the root 5, equals the remainder of the cube, 15875, and proves the correctness of the operation.

Out of the foregoing demonstration we derive the following

GENERAL RULE FOR EXTRACTING THE CUBE ROOT:

I. *Point off the number into periods of three places each, beginning at the decimal point and counting to the left for integers, and to the right for decimals.*

II. *Find the largest cube in the left-hand period, and place its root at the right. Subtract the cube from the left-hand period and annex the second period to the remainder.*

III. *Find three times the square of the first term of the root, considered as tens, and place it at the left as a trial divisor. Divide the second dividend by the trial divisor, and place the quotient as the second term of the root.*

IV. Find three times the product of the first and second terms of the root, considering the former as tens, and write it under the trial divisor. Square the second term of the root, and write the result under the preceding product. Find the sum of these three results, and multiply it by the second term of the root. Subtract the product thus found from the partial dividend and to the remainder annex the next period.

V. Find three times the square of the root already found, considered as tens, and write it at the left as a second trial divisor. Find the third term of the root and complete the divisor as before.

WHERE THE ROOT CONSISTS OF MORE THAN TWO FIGURES.

Required the cube root of 34,328,125.

OPERATION.

1st trial divisor $(30 \times 30) \times 3 = 2700$

$(30 \times 2) \times 3 = 180$

$2 \times 2 = 4$

1st complete divisor 2884

2d trial divisor $(320 \times 320) \times 3 = 307200$

$(320 \times 5) \times 3 = 4800$

$(5 \times 5) = 25$

2d complete divisor 312025

34328|125(325

27

7328

5768

1560 125

1560 125

EXPLANATION.—The first two figures of the root are found by the process already explained. The third period is then brought down and the portion of the root already obtained (32) is treated as tens, and a second trial divisor is found and completed as before. If there are other periods, they are in turn brought down and the process continued, the portion of the root already found being each time treated as tens in finding the next required figure.

FINDING APPROXIMATE ROOTS.

If the number of which the cube root is sought is not a perfect cube its root is a surd, and may be found approximately by annexing successive periods of three ciphers each, the resulting root figures being decimals.

Find the approximate cube root of 635.

OPERATION.

$$\text{1st trial divisor } (80 \times 80) \times 3 = 19200$$

$$(80 \times 5) \times 3 = 1200$$

$$5 \times 5 = 25$$

$$\text{1st complete divisor } \underline{20425}$$

$$\text{2d trial div. } (850 \times 850) \times 3 = 2167500$$

$$(850 \times 9) \times 3 = 22950$$

$$(9 \times 9) = 81$$

$$\text{2d complete divisor } \underline{2190531}$$

$$635.000 \overline{) 8.59}$$

$$512$$

$$\underline{123.000}$$

$$102.125$$

$$\underline{20\ 875\ 000}$$

$$19.714779$$

$$\underline{1.160221}$$

WRITTEN PROBLEMS.

Find the cube root of each of the following perfect cubes:

1. 262144.

2. 884736.

3. 1860867.

4. 15252992.

Find the cube roots of the following numbers correct to one decimal place:

5. 196.

6. 2147.

7. 19.875.

8. 248.7.

NOTE.—In forming decimal periods, annex enough ciphers for the period to have three places. Thus the last number should be written in this form: 248.700.

Find the cube root of the following correct to two decimal places:

9. 13.

10. 147.

11. 3.568.

12. What is the cube root of $5\frac{7}{8}$ correct to hundredths?

SUGGESTION.—Reduce the fraction to a decimal and find the cube root of the resulting decimal number.

13. Find the cube root of $15\frac{3}{4}$, carried to two decimal places.

14. If 160103007 cubical blocks of stone, each one foot square, were placed in a cubical pile, what would be the length of each edge?

15. There are 231 cubic inches in a gallon. Find the inside dimensions, correct to tenths of an inch, of a cubical tank that would hold 1000 gallons.

16. If a bar of iron 25 ft. 4 in. long, 1 ft. 2 in. wide, and 5 in. thick, were melted and cast in the form of a cube, what would be the length of each edge? Give the answer correct to tenths of an inch.

17. Find one of the three equal factors whose product equals 373248.

NOTE.—The cube root of a perfect cube may be obtained by resolving the cube into its prime factors, and then arranging these into equal groups; the product of each root will be the required cube root.

18. What is the cube root of $\frac{343}{729}$?

SUGGESTION.—The cube root of any common fraction may be expressed by writing the cube root of the numerator over the cube root of the denominator. Solve the above problem by inspection.

19. What is one of the three equal numbers that, multiplied together, will produce 28.934443?

20. Find the dimensions of a cubical bin that will have the same capacity as a bin 8 ft. 9 in. long, 5 ft. 3 in. wide, and 6 ft. 4 in. deep. Answer correct to tenths of an inch.

METRIC SYSTEM OF DENOMINATE NUMBERS.

In France, and other parts of Europe, and by scientific institutions generally, a system of denominate measures is used, which is known as the **Metric System**.

The system takes its name from the fact that all standards of weight and measurement are derived primarily from the unit of length called the *meter*.

Theoretically, the meter is one ten millionth part of a meridian quadrant, or the distance from the earth's equator to either pole, but owing to an error in the calculations at the time the meter was established in France, it varies slightly from this length.

The meter is somewhat longer than the English yard, being 39.374 in. It is divided into decimal divisions, decimeters

(tenths), centimeters (hundredths), millimeters (thousandths), etc.

In the tables of the Metric System the Latin prefixes *deci.*, *centi.*, *milli.*, indicate respectively *tenths*, *hundredths*, *thousandths*, while the Greek prefixes *deka*, *hecto*, *kilo*, respectively indicate *ten*, *one hundred*, *one thousand*.

The application of these prefixes will be seen in the following table of the linear units.

METRIC UNITS OF LENGTH.

TABLE.

1 kilometer (Km.=1000 meters)=.621 (about $\frac{5}{8}$) statute mi.

1 hectometer (Hm.=100 meters)=109.4 yd.

1 dekameter (Dm.=10 meters)=1.988 rd.

1 meter (m)=39.37 in.

1 decimeter (dm.= $\frac{1}{10}$ of a meter)=3.937 in.

1 centimeter (cm.= $\frac{1}{100}$ of a meter)=.393 in.

1 millimeter (mm = $\frac{1}{1000}$ of a meter)=.039 in.

In practice the hectometer and dekameter are rarely or never used.

The myriameter (100 kilometers, or 10000 meters) is sometimes employed to indicate long distances.

In microscopic measurements the *micron.*, or one millionth of a meter (about $\frac{1}{254000}$ of an inch), is used.

REDUCTION OF METRICAL DENOMINATE NUMBERS.

The reduction of metric numbers to higher or lower terms is very simply accomplished by merely moving the decimal point to the left or to the right. This simplicity of calculation is one of the chief advantages of the system. Thus, if it be desired to change 25678 meters to kilometers, all that is necessary is to point off three places, and we have 25 678 $\frac{4}{10}$ ometers. Conversely, kilometers may be changed to meters by simply moving the decimal point three places to the right, annexing ciphers if necessary; thus, 26.71 kilometers=26710 meters.

To add a series of metrical quantities, all that is necessary is to place the decimal points so that all the quantities will be

of the same denomination required in the answer, and then add the numbers as in Addition of Decimals.

EXAMPLE.—How many meters in 16712 decimeters, 34127 centimeters, 29.6 kilometers, and 47625 millimeters?

OPERATION.

$$16712 \text{ dm.} = 1671.2 \text{ meters.}$$

$$34127 \text{ cm.} = 341.27 \text{ meters.}$$

$$29.6 \text{ Km.} = 29600. \text{ meters.}$$

$$47625 \text{ mm.} = 47.625 \text{ meters.}$$

$$\hline 31660.095 \text{ meters. Ans.}$$

In reducing linear metrical measurements to English standards, and *vice versa*, when exact results are required, the equivalents given in the table should be used, but for approximate calculations it is well to remember the following:

1 meter is about $3\frac{1}{4}$ ft.

1 kilometer is about $\frac{5}{8}$ of a mile.

1 centimeter is about $\frac{1}{16}$ of an inch.

1 millimeter is about $\frac{1}{25}$ of an inch.

Use the above equivalents in solving the following practical problems:

1. The distance between two French cities is given as 122 Km. What is the distance in miles?

2. A table is 146 cm. wide and 258 cm. long; give its dimensions in inches.

3. A work on entomology gives the length of a certain beetle as 27.6 mm. What is its length expressed in decimals of an inch?

4. An American automobile manufacturer has a vehicle that can run at the rate of 46 miles an hour. A French customer writes to know its speed per hour in kilometers. What answer should be returned?

5. An account of a European battle states that the artillery was planted at a distance of 1600 meters from a given fort. What was the distance in yards?

6. The draught of a Belgian war vessel is given at 9.68 meters, her length at 126.15m. and her beam 15.27m. Give the dimensions of the vessel in feet and inches.

7. A Paris paper gives the speed of a new French mail

train at 72.46 Km. per hour. What is the speed of the train in miles?

In the following problems make exact calculations, using the equivalents given in the table.

8. A Paris firm places with an American foundry an order for a cargo of steel bridge frames 17.128m. in length. What is the required length in feet and inches, correct to hundredths?

9. A Chicago picture dealer wishes to order from Paris a frame for a canvas 42×38 inches. Required the dimensions of the frame in centimeters, correct to hundredths.

10. A bolt of imported silk is marked at 72.28m. A dress-maker purchased it at \$1.25 per yard. What is the amount of the bill?

11. A scientific journal gives the focal distance of a microscopic lens as 7 mm. What is the distance in decimals of an inch?

SURFACE MEASUREMENTS.

The area of ordinary surfaces is expressed in the Metric System in square meters, square decimeters, square centimeters, etc.

The standard unit for the measurement of land is the *Are*, which is a square, each side of which is 10 meters, and its area is, therefore, equal to 100 square meters.

The *Hectare* is also used; it is a square, each side of which is 100 meters, and its area is 10000 square meters.

TABLE OF EQUIVALENTS.

1 sq. cm. = .154 + sq. in.

1 sq. m. = 1.196 + (about $1\frac{1}{8}$) sq. yd.

1 are = 3.95 + (about 4) sq. rd.

1 hectare = 2.471 + (about $2\frac{1}{2}$) acres.

In calculating areas, the dimensions are multiplied together, and the result is reduced to the desired unit by moving the decimal point.

EXAMPLE.

12. What is the area in ares, of a lot 28.2 meters wide by 65.6 meters long?

OPERATION.

$$\begin{array}{r}
 28.2 \\
 65.6 \\
 \hline
 1692 \\
 1410 \\
 1692 \\
 \hline
 1849.92 \text{ sq. m.} = 18.49 \text{ ares.}
 \end{array}$$

WRITTEN PROBLEMS.

13. At 25 francs per are, what is the value of a lot 75 m. long and 32.6 m. wide?

14. What would the same land be worth at \$100 per acre?

NOTE.—In this and subsequent problems involving French exchange consider one franc as being equal to 19.3 cents U. S. money.

15. How many square yards in a room 4.15 m. x 6.21 m.?

16. What is the area in square inches of a piece of sheet metal 21.1 centimeters in length and 8.5 centimeters wide?

17. How many square yards of surface can be covered by a bolt of Belgian carpeting 70 cm. wide, containing 100 meters?

18. A lot 15 by 25 rods would contain how many ares?

19. A Paris paper represents that a tract of land, 521 x 475.5 meters, was sold at 1500 francs per hectare. Considering a hectare at $2\frac{1}{2}$ acres, how many acres of land were in the tract? What was the cost in U. S. money?

VOLUME MEASUREMENTS.

In the Metric System, volumes are expressed in cubic meters, cubic centimeters, etc., depending on the extent of the object to be measured.

For measuring excavations, stonework, wood, etc., the *stere*, or cubic meter, is used.

EQUIVALENTS.

1 cu. centimeter = .0607 cu. in.

1 stere (cu. meter) = 1.3 cu. yards or 35.1 cu. ft.

WRITTEN PROBLEMS.

20. How many steres is an excavation 4.5 m. x 6.2 m. x 2.8 m.?

21. How many cubic feet in a piece of masonry 5.7 m. long, 2.15 m. high, and 42 cm. thick?
22. When wood is sold in the Paris market at 3 francs per stere, what is the price per cord in United States money?
23. At 17 francs per stere, what will be the cost of the coal required to fill a bin 5.16 m. long, 1.8 m. wide, and 76 cm. deep?
24. An exporter ships to France a cargo of 650 cords of tan bark which cost \$11.50 per cord. The cargo was sold in Paris at 18 francs per stere. What was the gross gain?

MEASUREMENTS OF CAPACITY.

The standard measurement of capacity in the Metric System is the *Liter* (pronounced *leeter*), or cubic decimeter, the volume of which is 1000 cubic centimeters.

TABLE.

1 kiloliter (1000 liters)	=264 gal.
1 hectoliter (100 liters)	=26.4 gal.
1 dekaliter (10 liters)	=2 $\frac{5}{8}$ gal. (very nearly).
1 liter (1 cubic decimeter)	=1.057 qt.
1 deciliter ($\frac{1}{10}$ of a liter)	=.845 gi.

WRITTEN PROBLEMS.

25. Find the capacity in dekaliters of a box-tank 2.5 m. long, 1.6 m. wide, and 2.8 m. deep
26. A 200-liter cask of oil is purchased in Paris at 1.5 francs per liter, and retailed in New York at \$1.95 per gal. Allowing \$12 95 for duties and transportation, what sum was gained?
27. Find the capacity in liters (correct to hundredths) of a box 10x15x8 inches?
28. When the price of milk in Paris is 6 sous per liter, what is the price (correct to tenths of a cent) per quart in U. S. money?

NOTE.—A sou is equal to $\frac{1}{10}$ of a franc.

29. A 30-gallon cask of American alcohol is worth how much in Paris, if sold at 1.6 francs per liter?

METRIC WEIGHTS.

The standard is the *gram* which is the weight of a cubic centimeter of distilled water at a temperature of greatest density, or 4 degrees centigrade (about 39 degrees Fahrenheit).

TABLE.

1 tonneau (t)	1,000,000g. or 1000 Kg.	=2204.621 lb.
1 quintal,	100000g. or 100 Kg.	=220 lb. 7 oz.
1 kilogram (Kg.) or "kilo,"	1000 g.	=2.204 (practically 2½ lb.) Avoir.
1 hectogram (Hg.)	100g.	=3.537 oz. Avoir.
1 dekagram (Dg.)	10g.	=154.32 gr.
1 gram (g.)		15.432 gr.
1 decigram (dg.)	$\frac{1}{10}$ of a g.	=1.54 gr.
1 centigram (cg.)	$\frac{1}{100}$ of a g.	=.154 gr.
1 milligram (mg.)	$\frac{1}{1000}$ of a g.	=.015 gr.

The units in most common use are the tonneau or "ton," the quintal, the kilo, the gram, and the centigram.

The fact that the standards of weights are derived from a given volume of water, renders the weights of given volumes of other substances most easy of computation, when specific gravity tables are used.

NOTE.—By the *specific gravity* of a substance is meant the ratio of its weight to that of an equal volume of pure water.

ILLUSTRATION.—The weight of a cubic foot of water is 62.5 lb. Avoir., while the weight of a cubic foot of gold is 1206.25 lb.

Dividing the weight of the gold, 1206.25, by the weight of the water, 62.5, gives 19.3, the specific gravity of the gold.

The number of cubic centimeters in the volume of any given body is equal to the number of grams in the weight of an equivalent volume of water. Hence, to find the weight of any substance, multiply its volume in cubic centimeters by its specific gravity; the result will be its weight in grams.

In solving problems of this character, the accompanying reference table of specific gravities will be found convenient:

Metal.	Sp. gr.
Platinum	21.5
Gold.....	19.3
Mercury.....	13.6
Lead	11.3
Silver.....	10.5
Copper	8.9
Iron	7.6
Tin.....	7.2
Zinc.....	7.
Aluminum	2.7

WRITTEN PROBLEMS.

30. Find the weight in pounds avoirdupois, of the water contained by a tank 2.5 m. long, 1.8 m. wide, and 1.2 m. deep.

SUGGESTION.—The number of cubic decimeters equals the weight in kilos.

31. An importer bought an invoice of 129.6 kilos of gum at 28.5 francs per kilo. After adding \$28.75 for duties and transportation charges, what is the cost per Troy ounce (480 grains) in U. S. money?

32. What is the weight in Troy ounces and grains, of a bar of silver 15 cm. long, 7 cm. wide, and 42 cm. thick.

33. The dimensions of an aluminum paper weight are 11.2x 6.4x4.1 cm. Give its weight in pounds avoirdupois, correct to three decimal places.

NOTE.—There are 7000 grains in one pound avoirdupois.

34. A rectangular block of Italian marble is 85 cm. long, 56.4 cm. wide, and 35 cm. thick. Find its weight in pounds avoirdupois, the specific gravity of the marble being 4.8.

35. When wheat is quoted in the Brussels market at 21.4 francs per quintal, what is the equivalent price per bushel of 60 lbs.?

ANSWERS TO THE WRITTEN PROBLEMS PART I.

ADDITION—Page 16.

1 35 2 40 3 43 4 47 5 48 6 55 7 53 8 49
 9 42 10 38 11 51 12 115 13 105 14 115 15 113
 16 113 17 115 18 111 19 110 20 120 21 2660
 22 2509 23 3639 24 3665 25 3449 26 36681
 27 6361729 28 5883784 29 10292 30 183719 31
 237766 32 211213 33 184108 34 275274 35 602151045
 36 \$24006.44 37 94 38 102 39 102 40 89 41 103
 42 \$2871.71 43 \$4011.97 44 \$24006.34 45 108 46
 149 47 \$13.65 48 \$22 49 \$20.86 50 \$15.05 51
 \$98.50 52 \$54.65 53 \$86.50 54 \$143.20 55 \$3331.41
 56 \$535 57 20692 lbs. 58 5973 59 625 60 650 61
 356453 62 44650 63 Each total 68591.

SUBTRACTION—Page 26.

1 8267 2 718 3 15197 4 52858 5 893329 6 14047
 7 334438 8 74999263 9 \$5097.50 10 \$25539.83 11
 \$910.33 12 2184234 13 41972315 14 3317 15 7538
 16 \$34804.73 17 \$353148 18 2891741 19 24674546
 20 62376 21 3966 22 17101 lbs. 23 \$11071.17 24
 2717224 gal. 25 \$77349.77 26 166637 acres 27 68853
 28 \$470.93 29 14266 30 \$681.33 31 \$13706.48 32
 Debit \$1148.81 33 Credit \$225.28 34 Credit \$557.41 35
 Credit \$229.26 36 Debit \$2677.20 37 Credit \$3879.07 38
 Credit \$2157.94 39 Credit \$1294.39 40 Credit \$447.20.

MULTIPLICATION—Page 32.

1 3856; 29784; 36964; 98252; 39072 2 10230; 37435;
 462535; 354530; 99335 3 44628; 125436; 232320; 508614;
 2232402 4 300132; 628145; 341166; 635348; 410746 5 77400;
 55664; \$781.20; \$1515.84; \$75810.24 6 \$7.83; \$88.65; \$5378.58;
 \$88555.14; 420822 7 216456 8 542184 9 664652 10

139482 11 669768 12 699678 13 \$86323.02 14
 \$250709.67 15 50112 16 45360 17 306816 18 367416
 19 382270 20 3391152 21 768000 22 1591600 23
 92550000 24 3120300000 25 3222400000 26 20720000000
 27 62088 28 590688 29 2710656 30 \$63054 31
 121338 32 1978812 33 3982704 34 \$25701.20 35
 1022868 36 6644400 37 5914048 38 15012000 39
 7022366120 40 2488201600 41 19116000000 42
 2944861920 43 \$1268.25 44 560 lots; \$98000 45 \$3139.20
 46 247632 47 \$101616 48 \$3506.25 49 41280 rods
 50 76960 lbs. 51 \$4425 52 \$3412.50 53 \$531 25 Gain
 54 \$19.55 55 \$176.20 56 \$79.10 57 \$426 58 \$15.96
 59 \$502.15 60 \$1547.35 61 \$11.30.

DIVISION—Page 45.

1 282—4 R. 2 1496 3 1284—2 R. 4 1965311—3 R. 5
 4152240—1 R. 6 1352549—5 R. 7 \$2891.19 8 \$45248.05
 9 882401—10 R. 10 16802883 11 1146—12 R. 12
 3123—19 R. 13 70349—132 R. 14 1406084—30 R. 15
 1129470—443 R. 16 131287—354 R. 17 91439—11 R.
 18 111925—371 R. 19 569283—245 R. 20 5274—3469 R.
 21 17—439 R. 22 2030—6201 R. 23 85—266730 R. 24
 167—42013 R. 25 27403—6367 R. 26 11562500 miles per
 minute; 192708+ miles per second 27 32; 16 days 28
 \$16.75 29 19000 bu. 30 124 31 2358 lbs. 32 57 gal.
 33 795 bu. 34 12 ds. 10 hr. 35 15 yr. $1\frac{9}{11}$ mo.

COMBINATIONS OF PROCESSES—Page 55.

1 1000 bu. 2 $26\frac{2}{3}$ hr. 3 \$375 4 \$1050 5 \$338 6
 \$44.55 7 \$5510 8 \$1370 9 160 yds. 10 \$135.

PROMISCUOUS WRITTEN PROBLEMS—Page 57.

1 \$39 2 95 cts. 3 \$3.75 4 \$32 5 620 bu. 6 \$379.37
 7 \$16.50 8 45 mi. 9 \$432.60 10 \$37.49 11 22 cts. 12
 \$395.25 13 46 bicycles; \$28 unexpended 14 116 bbl.; \$8
 unexpended 15 \$869.50 16 18 bu. 17 \$4736.70 18
 13 mi. 19 \$12.85 20 \$58850.

ANSWERS TO THE WRITTEN PROBLEMS

PART II.

CANCELLATION—Page 13.

1 $22\frac{1}{2}$ 2 720 3 320 4 $25\frac{1}{2}$ 5 405 lb. 6 96 lb. 7 32 bu. 8 $6\frac{1}{4}$ pc. 9 $42\frac{3}{16}\phi$.

FACTORING—Page 14.

1 2 2 2 5 5 5 2 5 5 31 3 5 5 257 4 5
5 5 67 5 45's 28's 6 112's 7 5 5 67 8 2 5 3
3 7 7 9 5 3 3 163 10 5 2 2 7 41 11 5 3 3
7 11 12 7 7 11 13 13 5 7 11 13 14 2 3 7
11 13 15 2 11 53's 16 3 7 11 13 17 3 7 11
17 18 5 3 82's 19 17 7 11 13 20 19 7 11 13

COMMON DIVISORS—Page 16.

1 44 2 2 3 4 4 30 5 4 6 131 7 4 8 1001 9
25 10 111 11 25 12 4 13 139 14 14 15 75
16 327 17 Length 95 ft.; width 4 ft.

MULTIPLES—Page 18.

1 1800 2 600 3 720 4 2520 5 2520 6 1560 7 1125
8 840 9 3465 10 144 11 10080 12 360 13 900
14 \$600; 47 animals. 15 90 ft.

FRACTIONS—Page 20.

1 $\frac{9}{12}$ $\frac{8}{12}$ $\frac{10}{12}$ 2 $\frac{2}{24}$ $\frac{18}{24}$ $\frac{20}{24}$ $\frac{21}{24}$ 3 $\frac{68}{80}$ $\frac{30}{80}$ $\frac{36}{80}$ $\frac{38}{80}$ $\frac{25}{36}$
4 $\frac{128}{72}$ $\frac{45}{72}$ $\frac{180}{72}$ $\frac{51}{72}$ $\frac{62}{72}$ 5 $\frac{18}{24}$ $\frac{21}{24}$ $\frac{14}{24}$ 6 $\frac{28}{12}$ $\frac{45}{12}$ $\frac{34}{12}$ 7
 $\frac{126}{144}$ $\frac{81}{144}$ $\frac{162}{144}$ $\frac{44}{144}$ 8 $\frac{168}{336}$ $\frac{192}{336}$ $\frac{105}{336}$ $\frac{176}{336}$ 9 $\frac{315}{420}$ $\frac{96}{420}$
 $\frac{1120}{420}$ $\frac{260}{420}$ 10 $\frac{153}{180}$ $\frac{95}{180}$ $\frac{126}{180}$ $\frac{180}{180}$ 11 $\frac{135}{105}$ $\frac{24}{105}$ $\frac{70}{105}$ $\frac{65}{105}$
12 $\frac{9}{240}$ $\frac{42}{240}$ $\frac{44}{240}$ $\frac{156}{240}$ 13 $\frac{440}{80}$ $\frac{605}{80}$ $\frac{288}{80}$ 14 $\frac{2691}{9269}$ $\frac{4891}{9269}$
 $\frac{4433}{9269}$ 15 $\frac{1400}{1344}$ $\frac{196}{1344}$ $\frac{273}{1344}$ $\frac{400}{1344}$ 16 $\frac{2997}{3456}$ $\frac{936}{3456}$ $\frac{2016}{3456}$
 $\frac{26}{3456}$ 17 $\frac{2}{3}$ 18 $\frac{1}{3}$ 19 $\frac{2}{9}$ 20 $\frac{7}{18}$ 21 $\frac{1}{8}$ 22 $\frac{1}{2}$ 23 $\frac{1}{3}$
24 $\frac{1}{5}$ 25 $\frac{11}{20}$ 26 $\frac{7}{9}$ 27 $\frac{11}{12}$ 28 $\frac{37}{55}$ 29 $\frac{1}{2}$ 30 $\frac{1}{40}$
31 $\frac{5}{7}$ 32 $\frac{4}{11}$ 33 $\frac{15}{16}$ 34 $\frac{7}{8}$ 35 $\frac{3}{4}$ 36 $\frac{5}{123}$ 37 $\frac{79}{128}$
38 $\frac{121}{128}$ 39 $\frac{427}{15}$ 40 $\frac{8748}{27}$ 41 $\frac{297}{16}$ 42 $\frac{999}{8}$ 43 $\frac{2037}{16}$
44 $\frac{12013}{32}$ 45 $\frac{1877}{32}$ 46 $\frac{4314}{125}$ 47 $\frac{273}{7}$ 48 $\frac{16000}{125}$ 49

$\frac{53}{8}$ gi. 50 $\frac{219}{4}$ gal. 51 $\frac{437}{12}$ hr. 52 $\frac{2167}{30}$ mo. 53 $\frac{29602}{160}$
 rd. 54 $\frac{4903}{20}$ T. 55 $\frac{6343}{16}$ lb. 56 $\frac{25265}{27}$ cu. yd. 57 $\frac{50549}{60}$
 min. 58 $\frac{9409}{3}$ dollars 59 $\frac{227941}{1728}$ cu. ft. 60 $\frac{1963}{1}$ gal.
 61 $\frac{13895}{144}$ sq. ft. 62 $\frac{1155}{128}$ cd. 63 $\frac{2953}{32}$ bu. 64 $17\frac{1}{4}$
 65 $11\frac{9}{16}$ lb. 66 41 bu. 67 $13\frac{3}{8}$ rd. 68 72 cu. ft. 69
 $26\frac{3}{4}$ 70 $40\frac{5}{24}$ 71 $11\frac{6}{25}$ 72 $19\frac{31}{32}$ 73 $28\frac{7}{8}$ 74 $68\frac{1}{8}$
 75 $345\frac{1}{4}$ 76 $74\frac{77}{125}$ 77 $1\frac{5}{8}$ 78 $602\frac{1}{16}$ 79 $87\frac{7}{8}$ 80
 $6\frac{725}{1728}$ 81 $693\frac{3}{8}$ 82 $58\frac{392}{625}$ 83 $28\frac{117}{160}$ 84 $23\frac{1}{8}$ 85 $61\frac{1}{8}$
 86 $158\frac{31}{48}$ 87 $236\frac{1}{4}$ A. 88 $147\frac{3}{4}$ lb. 89 $1953\frac{1}{16}$ bu;
 $\$35\frac{25}{48}$ 90 1050 yd. 91 $111\frac{13}{60}$ 92 $5168\frac{5}{66}$ 93 $96\frac{9}{8}$
 94 $52\frac{19}{60}$ 95 $8\frac{3}{4}$ 96 $49\frac{7}{8}$ 97 $9\frac{1}{16}$ 98 $65\frac{5}{32}$ 99 $66\frac{1}{4}$
 100 $16\frac{9}{16}$ 101 $57\frac{7}{8}$ 102 $72\frac{5}{8}$ 103 $51\frac{7}{8}$ 104 $63\frac{1}{2}$
 105 $166\frac{1}{16}$ 106 $189\frac{9}{32}$ 107 $36\frac{1}{8}$ bu. 108 $13\frac{7}{16}$ lb. 109
 $95\frac{4}{11}$ 110 $54\frac{3}{17}$ 111 $7\frac{1}{12}$ 112 $45\frac{5}{11}$ 113 $15\frac{1}{2}$ 114
 $144\frac{1}{2}$ 115 $13\frac{5}{24}$ A. 116 $14\frac{43}{64}$ bu. 117 $35\frac{1}{16}$ 118 $45\frac{7}{8}$
 119 $9\frac{5}{7}$ 120 $125\frac{1}{2}$ 121 $137\frac{39}{64}$ 122 $152\frac{9}{16}$ 123 $70\frac{1}{8}$
 124 $51\frac{9}{24}$ 125 $41\frac{5}{4}$ 126 $228\frac{1}{7}$ 127 $81\frac{1}{2}$ 128 $12\frac{1}{16}$
 129 $60\frac{1}{16}$ 130 $4\frac{43}{48}$ 131 $56\frac{1}{12}$ 132 $51\frac{1}{2}$ 133 $50\frac{381}{440}$
 134 $88\frac{4}{112}$ 135 $121\frac{1}{48}$ 136 $22\frac{1}{8}$ gal. 137 $16\frac{1}{16}$ A.
 138 $11\frac{1}{8}$ 139 $110\frac{31}{48}$ 140 $115\frac{1}{8}$ 141 $19\frac{5}{48}$ 142 $17\frac{1}{8}$
 143 $214\frac{1}{16}$ 144 $360\frac{1}{48}$ 145 $319\frac{179}{192}$ 146 $28\frac{1}{2}$ A.
 147 $21\frac{1}{16}$ gal. 148 $9\frac{3}{8}$ 149 15 150 $15\frac{1}{8}$ 151 $93\frac{3}{4}$
 152 135 153 $10\frac{2}{25}$ 154 $21\frac{1}{8}$ 155 $92\frac{2}{3}$ 156 33
 157 $28\frac{1}{2}$ 158 $53\frac{1}{3}$ 159 $173\frac{1}{16}$ 160 $1171\frac{1}{8}$ 161 $79\frac{1}{2}$
 162 $348\frac{3}{4}$ 163 $583\frac{1}{8}$ 164 $2312\frac{1}{2}$ 165 $4866\frac{3}{8}$ 166
 906 $\frac{1}{2}$ 167 $\$20\frac{1}{4}$ 168 $\$27\frac{1}{8}$ 169 $168\frac{3}{4}$ mi. 170 $\$12\frac{1}{3}$
 171 $20\frac{1}{4}$ 172 $18\frac{3}{8}$ 173 $23\frac{7}{16}$ 174 $34\frac{1}{15}$ 175 $1\frac{2}{25}$
 176 $\frac{3}{5}$ 177 2 178 $10\frac{1}{16}$ 179 $7\frac{7}{8}$ 180 $24\frac{1}{16}$ 181
 $1\frac{25}{28}$ 182 $14\frac{5}{8}$ 183 $\frac{2}{3}$ 184 $129\frac{2}{3}$ 185 $117\frac{1}{2}$ 186
 $213\frac{1}{8}$ 187 $135\frac{1}{8}$ 188 $1214\frac{7}{8}$ 189 180 190 $\$4112\frac{1}{2}$
 191 $\$5985$ 192 $\$4921\frac{7}{8}; \frac{7}{32}$ 193 $\frac{3}{2}$ 194 $\frac{7}{24}$ 195
 $\frac{3}{48}$ 196 $\frac{27}{10}$ 197 $1\frac{1}{8}$ 198 $3\frac{1}{4}$ 199 $17\frac{5}{12}$ 200 $\frac{2}{3}$
 201 3 202 $17\frac{1}{4}$ 203 $\$3\frac{1}{2}$ 204 $\frac{9}{16}, \frac{1}{16}$ 205 $\$5\frac{5}{8}$
 206 $2128\frac{1}{2}, 4751\frac{1}{4}, 387\frac{3}{4}$ 207 $547\frac{1}{2}$ 208 1356 209 903
 210 6150 211 $227\frac{1}{4}$ 212 $1816\frac{1}{4}$ 213 2170 214
 156 $\frac{1}{4}$ 215 $650\frac{1}{4}$ 216 $1290\frac{3}{20}$ 217 $412\frac{1}{2}$ 218 $687\frac{1}{2}$
 219 $2772\frac{9}{10}$ 220 $1251\frac{1}{4}$ 221 $2738\frac{3}{2}$ 222 $\$27\frac{3}{4}$
 223 $\$126$ 224 $\$55\frac{5}{8}$ 225 $\$92\frac{1}{16}$ 226 $\$314\frac{5}{12}$ 227

\$69.42 $\frac{21}{32}$ 228 \$106.78 $\frac{1}{8}$ 229 \$112 $\frac{17}{32}$ 230 \$369 $\frac{27}{32}$ 231
 $\frac{35}{128}$ 232 50 $\frac{5}{8}$ 233 46406 $\frac{1}{4}$ 234 5937 $\frac{1}{2}$ 235 189 $\frac{1}{16}$
 236 326 $\frac{81}{4}$ 237 64 $\frac{19}{16}$ 238 997 $\frac{19}{32}$ 239 \$9 $\frac{41}{56}$
 240 2860 $\frac{43}{192}$ 241 126 $\frac{41}{512}$ 242 10 $\frac{1}{2}$ 243 21 $\frac{1}{3}$ 244
 27 $\frac{3}{7}$ 245 43 $\frac{1}{5}$ 246 128 247 19 $\frac{5}{32}$ 248 4 $\frac{4}{5}$ 249
 50 250 4 $\frac{2}{3}$ 251 8 252 5 $\frac{5}{17}$ 253 16 254 45
 255 14 256 20 257 720 258 248 $\frac{8}{9}$ 259 126
 260 27 $\frac{3}{7}$ 261 10 $\frac{1}{2}$ 262 9 $\frac{3}{5}$ 263 91 $\frac{1}{15}$ 264
 $\frac{3}{28}$ 265 $\frac{7}{40}$ 266 $\frac{15}{128}$ 267 $\frac{31}{96}$ 268 $\frac{5}{76}$ 269 $\frac{11}{204}$
 270 $\frac{2}{25}$ 271 $\frac{8}{85}$ 272 $\frac{2}{195}$ 273 $\frac{5}{144}$ 274 1 $\frac{6}{3}$ 275
 2 $\frac{6}{17}$ 276 2 $\frac{1}{2}$ 277 4 $\frac{1}{16}$ 278 10 $\frac{1}{16}$ 279 \$7 $\frac{11}{27}$ 280
 \$1 $\frac{1}{4}$ 281 \$12 $\frac{1}{2}$ 282 5 $\frac{1}{3}$ 283 2 $\frac{6}{7}$ 284 $\frac{35}{49}$ 285
 1 $\frac{9}{32}$ 286 \$ $\frac{1}{16}$ 287 24¢ 288 $\frac{9}{96}$ 289 57¢. 290
 1 $\frac{3}{32}$ 291 1 $\frac{1}{9}$ 292 $\frac{2}{64}$ 293 2 294 $\frac{1}{2}$ 295 1 $\frac{1}{8}$ 296
 $\frac{9}{40}$ 297 $\frac{3}{4}$ 298 2 $\frac{1}{4}$ 299 2 $\frac{3}{8}$ 300 4 301 8 $\frac{3}{4}$ 302
 33 $\frac{1}{3}$ 303 20 304 10 $\frac{5}{8}$ 305 1 $\frac{1}{4}$ 306 1 $\frac{1}{4}$ 307 2
 308 3 $\frac{1}{3}$ 309 2 $\frac{1}{2}$ 310 3 311 1 $\frac{30}{80}$ 312 8 $\frac{2}{3}$ 313
 18 lb. 314 \$7 $\frac{1}{2}$ 315 \$ $\frac{2}{5}$ 316 14 $\frac{11}{16}$ mi. 317 3 $\frac{29}{97}$
 318 2 $\frac{9}{100}$ 319 3 $\frac{8}{9}$ 320 4 321 26 $\frac{37}{166}$ 322 42 $\frac{26}{403}$
 323 17 $\frac{15}{127}$ 324 10 $\frac{15}{112}$ 325 25 $\frac{8}{1731}$ 326 \$8 $\frac{1}{2}$
 327 15 328 3981 $\frac{1}{4}$ bu. 329 36 cd. 330 1 $\frac{7}{8}$ 331
 4 $\frac{2}{3}$ 332 6 $\frac{3}{76}$ 333 1 $\frac{3}{30}$ 334 2 $\frac{31}{8}$ 335 5 $\frac{15}{212}$ 336
 9 $\frac{37}{222}$ 337 $\frac{33}{3019}$ 338 2 $\frac{37}{144}$ 339 17 $\frac{7}{8}$ 340 38 $\frac{5}{48}$ 341
 142 $\frac{1}{54}$ 342 19 $\frac{1}{8}$ 343 67 $\frac{9}{88}$ 344 862 $\frac{1}{5}$ mi. 345 28 $\frac{13}{16}$
 346 168 $\frac{3}{4}$ 347 91 $\frac{45}{46}$ 348 110 $\frac{5}{8}$ 349 20 $\frac{5}{6}$ 350 882 $\frac{1}{3}$
 351 8 $\frac{1}{2}$ da. 352 201 $\frac{3}{8}$ cu. in. 353 \$206 $\frac{1}{4}$ 354 $\frac{13}{16}$
 355 \$79 $\frac{9}{16}$ 356 17 $\frac{1}{3}$ cd. 357 \$312 $\frac{1}{2}$ 358 \$3000; \$4500
 359 \$57600; R. E. \$19200; B \$28800; S \$7200 360 \$8 loss
 361 \$32.06 $\frac{1}{4}$ 362 \$123 89 $\frac{55}{4}$ 363 \$2399 $\frac{5}{8}$ 364 \$2460 $\frac{1}{8}$
 365 \$20 366 B \$5142 $\frac{6}{7}$; C \$2057 $\frac{1}{7}$ 367 \$949 $\frac{1}{96}$ 368
 \$15 $\frac{5}{8}$ 369 \$261 $\frac{9}{16}$ 370 41 $\frac{35}{132}$ cd. 371 156 $\frac{3}{8}$ bu. 372
 40 $\frac{5}{8}$ lb. 373 100 $\frac{3}{52}$ T. 374 21 $\frac{1}{4}$ yd. 375 80 da. 376
 \$50 $\frac{13}{16}$ 377 \$10.72 $\frac{13}{32}$ 378 $\frac{2}{3}$ 379 $\frac{5}{72}$ 380 2 $\frac{26}{47}$ da.
 381 15 da. 382 $\frac{10}{17}$ da. 383 \$399 384 6 $\frac{23}{8}$ lb. 385
 \$21 $\frac{1}{16}$ 386 19 $\frac{1}{2}$ T. 387 \$17 $\frac{3}{8}$ 388 \$22400 389
 \$60000 390 \$23.93 $\frac{3}{8}$ 391 \$112.06 $\frac{5}{4}$ 392 24 $\frac{1}{4}$ mi. 393
 34 $\frac{2}{7}$ ft.; 45 $\frac{5}{7}$ ft.; 40 ft. 394 10 $\frac{1}{4}$ cd. 395 356 $\frac{5}{8}$ A. 396
 \$193.50 397 387 $\frac{10}{788}$ lb. 398 \$10000

DECIMALS—Page 55.

1 .875 2 .7 3 .25 4 .125 5 .75 6 .4375 7 .46875
 8 .84375 9 .390625 10 .348 11 .0625 12 .09375 13
 .078125 14 .0234375 15 .1625 16 .96 17 .01125 18
 .096 19 2.171875 20 .0009765625 21 14.75 22 5.875
 23 9.3125 24 7.09375 25 4.6875 26 3.140625 27
 21.0234375 28 5.064 29 4.0028 30 1.4007421875 31
 .3333+ 32 .2222+ 33 .4286- 34 .6667- 35 .7143-
 36 .6364- 37 .6154- 38 .0588+ 39 .1286- 40
 12.8333+ 41 8.3889- 42 9.0526+ 43 $\frac{1}{8}$ 44 $\frac{5}{25}$ 45
 $\frac{3}{4}$ 46 $\frac{5}{8}$ 47 $\frac{3}{4}$ 48 $\frac{3}{8}$ 49 $\frac{9}{16}$ 50 $\frac{1}{16}$ 51 $\frac{1}{16}$ 52 $\frac{1}{16}$
 53 $\frac{1}{80}$ 54 $\frac{1}{16}$ 55 $\frac{5}{16}$ 56 $\frac{1}{16}$ 57 $\frac{189}{400}$ 58 $\frac{1}{16}$ 59
 $\frac{333}{400}$ 60 $\frac{1}{320}$ 61 $\frac{9}{1600}$ 62 $18\frac{3}{4}$ 63 $9\frac{5}{16}$ 64 $31\frac{3}{400}$ 65
 $4\frac{3}{40}$ 66 $4\frac{37}{400}$ 67 $3\frac{3}{4}$ 68 $8\frac{3}{16}$ 69 $14\frac{1}{16}$ 70 $4\frac{1}{320}$ 71
 $\frac{2}{15}$ 72 $\frac{1}{15}$ 73 $\frac{5}{8}$ 74 $\frac{3}{8}$ 75 $12\frac{89}{100}$ 76 $6\frac{1}{8}$ 77 $14\frac{7}{80}$
 78 $19\frac{1}{7}$ 79 $4\frac{2}{9}$ 80 $34\frac{2}{11}$ 81 $\frac{1}{16}$ 82 $\frac{2}{3}$ 83 26 03825
 84 120.03754 85 340.912962 86 228 845495 87
 1737.92777625 88 69.21875 89 \$58.875 90 4.71626 91
 4.4375 92 $3\frac{863}{10000}$ 93 217.53125 A. 94 627.176875 mi.
 95 2119.210272365 96 95.928125 97 2.81204 98 17.296
 99 3.0895 100 31.041975 101 .024975 102 \$11.127
 103 .099999 104 12.2925hr. 105 2.5089375rd. 106 7.45
 mi. 107 10.35417- 108 19 6944-fur. 109 13.899898
 110 35.03125 yd. 111 970.142413 112 80.55623 113
 .216 114 .004796282 115 .08775; .01605 116 85.75;
 222.3 117 25.7925; 12.155 118 2401; 17.25 119 .9375
 120 .00525 121 824.4140625 122 155.25 123 .0004
 124 675 125 .0105105 126 55.3125 127 $\frac{3}{8}$ 128 $\frac{1}{16}$
 129 $\frac{3}{160}$ 130 $\frac{1}{2}$ 131 $\frac{1}{7}$ 132 $\frac{1}{20}$ 133 405.050625
 134 1 135 \$138.796875 136 \$111.78+ 137 \$129.75
 138 40 bu. 139 \$26.36 $\frac{3}{4}$ 140 \$13478.75 141
 13.3748475 142 Solved. 143 .0225 144 .0004928 145
 .0027847+ 146 .0110792 147 3.375 148 .4928. 149
 6.104 150 .38 151 1500 152 126 153 235750
 154 127000 155 483.3333+ ft. 156 1.169- mi. 157
 .0553+ yd. 158 2.80247- cd. 159 \$1.1076- 160
 .021- cu. ft. 161 .0278- mo. 162 .1121-qt. 163
 1.71+ bu. 164 1.6667- da. 165 4.051927+ mi. 166

2,549— rd. 167 .13 168 378.5 bu. 169 1848 bu.
 170 .001; 1000 171 1111.2211 172 3300606.333003
 173 40124804.08 174 1490210.82803 175 27780 0277775
 176 2573.37335+ 177 5145.955106 178 42261.727039
 179 .875; .5625; .09375; .109375 180 $\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{32}$; $\frac{1}{64}$; $\frac{1}{128}$
 181 .999999 182 40.6092 183 900; 9000; 900000 184
 \$6120 185 \$.40 186 \$193.96+ 187 \$11.13 188 \$100
 189 \$877.50 190 \$2500; \$2812.50 191 \$7500; \$4000;
 \$5000 192 \$6.37 $\frac{1}{2}$ 193 \$2700 194 \$3000 195
 \$3600, A; \$3200, B 196 \$7678.50, Wife; \$5119, Daughter.

UNITED STATES MONEY—Page 71.

1 32640 mills 2 \$156.84 3 \$684.26 4 \$23.05 5
 \$1019.02 6 \$125.64 7 \$946.10 8 \$1091.12 9 \$640 10
 \$212.50 11 42 cd. 12 \$19.08 $\frac{1}{3}$ 13 \$216.26 14 96 cd.
 15 \$486 16 \$.28 17 \$10.94 18 \$5.64 19 \$1.59 20
 \$2.06 21 \$130.63 22 \$104.22 23 \$306.25 24 \$93.66
 25 \$6.06 26 \$33.75 27 \$3.13 28 \$150 29 \$84.30 30
 \$525 31 \$13.50 32 \$35.70 33 \$5.25 34 \$224 35
 \$154.50 36 \$135 37 \$2583.33 38 \$558.75 39 35 T
 40 1260 bu. 41 145 bu. 42 25 da. 43 425 lb. 44 \$75
 45 \$1.77 46 \$6 47 \$6.50 48 25¢; 50¢. 49 \$211.90
 50 \$1593.75 51 \$134.90 52 \$984.16 53 \$692.72 54
 \$43.10 55 \$146.70 56 \$156 57 \$22.87 58 \$33 53
 59 \$30.56 60 \$253.18 61 \$1684.30 62 \$19 30 63
 64 $\frac{1}{2}$ T. 64 \$201.03 65 \$29.33 66 \$54.93 67 \$2310.25
 68 \$334 69 \$64.50 70 \$73.09 71 \$10.15— 72 5 $\frac{1}{2}$ yd.
 73 40 lb. 74 \$2.62 $\frac{1}{2}$ 75 \$560 18 76 \$226.62— 77
 \$2462.65 78 \$3012.30 79 \$793.30 80 Solved. 81
 \$2369.80 82 \$10105.55 83 \$96.16 84 \$1881.55 85
 \$23.94 86 \$665.72 87 \$4594.06 88 \$1416.58 89
 \$408.49.

DENOMINATE NUMBERS—Page 83.

1 11895 far. 2 8931 ms. 3 7529 pf. 4 £4 8s. 3d. 1 far.
 5 12 fr. 3 dc. 2 ct. 4 ms. 6 \$44.62 7 \$9.84 8 \$17 95 9
 11169 far. 10 19.54 marks 11 325044 in. 12 44694148
 sq. in. 13 75420 sq. l. 14 35487 cu. in. 15 1 mi. 309 rd.
 2 yd. 1 ft. 9 in. 16 9 A. 131 sq rd. 17 sq. yd. 5 sq. ft. 36 sq.

in. 17 92 cu. yd. 18 cu. ft. 1707 cu. in. 18 1 Tp. 14 sec.
 514 A. 6 sq. ch. 19 97 cd. 2 cd. ft. 12 cu. ft. 20 129.375
 mi. 21 4 hhd. 1 bbl. 17 gal. 2 qt. 22 131 bu. 2 pk. 7 qt. 1 pt.
 23 792 gal. 3 qt. 24 100 bu. 4 qt. 25 2925 pt. 26 502
 gi. 27 32030 gr. 28 86596 gr. 29 177440 oz. 30 46368
 gr. 31 51880 gr. 32 64800 oz. 33 4 lb. 2 oz. 6 dr. 34 7
 T. 11 cwt. 97 lb. 8 oz. 35 17 lb. 9 oz. 4 pwt. 36 559820 sec.
 37 73220" 38 3860 sheets 39 799 units 40 8784 hr.
 41 2678400 sec. 42 8 yr. 111 da. 10 hr. 24 min. 43 8 S.
 22° 52' 22" 44 18 hr. 45 186 rd. 3 yd. 2 ft. 46 17 da. 3
 hr. 25 min. 42 $\frac{6}{7}$ sec. 47 101 sq. rd. 24 sq. yd. 6 sq. ft. 108 sq.
 in. 48 11 cu. ft. 432 cu. in. 49 3 qt. 1 pt. $\frac{4}{5}$ gi. 50 $\frac{1}{2}$ pt.
 51 $1\frac{3}{8}$ in. 52 200 rd. 53 14 cwt. 54 lb. 54 39 gal. 1 qt.
 1 pt. 55 3 cd. ft. 6.08 cu. ft. 56 2 qt. 1.2 gi. 57 $5\frac{5}{7}$ 13 $8\frac{4}{7}$
 gr. 58 $\frac{5}{12}$ bu. 59 .00375 bu. 60 $\frac{1}{1728}$ sq. yd. 61 £.0375
 62 $\frac{29}{100}$ A. 63 .1125 T. 64 $\frac{3}{128}$ gal. 65 $\frac{3}{8}$ bu. 66
 $7\frac{1}{12}$ lb. 67 $1\frac{5}{8}$ 68 .0128+ 69 $\frac{1}{16}$ s. 70 .6962—lb. 71
 $\frac{187}{2880}$ mi. 72 $\frac{889}{9801}$ sq. rd. 73 228 lb. 4 oz. 10 pwt. 9 gr.
 74 23 bbl. 27 gal. 2 qt. 2 gi. 75 142 A. 58 sq. rd. 19 sq. yd.
 3 sq. ft. 34 sq. in. 76 15 ft. $8\frac{1}{2}$ in. 77 122 rd. 1 ft. $10\frac{7}{8}$ in.
 78 6 gal. 3 qt. .875 pt. 79 .5 gal. 1 qt. 1 pt. 2 gi. 80 23
 yr. 349 da. 22 hr. 54 min. 12 sec. 81 88 mi. 179 rd. 1 yd. 10
 in. 82 125 A. 8 P. 83 12s. 5d. 2 far. 84 112 rd. 2 ft. $\frac{3}{4}$
 in. 85 0 86 296 bu. 3 pk. 2 qt. 87 36 da. 22 hr. 3 min.
 12 sec. 88 45 A. 77 P. 26 sq. yd. 6 sq. ft. 108 sq. in. 89
 \$519.12 90 \$146.55 91 \$678.68 92 72 rd. 1 yd. $6\frac{1}{5}$ in.
 93 1 bu. 2 pk. 3 qt. $1\frac{1}{20}$ pt. 94 292 da. 12 hr. 2 min. 1 sec.
 95 1 bbl. 17 gal. $1\frac{3}{5}$ pt. 96 8 cwt. 57 lb. $3\frac{1}{5}$ oz. 97 W.
 426 A. 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in.; C. 213 A. 53 sq.
 rd. 10 sq. yd. 108 sq. in. 98 10 rings 99 $22\frac{1}{2}$ bu. 100 \$4
 101 40¢ 102 \$7.50 103 $\frac{1}{2}$ ¢ 104 $47\frac{1}{7}$ lb.

PRACTICAL MEASUREMENTS—Page 102.

1 432 sq. ft. 2 $170\frac{2}{3}$ sq. yd. 3 \$128 4 864 sq. ft.; 1440
 sq. ft.; 960 sq. ft.; $362\frac{2}{3}$ sq. yd.; \$68.60 5 \$45.51 6 96 sq. ft.
 7 6400 sq. rd. 8 24 A. 9 \$1875 10 \$210.60 11 7200
 12 \$82.13 13 \$51.37 14 \$202.67 15 48 yd.; 96 yd.; 72
 yd.; $57\frac{3}{8}$ yd. 16 $73\frac{1}{8}$ yd.; 144 yd.; 24 yd. 17 \$90; \$120; $93\frac{1}{2}$

18 5120 sq. rd. 19 60 A. 20 23040 A. 21 960 A. 22
 400 A. 23 \$40.20 24 \$12245 25 200 A. 26 520 A.
 27 \$16000; \$2680 28 \$6000 29 572 $\frac{2}{5}$ A. 30 192 cu. ft.
 31 175 cu. ft. 32 3840 cu. ft. 33 \$6600 34 8 $\frac{1}{2}$ cu. yd.
 35 2 hr. 19 min. 13 $\frac{1}{2}$ sec. 36 \$1368.64 37 960 cu. ft. 38
 98 cd. ft. 39 45 cd. 40 5 cd. 41 36 $\frac{9}{16}$ cd. 42 \$46.41
 43 480 ft. 44 \$1320 45 960 ft. 46 14 ft. 47 37 $\frac{1}{2}$ ft.
 48 600 ft. 49 \$36.17 50 \$691.20 51 \$293.83 52
 \$41 53 \$240 54 \$1325.74 55 2144 ft. 56 8 in.
 57 3200 58 2845 59 3902 Mil. 9856 Com. 60 960 cu.
 ft. 61 \$243 62 72 $\frac{8}{11}$ p. 63 14 $\frac{1}{2}$ $\frac{8}{5}$ cd. 64 \$80.81 65
 \$109.09 66 \$442.91 67 \$435.05 68 64 bu. 69 771.43
 bu. 70 301.87 bu. 71 5303.6—lb. 72 5—ft 73 134
 gal. 74 1077.19+gal. 75 598 $\frac{3}{4}$ gal. 76 2 ft. 11 $\frac{1}{4}$ in.
 77 91.19+hhd. 78 232 $\frac{8}{11}$ gal. 79 698 $\frac{2}{11}$ gal. 80
 59 08+ bu. 81 297+ gal. 82 $\frac{7}{9}$ 1 $\frac{6}{5}$ $\frac{8}{5}$ 83 45¢ 84 \$9.62
 85 5 yr. 5 mo. 3 da. 86 3—3—19 87 3—1—20 88 3—
 2—12 89 2—3—3 90 3—10—3 91 3—10—16 92 4—
 8—9 93 4—10—16 94 4—5—23 95 2 yr. 3 mo. 20 da.
 2 hr. 16 min. 96 2 yr. 8 mo. 19 da. 11 hr. 52 min. 97 4
 yr. 2 mo. 10 da. 10 hr. 33 min. 98 211 da. 99 241 da.
 100 288 da. 101 262 da. 102 232 da. 103 149° 54'
 15" 104 84° 27' 15" 105 175° 56' 47" 106 5° 45'
 44" 107 48° 23' 45" 108 164° 39' 15" 109 5 hr. 8
 min. 1 sec. 110 2 19 16 111 1 4 49 112 9 35
 50 113 11 43 47 $\frac{2}{15}$ 114 6 31 33 $\frac{1}{15}$ 115 10 hr. 37
 min. 30 $\frac{2}{5}$ A. M. 116 10 hr. 58 min. 37 sec. A. M. 117 5 hr.
 28 min. 26 $\frac{1}{5}$ sec. P. M. 118 12 hr. 58 min. 42 $\frac{1}{5}$ sec. P. M.
 119 9 hr. 40 min. 44 sec. A. M. 120 10 hr. 55 min. 11 sec.
 A. M. 121 5 hr. 58 min. 24 sec. A. M. 122 8 hr. 46 min.
 25 sec. A. M.

INVOLUTION—Page 118.

1 625 2 421875—614125—857375—274625—166375 3
 256—4096—65536—50625 4 $\frac{27}{64}$ — $\frac{64}{125}$ — $\frac{125}{216}$ — $\frac{216}{343}$ 5 125—
 36—4096—100000—15625 6 $\frac{343}{512}$ — $\frac{625}{1296}$ —15.625—204 $\frac{4}{15}$ —
 129.746337890625 7 14025

EVOLUTION—Page 118.

1 35 2 45 3 55 4 65 5 75 6 85 7 123 8 321
 9 231 10 132 11 312 12 213 13 $\frac{1}{2}$ 14 $\frac{1}{7}$ 15
 .35 16 .085

MENSURATION—Page 120.

1 96 A. 2 $258\frac{1}{2}$ sq. ft. 3 $5\frac{1}{4}$ A. 4 32 rd. 5 103680 A.
 6 240 sq. ft. 7 14256 sq. ft. 8 $337\frac{1}{2}$ sq. ft. 9 9 A. 10
 15 ft. 11 $6\frac{1}{4}$ rd. 12 640 rd. 13 64 rd. A; 40 rd. B 14
 50 ft. 15 79.05+ ft. 16 30 ft. 17 54.54+ rd. 18
 40 $3\frac{1}{2}$ + rd. 19 93.67+ rd. 20 60 A. 21 100 rd. 22 20 ft.
 23 17.57+ mi. 24 Solved. 25 489.89+ sq. rd. 26
 \$400.74 27 1558.84+ sq. ft. 28 6.29+ A. 29 1200 sq. ft.
 30 2417.5 sq. ft. 31 31.83+ ft. 32 57.29+ rd. 33
 101.85+ rd. 34 314.16 ft. 35 18.85- ft. 36 157.08+ rd.
 37 125.66+ ft. 38 Solved. 39 1017.87 sq. ft. 40
 23 27+ sq. yd. 41 490.875 sq. yd. 42 4417.9 sq. ft. 43
 4973.58+ sq. ft. 44 .11+ A. 45 17.67+ ft. 46 84.85+ ft.
 47 135.04+ ft. 48 10 A. 49 Solved. 50 600 sq. ft.;
 635 80+ sq. ft. 51 576 sq. ft.; 624 sq. ft. 52 127.23+ sq.
 ft. 53 120 sq. ft. 54 216 cu. ft. 55 144 cu. ft. 56
 83 04+ cu. ft. 57 141.37+ cu. ft. 58 4080 gal. 59
 375 sq. ft.; 446.62+ sq. ft. 60 36 sq. ft.; 37 sq. ft. 61
 \$116.67 62 478.93+ sq. ft. 63 67.02+ cu. ft. 64 168
 cu. ft. 65 26.92+ bu. 66 78.54 sq. in. 67 \$33 68
 904.78+ cu. ft. 69 29.18+ cu. ft. 70 3916.8 gal. 71 1683
 bu. 72 28.84+ ft. 73 167.7+ ft. 74 \$260.42 75 \$22 22
 76 83.3+ ft. 77 325 men 78 639.66+ in. 79 50.39+
 ft. 80 30.74+ ft. 81 452.54+ rd. 82 50.8+ ft. 83
 \$235.62 84 72 ft. 85 96 ft. 86 \$43.3+ ft.; C 48.86+
 ft. diameter 87 5.51; 7.17; 17.32 in. 88 32 rd. 89 14.19
 A. 90 47.12+ ft. 91 159.6+ cu. ft. 92 402.12+ cu. ft.;
 100.53+ cu. ft. 93 268.08+ cu. ft. 94 489.89+ ft. 95
 60 A. 97.5 sq. rd. 96 3774.375 ft.

PROPORTION—Page 137.

1 72 2 96 3 108 4 40 5 $16\frac{5}{8}$ 6 \$6 7 $3\frac{1}{8}$ 8 $\frac{9}{16}$
 9 $4\frac{1}{2}$ 10 54 11 17 12 \$23 $\frac{1}{3}$ 13 Solved. 14 Solved.

15 \$43 $\frac{1}{4}$ 16 66 $\frac{3}{4}$ yd. 17 720 bu. 18 107.42 bu. 19
\$1.20 20 \$600 21 \$1894 $\frac{2}{7}$; \$1605 $\frac{1}{7}$ 22 3 $\frac{5}{8}$ ft. 23 200
rd. 24 30 men 25 \$4 $\frac{3}{8}$ 26 3347 bricks 27 1320 lb.
28 20 men.

PERCENTAGE—Page 139.

1 134.40 2 61.95 lb. 3 362.88; 404.04 4 \$92; \$144 5 90
bu.; 891 bu. 6 1098 bu.; 1782 bu. 7 1620 lb.; 3100 lb. 8
\$9364.50; \$8154 9 20c; 30c; 35c. 10 $\frac{1}{200}$ 11 \$4320 R E.;
\$6480 bank stock; \$6480 city lot 12 \$225 13 \$3562 50
14 \$11760 15 \$3784 16 \$70 17 \$1890 18 \$4500
19 478 bu. 20 \$1760 21 \$10500 22 33 $\frac{1}{3}$ % 23 80%;
11 $\frac{1}{2}$ % 24 66 $\frac{3}{8}$ %; 37 $\frac{1}{2}$ % 25 60%; 33 $\frac{1}{3}$ % 26 25%; 11 $\frac{1}{5}$ %
27 50% 28 14 $\frac{3}{4}$ % 29 18% 30 12 $\frac{1}{2}$ % 31 200%;
166 $\frac{3}{8}$ %; 33 $\frac{1}{3}$ % 32 20% G; 32% R; 48% A 33 33 $\frac{1}{3}$ % 1st;
25% 2d; 41 $\frac{3}{8}$ % 3d 34 250 bbl.; 50% 35 20% 36 37 $\frac{1}{2}$ %;
204 bbl. 37 40%; 60%; 66 $\frac{3}{8}$ % A's; 150% B's 38 25% 39
187 $\frac{1}{2}$ %; 53 $\frac{1}{3}$ % 40 25% 41 16 $\frac{3}{8}$ %; 33 $\frac{1}{3}$ %; 50% 42
1500 43 300 44 22; 13.75 45 $\frac{5}{8}$; 1 $\frac{7}{8}$ 46 120 47 64
48 400 49 300 50 300 51 100 52 48 53 \$2100
54 \$2918.25 55 \$6000 56 \$3325 57 \$4200; \$5208 58
\$33000 59 \$4000; \$4800 60 \$12000; \$20000; \$40000 61
\$47200 62 \$10800 63 \$12000 64 1.76 65 2.52
66 .1875 67 .005 68 3 $\frac{1}{8}$ % 69 111 $\frac{1}{8}$ % 70 28 $\frac{1}{4}$ % 71
104 $\frac{1}{8}$ % 72 1062 $\frac{1}{2}$ % 73 1500 74 4000 75 78.12 $\frac{1}{2}$ 76
80 77 3240 78 .135 79 80 80 .00036 81 8 $\frac{1}{8}$ %
82 116 $\frac{3}{8}$ % 83 60 84 4 $\frac{1}{5}$ % 85 79 $\frac{1}{6}$ % 86 7 $\frac{1}{2}$ % 87
37 $\frac{1}{2}$ %; 240 A. 88 \$1485 89 \$16416 90 20% 91 120%
92 80% 93 \$28000 94 \$30000 95 80% 96 \$4800
97 \$2400; \$4000 98 \$15000; \$7500 99 \$7500 A; \$9000 C
100 \$30 M.; \$10 T.; \$18 Mk. 101 \$750 102 \$400 A;
\$500 B; \$600 C; \$1440 D 103 1980 A; 900 P. 104
\$1600 1st; \$1920 2d; \$2112 3d 105 66 $\frac{3}{8}$ % A's; 150% B's
106 \$7500; 33 $\frac{1}{3}$ % 107 \$1500 B; \$600 C; 108 \$11375 O;
\$16250 Y 109 \$11085 A's; \$7390 B's 110 \$9000 111
\$45000 112 \$450 H.; \$150 S. 113 \$300 A 114
\$546.59 115 \$9000 116 \$50203.13 117 \$7893 118
\$9048

PROFIT AND LOSS—Page 157.

1 \$172.80 2 \$140 3 \$1 4 \$40 5 \$90 6 \$12 7 \$.12½
 8 \$.10½ 9 \$165 10 \$1 11 \$88 12 \$5.60 13 \$220
 14 \$57 15 \$.76 16 \$106.20 17 12½% gain 18 11¼%
 loss 19 7½% gain 20 6¾% loss 21 \$608; \$684 22
 \$399; \$285 23 \$252; \$336 24 \$2900; \$2755 25 \$1200 26
 \$3600 27 \$90 28 \$1263.36 29 \$225 30 \$96 31 \$25
 G; 20% 32 \$25 L; 9½% 33 \$20 G; 7½% 34 \$120 L
 25% 35 \$300 S; 7⅓% L 36 \$180 S; 25% L 37 \$360.25
 S; 10% G 38 \$346.08 S; 5% G 39 \$1500 C; \$1425 S 40
 \$2000 C; \$1875 S 41 \$4100 C; \$4428 S 42 \$726 C; \$847 S
 43 \$4 C; \$.66⅔ S 44 \$216¼ C; \$129⅓ S 45 \$150 C; \$30 G
 46 \$912 C; \$228 G 47 \$750 C; \$330 G 48 \$9.09⅙ C;
 \$.75⅔ L 49 \$8224.29 C; \$1370.71 G 50 \$700 C; 7½% L
 51 \$2560 C; 15% G 52 \$4000 C; 20% L 53 \$2000 C;
 28% L 54 \$600 C; 46⅓% G 55 20% 56 \$450 57 11½%
 58 37½%; 72 sq. rd. 59 205% 60 50% 61 \$11458.33
 62 \$33250 63 16⅘% 64 14⅙% 65 \$800 loss
 66 Even 67 17⅓% 68 5⅝¢ 69 33⅓% 70 4⅙%
 A 71 \$1.65 72 8⅙% 73 95 yd. 74 13.4% 75
 6¼% L 76 \$5696 77 \$7.33⅓ 78 \$1381.94; \$1796.52 79
 \$10800 80 25% 81 \$112.50 82 37½% 83 45⅝% 84
 11⅙% gain 85 20% 86 \$40 87 \$4888.89 88 \$2187.50
 89 17⅔%; 21⅓% 90 Tea, 56¼ lb.; Coffee, 117⅓ lb.; Tailor,
 \$8.44 91 2¼% loss 92 \$2000 93 \$4.37½ gain 94 \$3000
 each 95 \$17600

COMMISSION AND BROKERAGE—Page 167.

1 \$28.80 2 \$78.13 3 \$7.81 4 \$20.08 5 \$720 6 \$835.92
 7 \$216.86 8 \$895.56 9 \$1000 10 \$300 11 \$1200 12
 \$1600 13 3% 14 4% 15 1% 16 2⅔% 17 3% 18
 2½% 19 ½% 20 2% 21 2% 22 \$378 23 \$109.50
 24 \$640.34 25 \$325 26 \$1955 27 \$13977.50 28
 \$400 29 \$2450 30 \$4000 31 \$50 Com.; \$2450 Pro. 32
 \$37.46 Com.; \$899.04 Pro. 33 \$14.61 Com.; \$545.29 Pro. 34
 \$27.82 Com.; \$864.36 Pro. 35 \$3110 S.; \$3016.70 Pro. 36
 \$6000 S.; \$5850 Pro. 37 \$1200 S.; \$1149.40 Pro. 38 \$2110
 S.; \$2009.40 Pro. 39 \$625 S.; 2% 40 \$600 S.; 3% 41

\$500 S; 3% 42 \$820 S; 2% 43 $\frac{1}{2}\%$; \$2388 Pro. 44 $\frac{1}{2}\%$
 \$1293.50 45 5%; \$428.65 Pro. 46 $2\frac{1}{2}\%$; \$353.06 Pro. 47
 $2\frac{1}{2}\%$; \$18.90 Com. 48 $1\frac{1}{4}\%$; \$5.45 Com. 49 $2\frac{1}{2}\%$; \$7.10 Com.
 50 3%; \$24.75 Com. 51 2%; \$9.60 Com. 52 1%; \$5.75 Com.
 53 $\frac{3}{4}\%$; \$4.68 Com. 54 3%; \$9.75 Com. 55 \$6.49 Com.;
 \$330.99 G. C. 56 \$7.51 Com.; \$195.26 G. C. 57 \$14.78
 Com.; \$322.28 G. C. 58 \$8.11 Com.; \$336.88 G. C. 59 $2\frac{1}{2}\%$;
 \$448.95 G. C. 60 2%; \$369.75 61 5%; \$589.23 G. C. 62
 $2\frac{1}{2}\%$; \$262.41 G. C. 63 \$10 Com.; \$500 P. C. 64 \$9 Com.;
 \$600 P. C. 65 \$16.23 Com.; \$324 60 66 \$6.96 Com.; \$278.40
 67 3%; \$375 P. C. 68 $2\frac{1}{2}\%$; \$382 P. C. 69 4%; \$263 P. C.
 70 $2\frac{1}{2}\%$; \$336 P. C. 71 \$37.50 72 2% 73 \$3825 74
 \$1200 75 \$1650 76 $2\frac{1}{2}\%$ 77 \$9700 78 \$1250 79
 \$700 80 20005+lb. 81 50% 82 \$197.38 83 534 bbl.;
 \$1.42 84 \$227.78 85 \$32.16 86 \$4080 87 \$15468.75;
 \$14437.50 88 41; \$235; \$965 89 \$1732.75 90 \$1060.97

MARKING GOODS—Page 176.

1 $\frac{D. oy}{O. we}$	2 $\frac{N. ty}{E. oe}$	3 $\frac{.be}{.ab}$	4 $\frac{.ne}{.ho}$	5 $\frac{D. by}{O. bp}$	6 $\frac{D. tx}{N. sa}$	7 $\frac{.ne}{.ba}$
8 $\frac{N. by}{W. ae}$	9 $\frac{H. ms}{A. xm}$	10 $\frac{A. bs}{M. ab}$	11 $\frac{U. xe}{M. dx}$	12 $\frac{H. sa}{R. sp}$		

TRADE DISCOUNT—Page 177.

1 19% 2 28% 3 $14\frac{1}{2}\%$ 4 40% 5 40% 6 75% 7
 25% 8 $62\frac{1}{2}\%$ 9 $78\frac{1}{8}\%$ 10 $50\frac{1}{2}\%$ 11 46% 12 55%
 13 $42\frac{2}{3}\%$ 14 62% 15 $49\frac{2}{3}\%$ 16 $29\frac{1}{2}\%$ 17 $27\frac{1}{6}\%$
 18 $16\frac{2}{10}\%$ 19 $25\frac{9}{10}\%$ 20 $37\frac{1}{2}\%$ 21 \$506.25 22
 \$194.40 23 \$300 24 \$400 25 \$143.43 26 \$188.03 27
 \$336.80 28 \$428.40 29 \$192.25 30 \$106.70 31 \$136.80
 32 \$14.20 33 \$34.02 34 \$155.52 35 \$36.20 36
 \$128 30 37 \$319 34 38 \$219.90 39 \$118.12 40 \$243.72
 41 \$500 42 \$700 43 \$1262.25 44 \$1300 45 \$241
 46 \$247 47 \$145.60 48 \$124.25 49 \$124 50 \$135
 51 \$136.50 52 \$260.20 53 \$196.30 54 \$395.43 55
 \$600 56 \$250 57 \$190 58 \$397.66 59 \$225 60 \$550
 61 \$512; \$128 62 \$192; \$128 63 \$160; \$80 64 \$72; \$48
 65 \$133.92; \$114.08 66 \$162; \$198 67 \$233.92; \$46.68
 68 \$300; \$60 69 \$400; \$112 70 \$800; \$320 71 \$2500;
 \$475 72 \$540; $16\frac{2}{3}\%$ 73 \$400; 10% 74 \$486; $33\frac{1}{3}\%$ 75

\$325; 30% 76 \$800; \$612 77 \$278; \$237.69 78 \$324;
 \$116.64 79 \$600; \$437.40 80 20%; \$387.20 81 30%;
 \$262.50 82 15%; \$70.80 83 33½%; \$174 84 5%; \$437
 85 40%; \$384 86 \$76 87 \$683.48 88 \$946.17 89
 \$456.30 90 \$139.50 91 \$835.13 92 \$1213.29; \$1226.19;
 \$1239.10 93 \$4000 94 20% 95 11% 96 50% 97 \$480;
 \$45 98 20% 99 \$200 100 21½% 101 20% 102 17½¢
 103 \$247.86 104 \$36 105 \$1203.07; \$1191.04 106
 \$2647.30; \$2514.93; \$2594.35 107 \$897.30 108 \$197.86
 109 \$355.69; \$341.46 110 \$223.24; \$218.93 111 24.2+ %
 112 \$1114.58 113 \$4810.31; \$4249.91

INSURANCE, FIRE—Page 188.

1 \$126 2 \$75.60 3 \$3558.10; \$81.90 4 \$57.38 5 \$175
 6 \$46915 7 \$6547.32 8 \$3929.45 9 \$5000 10 \$76 50
 11 \$5273 28 12 \$2180; \$3820 13 \$157.50 14 \$3450;
 \$2875; \$4312½; \$5750; \$2300 15 \$2133.33; \$1280; \$2986.67
 16 \$319.17

INSURANCE, LIFE—Page 192.

1 \$26.40; \$102 51 2 \$1177.80 3 \$9288 4 \$9571.20 5
 \$5942.40 6 \$3855.20 7 \$4636.40

TAXES—Page 196.

1 \$40 2 \$28.13 3 \$62.50 4 \$41.60 5 \$352 6
 \$16835 50 7 \$19462.50 8 \$58024.50 9 \$15229 10 (.0484
 rate); \$250031.97 levy; \$518.24 A's 11 ½% 12 7½% 13
 1% 14 1⅞% 15 2¼% 16 2½% 17 1.63+ % 18
 .022 rate; \$565.50 19 \$123.08

INTEREST—Page 198.

1 \$648 2 \$384 3 \$283 50 4 \$42.12 5 \$67.20 6
 \$134.61 7 \$118.47 8 \$15.83 9 \$12 40 10 \$15.82 11
 \$28.42 12 \$9.52 13 \$1.97 14 \$30 10 15 \$107.91 16
 \$321.30; \$481.95 17 \$194 24; \$164 36 18 \$3765.33; \$4048
 19 \$4514.20; \$4775.40 20 \$1421.63 21 \$156 22 \$187.50
 23 \$75 24 \$88.40 25 \$7.83 26 \$33.42 27 \$6.12
 28 \$10.14 29 \$32.11 30 \$13.44 31 \$12.88 32 \$19.66
 33 \$263.34 34 \$165.71 35 \$6735 36 36 \$160.53 37

\$282	38	\$160.23	39	\$23.07	40	\$17.08	41	\$40.71	42
\$348.29	43	\$74.05	44	\$258.84	45	\$296.25	46	\$8.82	
47	\$12	48	\$5.12	49	\$25.83	50	\$35.13	51	\$11.20
52	\$23.21	53	\$55.20	54	\$7.88	55	\$1.44	56	\$11.68
57	\$6.44	58	\$14.73	59	\$16.31	60	\$73.03	61	\$454.88;
\$455.69	62	\$865.97;	\$867.76	63	\$323.73;	\$327.47	64		
\$1303.79;	\$1298.94	65	\$514.50;	\$507.30	66	\$1597.27;			
\$1642.94	67	\$2648.78;	\$2758.30	68	\$8594;	\$8702	69		
\$7247.03;	\$7546.18	70	\$471.66;	\$477.49	71	\$972.60;	\$978.52		
72	\$913.46;	\$919.14	73	\$2687.85;	\$2719.75	74	\$1291.77;		
\$1299.50	75	\$4725.45;	\$4841.56	76	\$827.20	77	\$2541.60;		
\$2636	78	\$722.55;	\$743.28	79	\$664.36;	\$672.62	80		
\$1097.84;	\$1122.13	81	\$1434.30;	\$1440.02	82	\$595.50;			
\$585.36	83	\$871.96;	\$877.68	84	\$399.25;	\$411.38	85		
\$605.43;	\$620.53	86	\$987.50	87	\$2515.39	88	£73 3s. 9d.		
89	£709 12s 11.69d	90	\$12720	91	\$4192.95	92	\$19.99	93	
\$165.19	94	\$1058.65	95	\$77.83	96	\$5845.73	97	\$7200	
98	\$3600	99	\$2400	100	\$2684.91	101	\$6615.77	102	
\$1919.68	103	\$2765.82	104	\$3122.36	105	\$2250.84			
106	\$2746.34	107	\$6400	108	\$7200	109	\$3600		
110	\$2553.10	111	\$7039.07	112	\$1218.73	113			
\$370.80	114	\$845	115	\$866.67	116	\$3442.31	117		
\$955.71	118	\$4448.75	119	6%	120	7%	121	6%	
122	3½%	123	6%	124	4%	125	9%	126	6%
127	6%	128	9%	129	9%	130	1 yr. 6 mo.	131	
8 mo. 29 da.	132	1 yr. 1 mo. 1 da.	133	2 yr. 6 mo.					
134	2 yr. 6 mo. 25 da.	135	2 yr. 9 mo. 20 da.	136	3 yr. 7 mo. 6 da.	137	2 yr.	138	3 mo.
139	3 yr. 2 mo. 17 da.	140	8 mo. 24 da.; Apr. 25, 1897	141	\$108;	\$828			
142	\$26.22;	\$486.22	143	\$303.98;	\$1563.98	144	\$38.48;		
\$362.48	145	\$15;	\$177.75	146	2 yr. 3 mo.;	\$726.40	147		
2 yr. 4 mo. 15 da.;	\$1074	148	2 yr.;	\$2592	149	3 yr. 5 mo. 22 da.;	\$3445.60	150	2 yr. 6 mo. 2 da.;
\$1402.41	151	6%;	\$404.23	152	7%;	\$556.72	153	9%;	\$178.35
154	6%;	\$2587.74	155	8%;	\$4479.23	156	\$3600;	\$3938.40	
157	\$2795.03;	\$3245.03	158	\$3104.70;	\$3747.20	159			
\$4918.45;	\$5234.87	160	\$585.09;	\$710.69	161	\$4500;			
\$684	162	\$2403.76;	\$791.44	163	\$3689.47;	\$471.03	164		

\$327.46; \$32.54 165 \$2991.34; \$248.66 166 3%; \$483.20
 167 9%; \$152.52 168 6%; \$125.38 169 4½%; \$115.76
 170 6%; \$96.97 171 7 yr. 7 mo. 20 da.; \$660 172 4
 yr. 5 mo. 20 da.; \$300.50 173 5 yr. 3 mo. 11 da.; \$329 174
 3 yr. 1 mo. 28 da.; \$860.25 175 6 mo. 11 da.; \$293

EXACT INTEREST—Page 214.

1 \$68.76 2 \$9.50 3 \$47.97 4 \$9.70 5 \$47.34 6 \$11.52
 7 \$169.15 8 \$186.99 9 \$9.23 10 \$21.30 11 \$34.36
 12 \$347.77 13 \$2

COMPOUND INTEREST—Page 216.

1 \$1551.22 2 \$1901.56 3 \$4877.40 4 \$55627.50 5
 \$47398.38 6 \$1599.94 7 \$2500 8 \$2600 9 \$7707.96

ANNUAL INTEREST—Page 218.

1 \$9083.52 2 \$5511.15 3 \$12686.20 4 \$3884.82 5
 \$10045.31 6 \$10944 7 \$4872 8 \$3171.25 9 \$5758.62
 10 \$3528.84 11 \$1667.25 12 \$705.88 13 \$401.43

PARTIAL PAYMENTS, U. S. RULE—Page 220.

1 \$305.53 2 \$732.69 3 \$666.02 4 \$1823.62 5 \$6217.90
 6 \$583.20 7 \$1913.75 8 \$5162.50 9 \$1896.03; \$2067.28
 10 \$5000 11 \$460.25 12 \$1176.94 13 \$1815.07

MERCHANTS' RULE—Page 224.

1 \$1220.82; \$1341.63 2 \$749.38; \$771.99 3 \$136.66 4
 \$2929.48; \$3459.80 5 \$212.70

TRUE DISCOUNT—Page 226.

1 \$84; \$700 2 \$167.40; \$620 3 \$63.22; \$687.18 4 \$20.70;
 \$900 5 \$400.50; \$360 6 \$546.78; \$520 7 \$7460.40; \$7200
 8 \$1005.25; \$1000 9 \$1065.89; \$1026 21 10 \$784.80; 6% 11
 \$1027.80; 6% 12 \$699.52; 6% 13 \$3074; 8% 14 \$620; 4
 mo. 18 da. 15 \$625; 4 mo. 9 da. 16 \$850; 1 yr. 8 mo. 18 da.
 17 \$460; 2 mo 25 da. 18 3 mo. 23 da.; \$637.95 19
 3 mo. 22 da.; \$933.76 20 2 yr. 9 mo. 11 da.; \$445.64 21
 \$400 22 \$380 23 \$650 24 \$550 25 \$960 26 \$6, cash
 27 \$7 28 \$12.99, cash 29 \$231.02 30 \$255.07 31
 \$2127.57 32 B's 41¢ 33 \$495.38, loss 34 \$480 35
 \$166.51 36 \$40.20 37 \$86.47, gain 38 \$2498.76

BANK DISCOUNT—Page 229.

1 May 19, 1892; 93 da.	2 June 14, 1892; 95 da.	3 Dec. 3, 1892; 186 da.	4 Oct. 2, 1892; 63 da.	5 Mar. 3, 1897; 64 da.	6 Mar. 3, 1897; 123 da.	7 Mar. 3, 1897; 63 da.	8 Mar. 3, 1897; 62 da.	9 June 18, 1895; 63 da.	10 Aug. 6, 1895; 95 da.	11 Aug. 17, 1896; 93 da.	12 Dec. 17, 1896; 186 da.	13 Nov. 28, 1896; 116 da.	14 Nov. 6, 1896; 66 da.	15 Dec. 26, 1896; 72 da.	16 July 2, 1893; 190 da.	17 Mar. 3, 1896; 94 da.	18 Sept. 18, 1896; 187 da.	19 \$708.84	20 \$963.05	21 \$914.08	22 \$1250.08	23 \$751.35	24 \$2358.83	25 \$5570.66	26 \$407.15	27 \$849.62	28 \$914.70	29 \$833.98	30 \$1289.71	31 \$742.42	32 \$851.33	33 \$674.11	34 \$491.77	35 \$1238.18	36 \$5128.20	37 \$3325.20	38 \$2071.21	39 \$1575.33	40 \$739.43	41 \$967.72	42 \$565.05	43 \$826	44 \$346.30	45 \$727.70	46 \$6318.22	47 \$1289.69	48 \$2441.73	49 \$885.01	50 \$763.02	51 \$1265.68	52 \$626.61	53 \$1357.79	54 \$2389.21	55 \$1456.91	56 \$3152.20	57 \$3958.55	58 \$3409.90	59 \$3184.03	60 \$1226.30	61 \$3377.27	62 \$5000.82	63 \$2111.54	64 \$1397.82	65 \$1353.15	66 \$1853.21	67 \$1711.98	68 \$4363.66	69 \$2009.87	70 First, \$1.83	71 \$4930.71	72 \$5153.77	73 \$9920	74 \$24858.33	75 \$3853.82	76 \$586.87	77 \$464.75; 1289.75	78 Solved	79 \$29.94	80 \$740.23	81 \$1631.61	82 \$964.83	83 \$680.30	84 \$3001.39	85 \$1500.84	86 \$6003.59	87 \$875.59	88 \$903.15	89 \$975.11	90 \$4604.60
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STOCKS AND BONDS—Page 241.

1 \$150; \$1350	2 \$78.75; \$1421.25	3 $7\frac{1}{2}\%$	4 $4\frac{1}{4}\%$	5 160 half shares	6 148 quarter shares	7 125	8 64	9 8%	10 $1\frac{7}{8}\%$	11 \$350	12 \$228	13 \$8611.75	14 \$175	15 \$922.50	16 $177\frac{7}{8}$	17 72 shares	18 800 shares	19 $12\frac{1}{2}\%$	20 $9\frac{1}{11}\%$	21 $83\frac{5}{11}$	22 First, $\frac{4}{15}\%$	23 \$80080	24 Yes, \$20	25 Increased \$50	26 \$22161.88	27 700 shares.
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EXCHANGE—Page 245.

1 \$1602	2 \$997.50	3 \$951.12	4 \$1243.62	5 \$800	6
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\$240 7 \$800 8 \$600 9 \$3208 10 \$746.25 11 \$2002.50
 12 \$958.80 13 \$3999 14 \$2400.96 15 \$1196.40 16
 \$948.72 17 \$345.63 18 \$2392.80 19 \$1243.62 20
 \$2463.75 21 \$500 22 \$940 23 \$620 24 \$450 25
 \$650 26 \$5200 27 \$415.78 28 \$550 29 \$609.31 30
 \$540 31 \$620 32 \$320 33 \$2822.55 34 \$5076.72 35
 \$1173.66 36 \$21696.50

SAVINGS BANKS—Page 248.

1 \$1458.88; \$1463.83 2 \$3424.06; \$3433.83 3 \$1454.01;
 \$1458.97 4 \$3015.93; \$3023.10 5 \$5000.26, \$56 loss;
 \$5071.72, \$33.32 loss.

EQUATION OF ACCOUNTS—Page 250.

1 3 mo. 14 da. 2 Aug. 30, 1896 3 Oct. 16, 1896 4 Mar.
 24, 1896 5 Feb. 27, 1896 6 May 13, 1895 7 May 7, 1896
 8 Sept. 15, 1895 9 June 6 10 Oct. 13, 1896 11 Jan. 3,
 1897 12 July 21, 1896 13 June 23, 1896 14 July 23, 1896
 15 July 27, 1896 16 July 15, 1896 17 Oct. 18, 1896 18
 Dec. 8, 1894 19 Feb. 23, 1895 20 Mar. 21, 1895 21 May
 15, 1896 22 Aug. 4, 1895 23 Feb. 21, 1896 24 Oct. 23,
 1895 25 Aug. 25, 1895 26 June 28, 1896 27 Apr. 10,
 1896 28 \$917.48 29 Feb. 27; Apr. 27, 1896 30 Sept. 4,
 1896 31 Nov. 6, 1896 32 \$348.36 33 \$219.95 34
 \$349.11 35 \$950.77 36 \$988.70 37 \$812.44 38 Apr.
 9, 1896

PARTNERSHIP—Page 264.

1 \$2560 2 \$1100 3 \$5160; \$2580 each 4 \$10460 5
 \$12277.50 6 \$2890; \$7445; \$8945 7 \$5166.67; \$10333.33 8
 \$4900; \$5600 9 \$4230; \$5170; \$6110 10 \$4695.12; \$5121.95;
 \$7682.93 11 \$5028; \$6285; \$5447 12 \$6000; \$8000; \$12006
 13 \$17010; \$29700; \$22010; \$40700 14 \$4500; \$3900 15
 \$800; \$1200; \$1000 16 \$2177.48; \$1451.61; \$120.96 17
 G pays H \$2966 $\frac{1}{2}$ 18 \$13714.29; \$13885.71 19 \$7001.25;
 \$6998.75 20 \$120.42; \$157.08 21 P pays \$10.50 and R
 pays \$15 to Q 22 \$3685; \$3685; \$3235; \$15570 23 Re-
 ceive, A—\$4329.22; D—\$2963.91; Pay, B—\$3336.49; C—
 \$1179.87; E—\$2776.77 24 \$5626.44; \$6430.23.

100.

